ELECTRIC CHARGES AND FIELDS

Electrostatics
charge   rest

→ __Electrostatics__ deal with the study of forces, fields potentials arising due to static charges.

→ __Static electricity__ is the imbalance of charge on a surface of material. It is used to dynamic (move) electricity.

e.g (i) We see spark when take off synthetic clothes or sweater.

(ii) Experincing a shock while opening the door of a car.

→ __Electric charge__ is the physical properties of matter that causes electrostatic force.

- Two kinds of charges - positive and negative.
- Like charges repel and unlike charges attract.
- Charge is conserved
- Charge is quantised
- SI unit of charge - coulomb (c)
- It is scalar quantity.
- charged objects exert electric force on each other.
- Loss of electron - getting +ve charge
- Gain of electron - getting -ve charge
- If an object has an electric charge, it is said to be electrified.

→ __Gold-leaf electroscope__ is a simple apparatus to detect charge on a body. The degree of divergence of leaves is an indicator of the amount of charge.

→ <u>Method of charging -</u>
 (1) Friction (by rubbing) → Tribo electricity
 (11) By touching (electrons tranfer charged body
                     to neutral body)
 (111) Induction (Polarisation)

 Conductors and Insulators
        ↓   ↓
 electricity can pass   electricity cannot pass

 (e.g. metals, water   (e.g. glass, plastic, wood
 earth, human body etc.)   nylon, porcelain etc)
 App^n- grounding or earthing   App^n- insulation

→ <u>Basic Properties of Electric charge.-</u>

 ✳ Point charge - if size of charge bodies
 are very small as compared to the distance
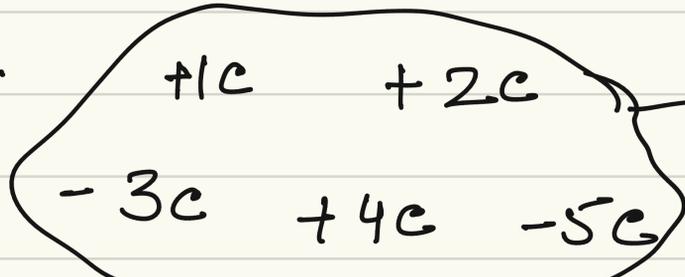 between them, the bodies are called
 point charges.

1. <u>Additivity of charges</u>
    If a system contains n charges $q_1, q_2 \cdots q_n$
 then the total charge of the system is
 given
              $$\boxed{q_{net} = q_1 + q_2 + q_3 + \cdots q_n}$$
                     = Algebraic sum of all the
                        individual charges

 e.g.   ⟨ +1c   +2c ⟩ → q_{net} = 1+2-3+4-5
          ⟨ -3c +4c -5c ⟩   = -1c

 Charge and mass both are scalar but mass
 is always +ve whereas charge can be -ve and +ve.

2. <u>Conservation of Charges</u>

charge of an isolated system is always conserved.

It is not possible to creat or destroy net charge carried by an isolated system.

* Charge carrying particles may be created or destroyed in a process.

e.g

neutron $<$ $\left.\begin{array}{l}\text{proton (+e)} \\ \text{electron (-e)}\end{array}\right]$ $q = 0$

$\downarrow$
$q = 0$

3. <u>Quantisation of Charge -</u>

The charge on a body is always given by

$$q = ne \qquad n \rightarrow \text{integer (+ve or -ve)}$$

Electric charge is always integral multiple of e is termed as quantisation of charge.

* The value of basic unit of charge is

$$e = 1.602192 \times 10^{-19} C$$
$$= 1.6 \times 10^{-19} C$$

Thus $-1C = 6 \times 10^{18}$ electrons $\qquad \left[n = \frac{q}{e} \right]$

* coulomb is very big unit of charge, so we use $1 \mu c = 10^{-6} c$ or $1 mc = 10^{-3} c$.

* At macroscopic level charged are enormous compared to e thus quantisation of charge can be ignored.

* Quarks have charges $\pm \frac{e}{3}$ or $\frac{2e}{3}$ but cannot in free state, hence not taken as elementry charge.

→ **Coulomb's law** -

This law states that the magnitude of electrostatic force between two point charges is directly proportional to the product of magnitude of the charges and inversely proportional to the square of the distance between them. i.e

$$F \propto \frac{|q_1 q_2|}{r^2}$$

$$\boxed{F = k \frac{q_1 q_2}{r^2}} \quad - ①$$

Where k is coulomb constant and $k = \frac{1}{4\pi \epsilon_0}$

In SI unit $k = 9 \times 10^9 \, Nm^2 c^{-2}$.

* If $1c$ of charge when placed at a distance $1m$ from another $1c$ of charge in vacuum experiences an electrical force of repulsion of magnitude $9 \times 10^9 \, N$.

* $1c$ is very big unit of charge to be used. In practice $1 mc \, (= 10^{-3} c)$ or $1 \mu c (= 10^{-6} c)$, smaller units are used.

In eqn(1) $k = \frac{1}{4\pi \epsilon_0}$, then coulomb's law is written as
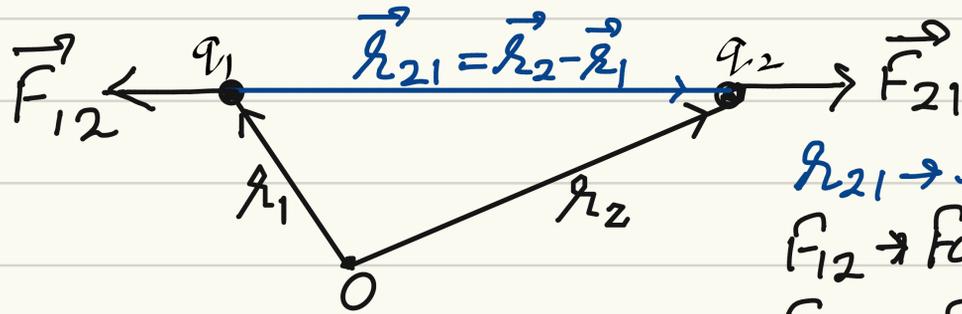
$$\boxed{F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}}$$

Here ϵ_0 is called permittivity of free space.

In SI unit $\epsilon_0 = 8.854 \times 10^{-12} \underset{\text{SI unit}}{N^{-1} m^{-2} c^2}$

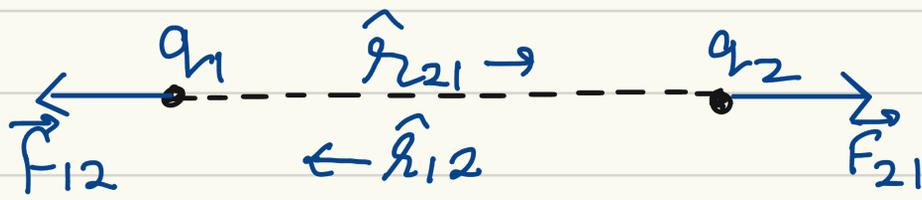*This value of ϵ_0 is taken for free space vacuume/air

Vector form of coulomb's law:

Let q_1 and q_2 are point charges with position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively.
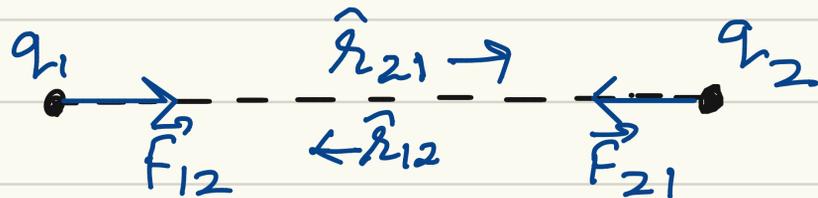

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

$r_{21} \rightarrow$ leading from 1 to 2
$F_{12} \rightarrow$ Force on q_1 due to q_2
$F_{21} \rightarrow$ Force on q_2 due to q_2

$q_1 q_2 > 0$
like charges

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$$

$q_1 q_2 < 0$
unlike charges

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}}$$

The coulomb force between q_1 and q_2 is given
as $\quad \vec{F}_{21} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_1 q_2}{r_{21}^2} \hat{r}_{21} \quad$ —①

and $\quad \vec{F}_{12} = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_1 q_2}{r_{12}^2} \hat{r}_{12} \quad$ —②

Eq^n's (1) and (2) are the vector form of coulomb's law.
Here $\quad \vec{r}_{12} = -\vec{r}_{21} \quad$ or $\quad \hat{r}_{12} = -\hat{r}_{21}$
then

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

Thus coulomb's law agrees with Newton's 3rd law.

* Equations (1) and (2) are valid for any sign of q_1 and q_2.

* Coulomb's law gives the forces between two charges q_1 and q_2 in vacuum.

<u>Permittivity (ε)</u> - A property of a material that measures the opposition it offers against an electric field.

It is an ability of a substance to store electrical energy in an electric field.

<u>Relative Permittivity ($ε_r$)</u>

By coulomb's law, force between charges q_1 and q_2 in free space

$$F = \frac{1}{4\pi ε_0} \frac{q_1 q_2}{r^2} \qquad -(1)$$

When charges q_1 and q_2 are placed in a medium of permittivity ε

$$F_m = \frac{1}{4\pi ε} \frac{q_1 q_2}{r^2} \qquad -(2) \quad F_m \rightarrow \text{force in medium}$$

divide (2) ÷ (1), we get

$$\boxed{\frac{F}{F_m} = \frac{ε}{ε_0} = ε_r}$$

here $ε_r$ is called relative permittivity.

It is defined as the ratio of permittivity of the medium to the permittivity of vacuum.
* It has no unit.
* $ε_r = K$, <u>K is dielectric constant</u>

<u>Dielectric constant</u> of a substance is defined as the ratio of substance's permittivity to the permittivity of free space.

$$\boxed{ε_r = \frac{ε}{ε_0} = K}$$

or $\quad K = ε_0 ε_r$
* K has no unit.

$ε_r$ and K has no unit

Forces between multiple charges

Principle of Superposition - force on any charge due to a number of other charges is the vector sum of all the forces on the charge.

The individual forces are unaffected due to the presence of other charges.

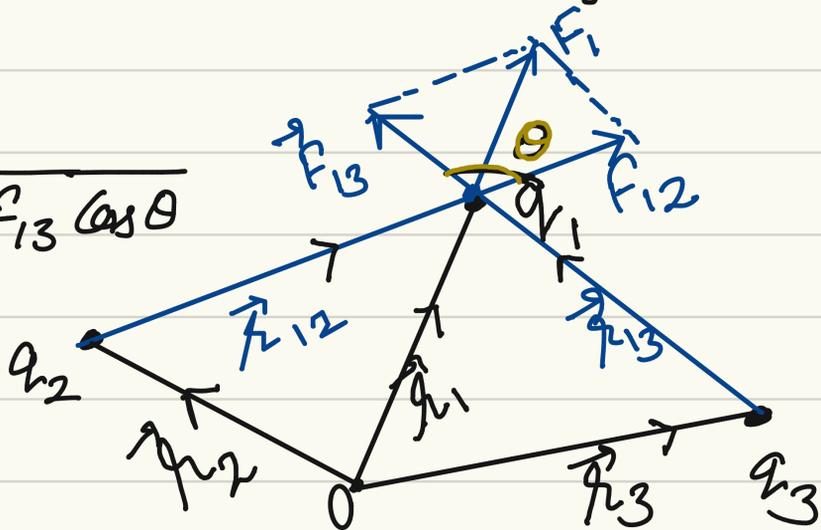e.g. for a system of three charges q_1, q_2 and q_3
force on q_1 due to q_2 and q_3

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}}$$
$$|\vec{F_1}| = \sqrt{F_{12}^2 + f_{13}^2 + 2 f_{12} f_{13} \cos\theta}$$

here,
$$F_{12} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{R_{12}^2} \hat{R}_{12}$$

$$F_{13} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_3}{R_{13}^2} \hat{R}_{13}$$

for four system of charges q_1, q_2, q_3 and q_4
net force on q_1 due to q_1, q_2, q_3 and q_4

$$\vec{F_{123}} = \vec{F_{12}} + \vec{F_{13}}$$
net force on q_1
$$\vec{F_1} = \vec{F_{123}} + \vec{F_{14}}$$
$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}}$$
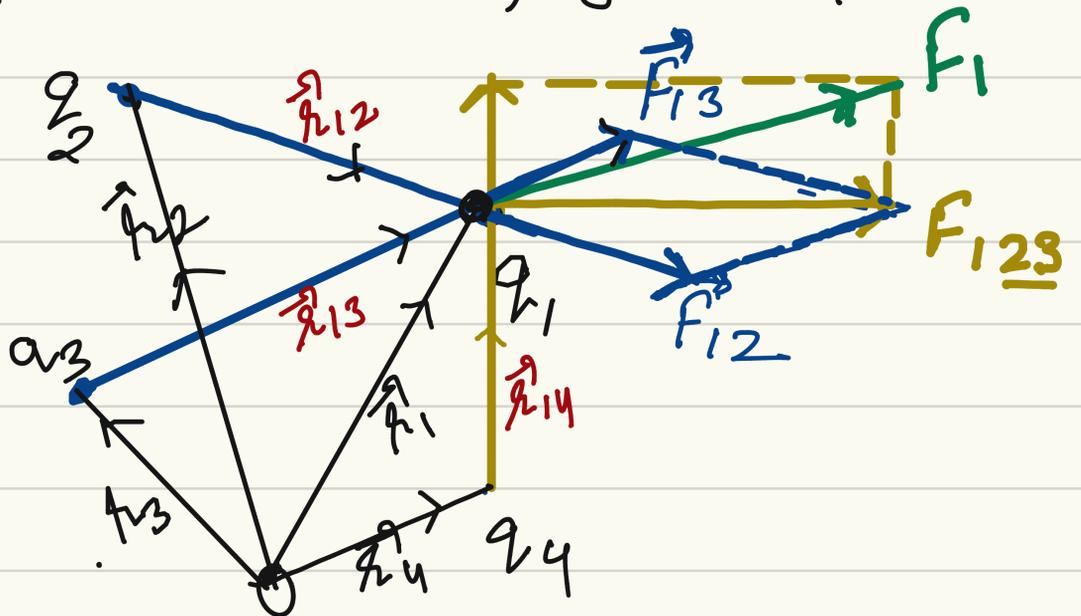
for n system of charges, net force on q_1
$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \cdots \vec{F_{1n}}$$

$$f_1 = \frac{1}{4\pi \epsilon_0} \left[\frac{q_1 q_2}{R_{12}^2} \hat{R}_{12} + \frac{q_1 q_3}{R_{13}^2} \hat{R}_{13} \cdots \frac{q_1 q_n}{R_{1n}^2} \hat{R}_{1n} \right]$$

* Vector sum is obtained by 11^{gm} law.

<u>Electric field</u> - Electric field due to a charge Q at a point in space may be defined as the force experienced by unit charge at that point.

$$\vec{E} = \frac{\vec{F}}{q}$$

$$= \frac{\frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{r}}{q}$$

$$\left[\because F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\right]$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$*$ Direction of $\vec{E}$ is $+ve$ charge to $-ve$ charge.

$*$Charge 'Q' which produces electric field is called <u>source charge</u> and the charge 'q' which tests the electric field is called <u>test charge</u>.

$*$ Test charge means a negligibly small charge which does not apply any force on source charge but experiences a force due to the source charge.

$$E = \lim_{q \to 0}\frac{F}{q}$$

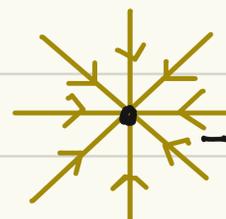$*$ Electric field E is independent of q. It depends on source charge Q and r.

$*$ The field exists at every point in three-$\dim^n$ space.

$*$ For $+ve$ charge E will be directed radially outwards from the charge and for $-ve$ charge it is radially inwards.
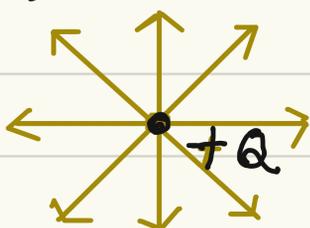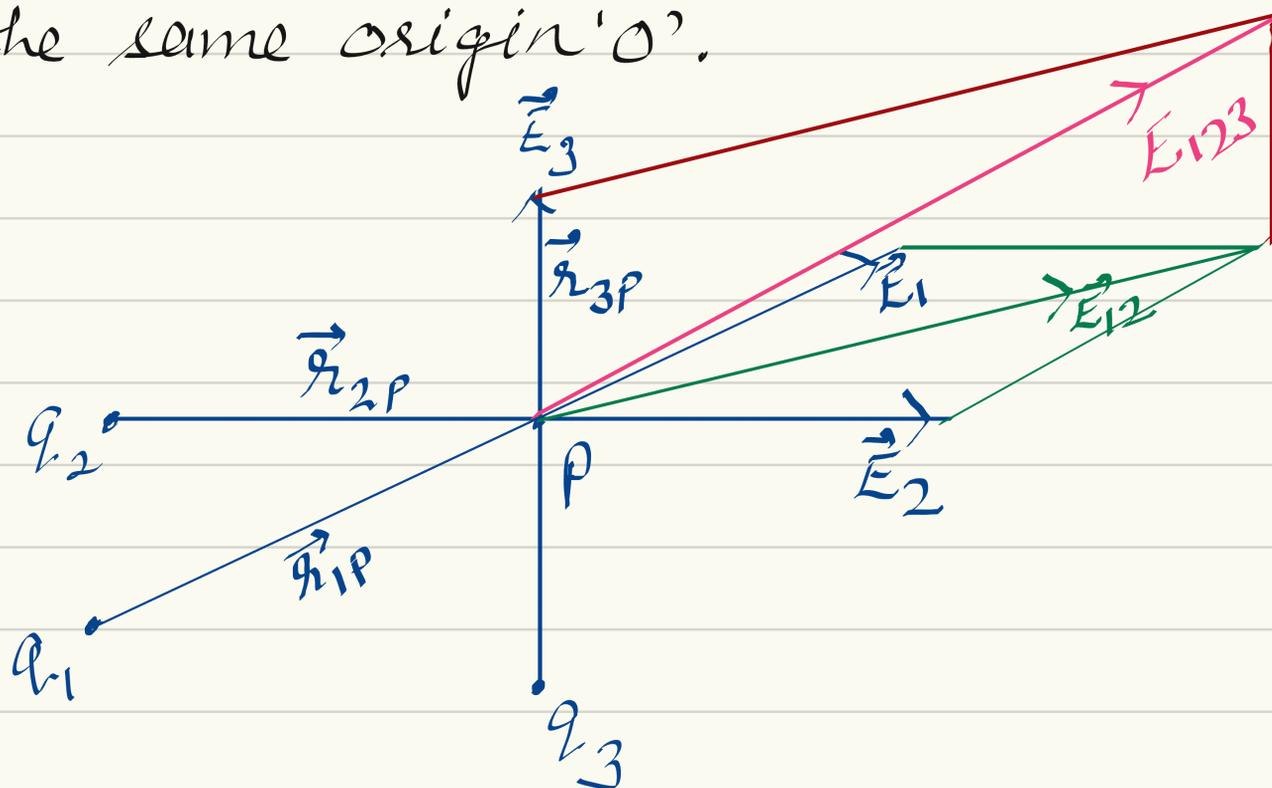
$\vec{E}$ radially outwards $+Q$

$\vec{E}$ radially inwards $-Q$

* Like force, electric field E also depends on the distance r. Thus at equal distance from Q, the magnitude of $\vec{E}$ is also same. E has spherical symmetry.

Electric field due to a system of charges

consider a system of charges $q_1, q_2 \cdots q_n$ with position vector $r_1, r_2 \cdots r_n$ relative to the same origin 'O'.

By the principle of superposition the electric field $\vec{E}$ at $\vec{r}$ due to the system of charges is

$$\vec{E} = \vec{E_1} + \vec{E_2} + \cdots \vec{E_n}$$

$$= k \frac{q_1}{r_{1P}^2} \hat{r}_{1P} + k \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \cdots k \frac{q_n}{r_{nP}^2} \hat{r}_{nP}$$

or $\boxed{E = k \sum_{i=1}^{n} \frac{q_i}{r_{iP}^2} \hat{r}_{iP}}$

Physical Signification of Electric field

1. <u>Under Static condition</u>
- Electric field is an elegant way of characterising electrical environment of a system of charges.
- Electric field at a point in the space around a system of charges tell the force on unit positive test charge placed at that point.
- Electric field is independent of test charge.
  The term field refers to a quantity that is defined at every point in space and may vary from point to point.
- The true physical significance of the concept of electric field deals with time dependent electromagnetic phenomena.

2. <u>Electromagnetic Non-Static Condition</u>

- Even though electric and magnetic field can be detected only by their effects (forces) on charges, they are regarded as physical entites, not merely methematical construct.
- Concept of field was first introduced by Faraday and is now among the central concept of physics.

<u>Electric Field lines</u>: These are the imaginary lines which represent electric field pictorially and give the direction of electric field at any point.
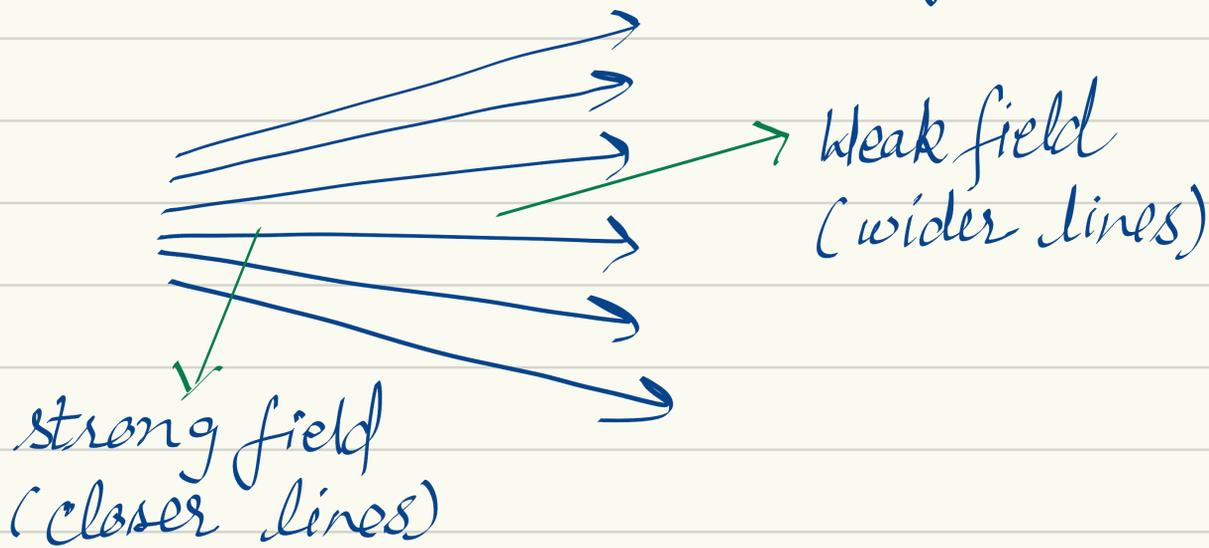
<u>Properties of Electric field lines</u>

1. Start from +ve charge, end at -ve charge.
2. Do not form closed loop because starting and ending point are not same.

3. Two field lines from one source never intersect each other because if they do so there will be two directions of electric field at one point. Which is not possible.

tangent of E_1

E_1

tangent of E_2

E_2

O → point of intersection
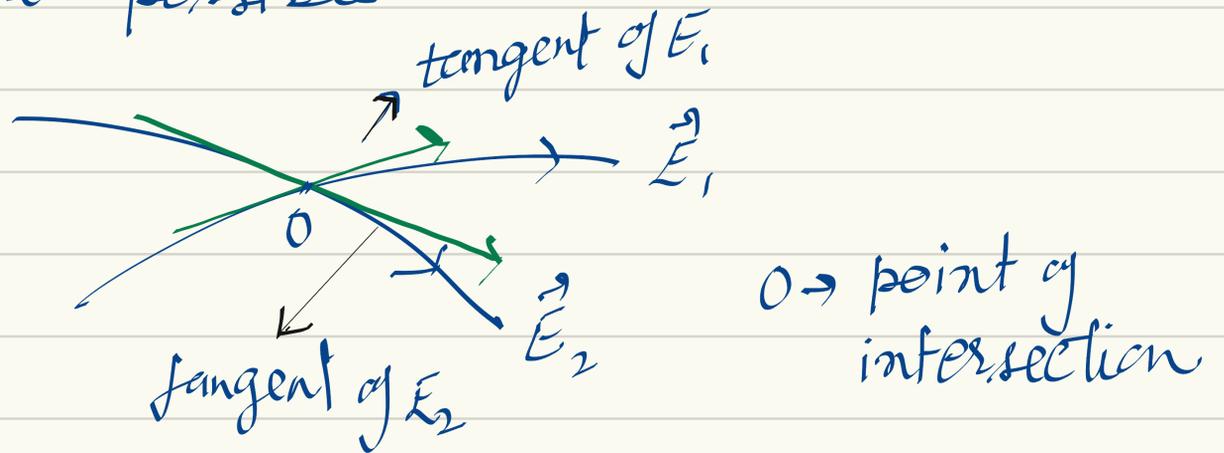
4. Closer the lines, stronger the field and wider the lines weaker the field.

Weak field (wider lines)

strong field (closer lines)

5. Always perpendicular to the surface of charge.

$\vec{E}$
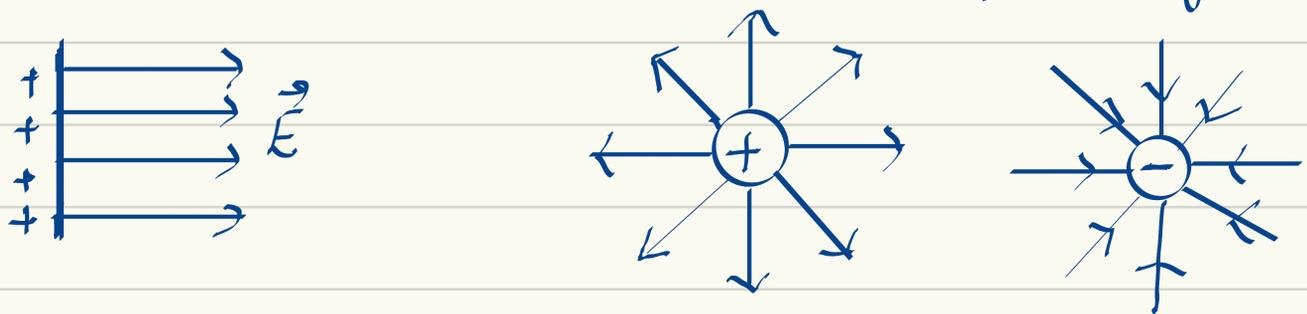
6. Do not pass through a conductor, as electric field inside a conductor is zero.

7. Pass through a dielectric. (Electric field polarises the molecules or atoms in a material.)
   [* Dielectrics are insulators.
   * Separation of centre of +ve and −ve charge is called polarisation.]

<u>Dependence of Electric Field Strength On The Distance And Its Relation To The Number Of Field Lines</u> (<u>Inverse Square Law</u>)

Field lines carry information about the direction of electric field at different points in space.
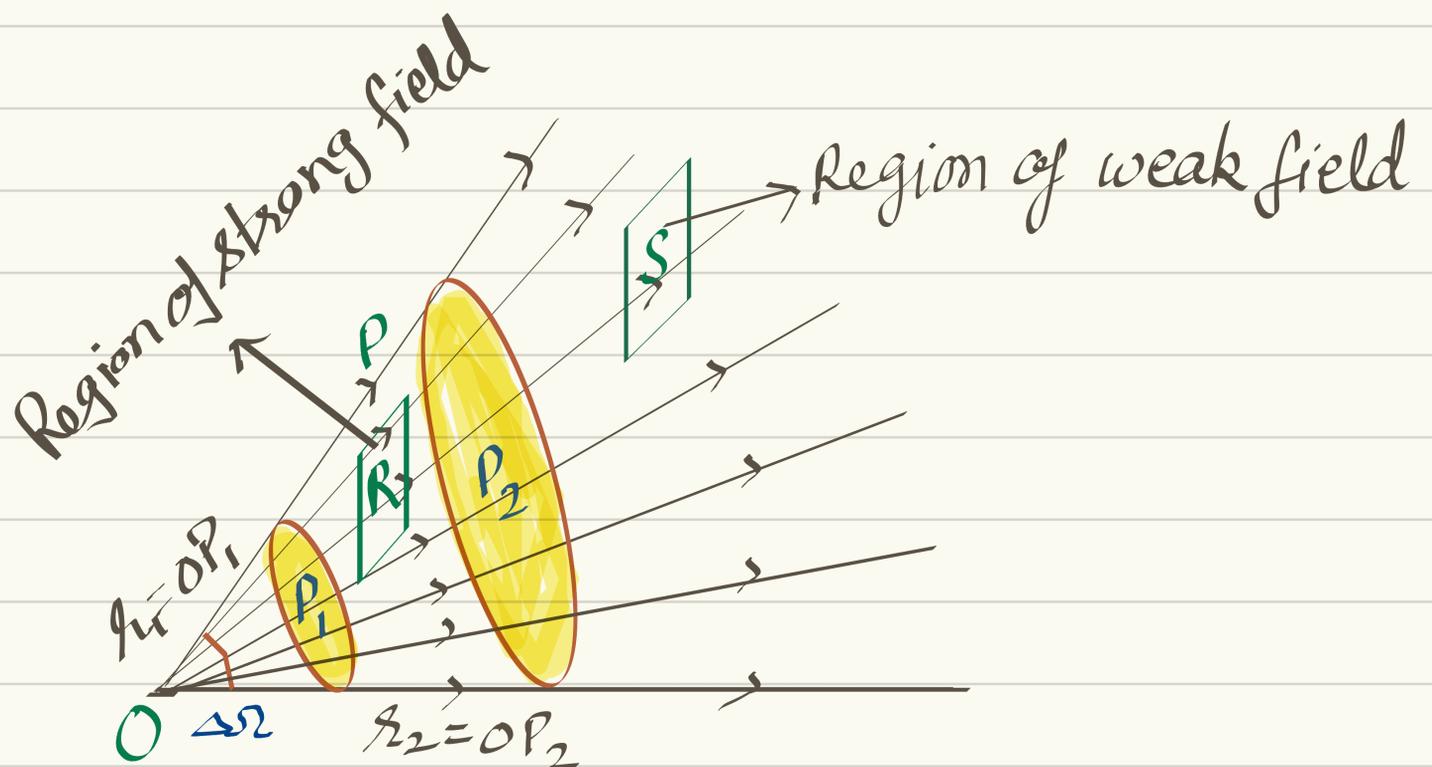
In fig.
consider a point charge (+ve) at 'O'

$R_1 = OP_1$

$R_2 = OP_2$

Area of R element = Area of S element

$\Delta \Omega$ = solid angle

From fig it is clear that equal number of field lines are passing through P_1 and P_2 surfaces.

Now by   Area = R^2 × solid angle

Area of surface P_1

$$A_1 = R_1^2 \Delta \Omega$$

Area of surface P_2

$$A_2 = R_2^2 \Delta \Omega$$

Let n no. of lines are passing through each area P_1 and P_2.

So,
number of field lines cutting unit area element at P_1

$$E_1 = \frac{n}{R_1^2 \Delta \Omega}$$

also at P_2,

$$E_2 = \frac{n}{R_2^2 \Delta \Omega}$$

Now $\dfrac{E_1}{E_2} = \dfrac{\text{Strength of field at } P_1}{\text{strength of field at } P_2} = \dfrac{n/R_1^2 \Delta \Omega}{n/R_2^2 \Delta \Omega}$

or $\dfrac{E_1}{E_2} = \dfrac{R_2^2}{R_1^2}$
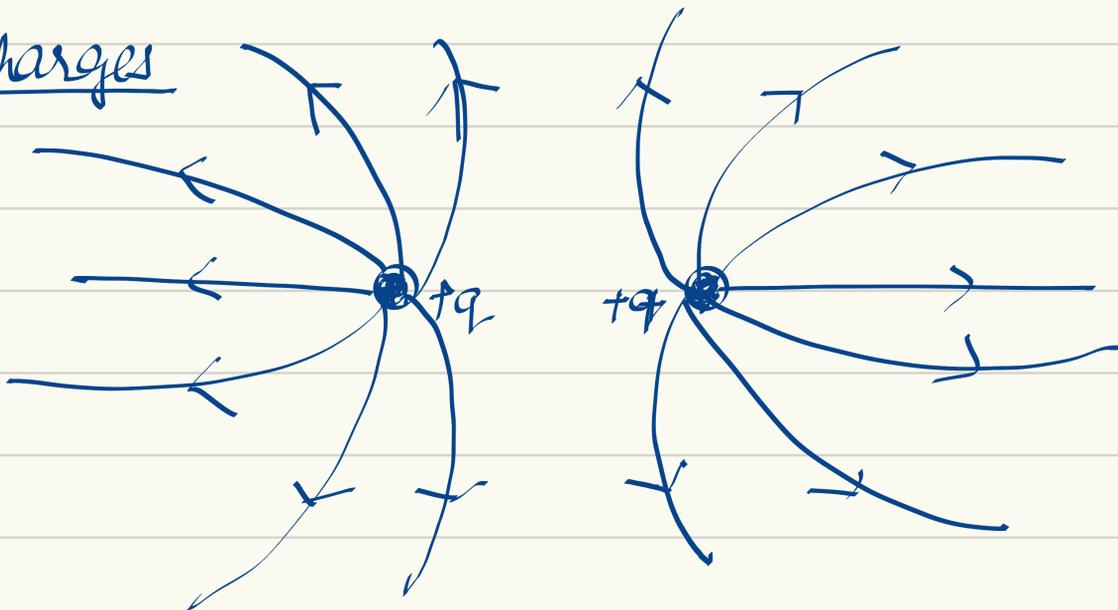
or $$\boxed{E \propto \frac{1}{R^2}}$$

This is the inverse square dependency of electric field.

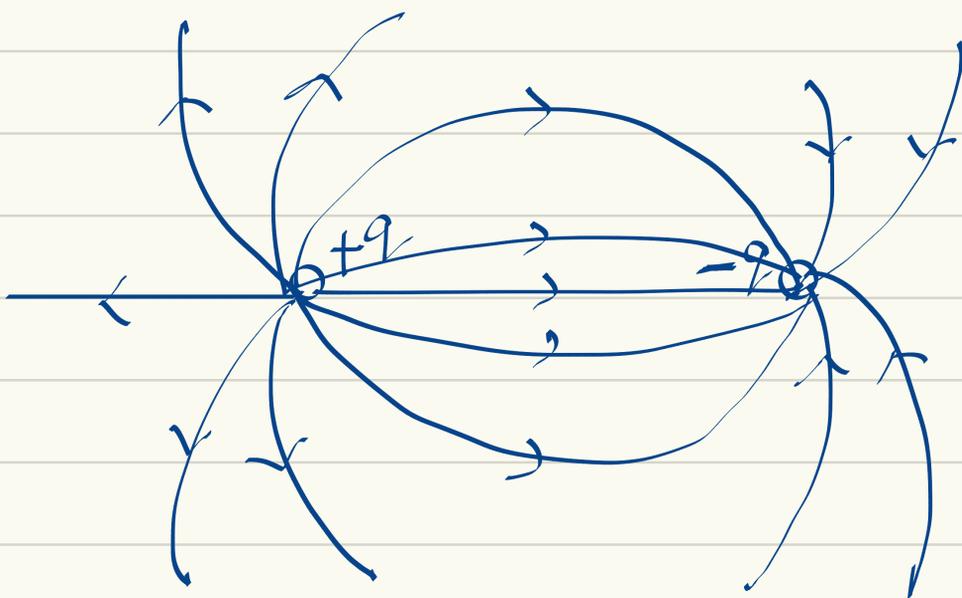* The picture of field lines was invented by Faraday.

* Electric field lines are also known as lines of force.

Field lines due to some simple charge configuration

(1) Like charges

(11) Unlike charge   (Electric dipole)

Electric Flux (ϕ): The rate of flow of electric field through an area is called electric flux.

No. of electric field lines intersecting a given area

Electric flux $\Delta\phi$ through an area ΔS is given as

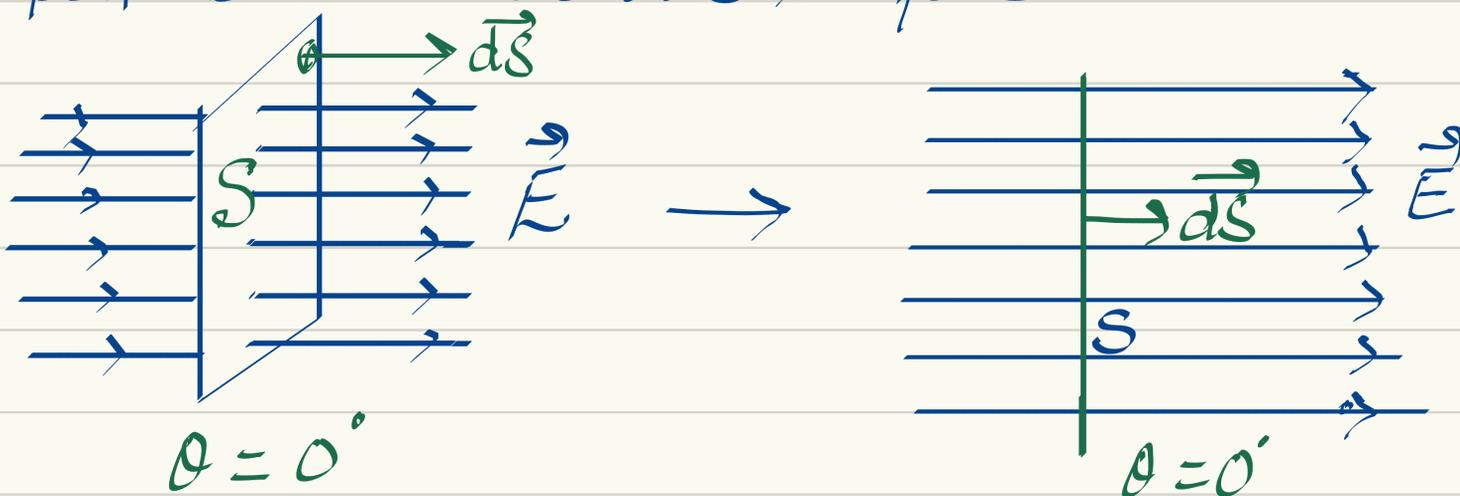$$\Delta\phi = \vec{E}\cdot\vec{\Delta S} = E\Delta S \cos\theta \qquad \left[\vec{\Delta S} = \Delta S\cdot\hat{n}\right]$$

or $$d\phi = (E\cos\theta)dS = E dS\cos\theta = \vec{E}\cdot\vec{dS}$$

$$\boxed{\phi = \int d\phi = \int\vec{E}\cdot\vec{dS} = \int E dS\cos\theta}$$

Here θ is the angle between $\vec{E}$ and $\vec{dS}$.

* Area vector $(\vec{dS})$ – A vector whose magnitude is equal to the area of a surface and whose direction is perpendicular to the surface.
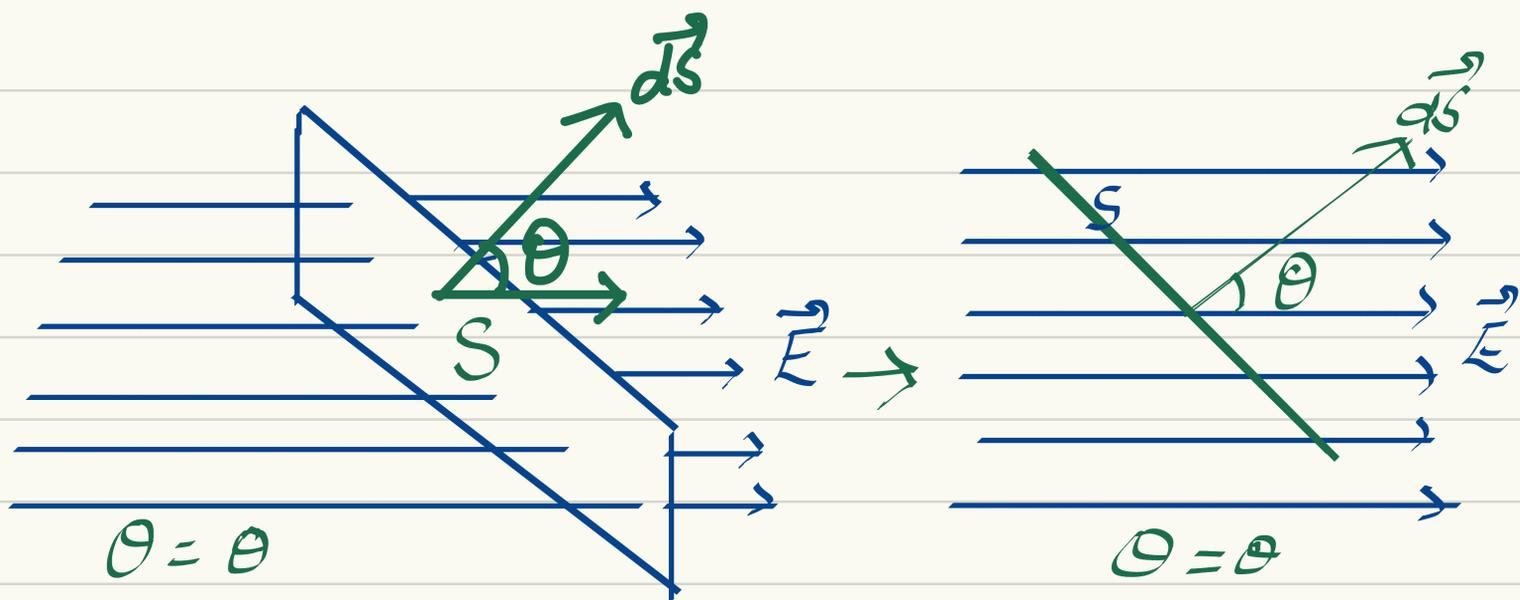
(1)

$\theta = 0°$ $\theta = 0°$

When $\theta = 0°$

$$\phi = \int E dS\cos 0° = \int E dS = ES$$

or $$\boxed{\phi_m = \vec{E}S} \qquad \phi_{max} \rightarrow max^m \text{ flux}$$

(II)
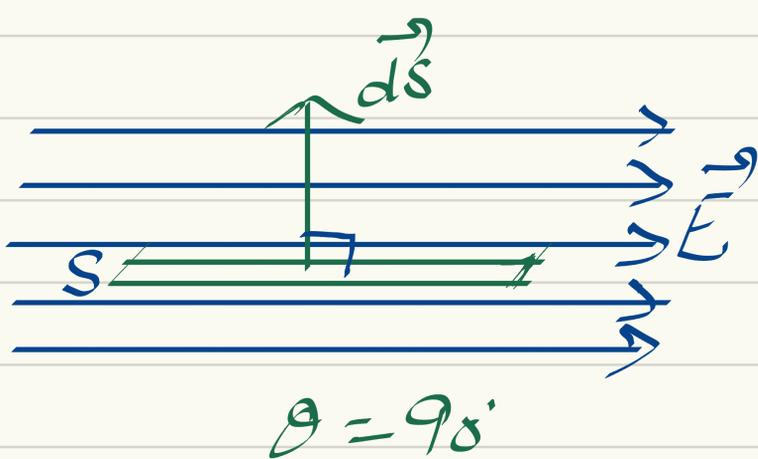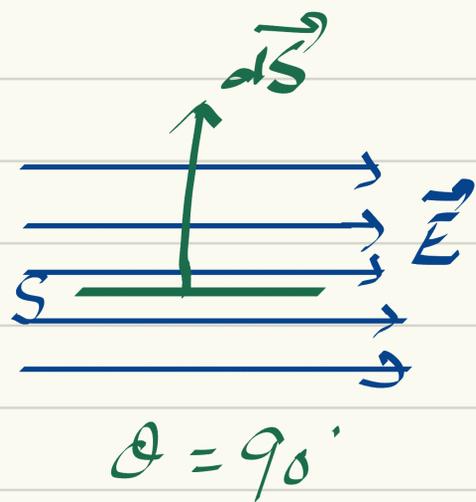

$\theta = 0$ $\theta = 0$

$$\phi = \int E \, ds \cos\theta$$

$$\boxed{\phi = E S \cos\theta}$$

(iii)

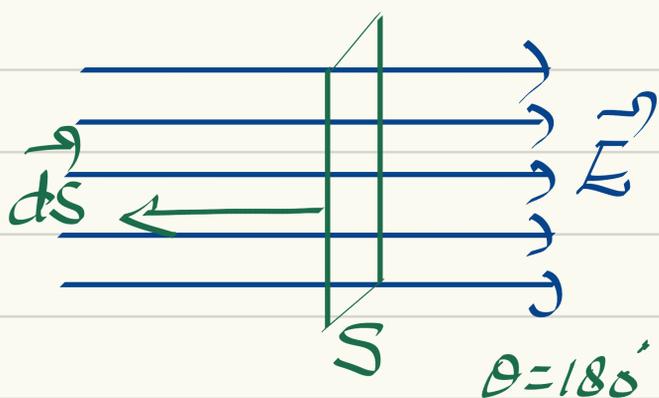$\theta = 90°$ $\theta = 90°$

$$\phi = \int E \, ds \cos 90°$$

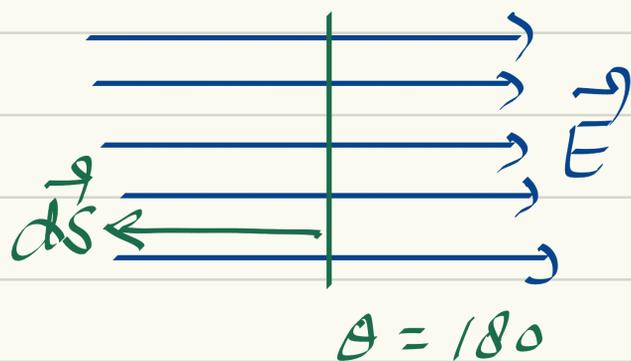$$\phi = 0 \qquad [\text{no flux}] \qquad [\cos 90° = 0]$$

No field lines passes through the surface S.

(iv)

$\theta = 180°$ $\theta = 180°$
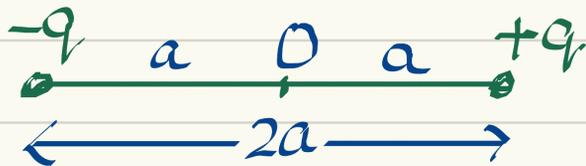
$$\phi = \int E \, ds \cos 180°$$

$$\phi = - E S \quad [-ve \ flux] \quad [\cos 180° = -1]$$

* SI units - Volt-meters (V-m)
        - N m² c⁻¹
* Flux is a scalar quantity.

* For closed surface the direction of area vector is taken outwards $\perp$ to the surface by convention.

Electric Dipole : It is a pair of equal and opposite point charges q and -q separated by a distance 2a. e.g. H_2O, HCl etc.

$$-q \quad a \quad O \quad a \quad +q$$
$$\longleftrightarrow \quad 2a \quad \longrightarrow$$

Electric Dipole moment (p) It is equal to the product of magnitude of either charge and length of dipole.
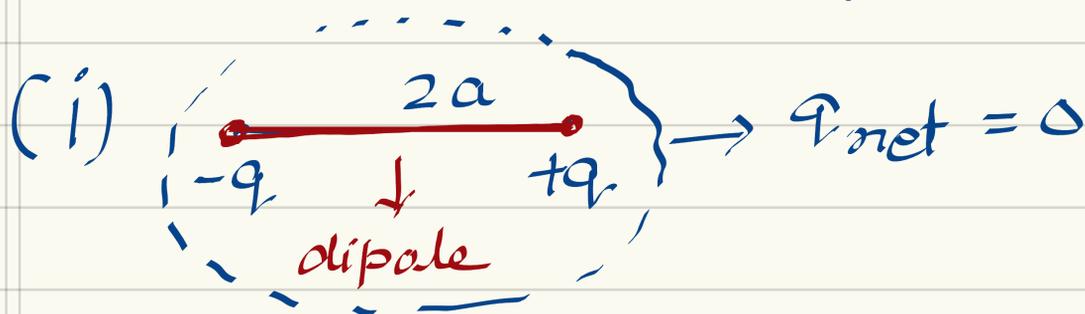
$$p = |q| \times 2a$$
$$\boxed{p = 2qa}$$
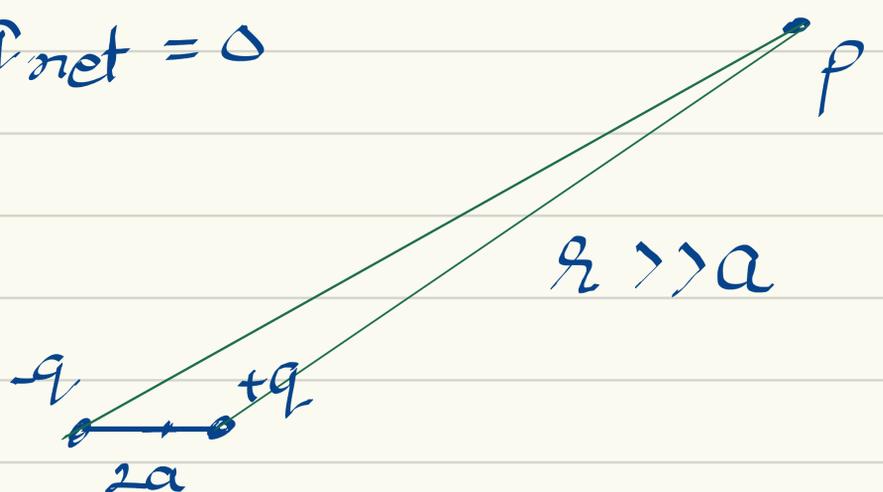
* Dipole moment is a vector quantity.
* Direction of dipole moment is -ve to the +ve by convention.
* The net charge on an electric dipole is zero. This does not mean that the field of dipole is zero.
* Electric field near the dipole is not zero as the two charges are separated by some distance. but for $r \gg 2a$ the fields due to +q and -q nearly cancel out.

(i)
$$2a$$
$$-q \quad \downarrow \quad +q \longrightarrow q_{net} = 0$$
$$dipole$$

(11) $\quad r \gg 2a$
$$E_p \simeq 0$$

$$-q \quad +q$$
$$\longleftrightarrow$$
$$2a$$
$$r \gg a$$
$$P$$

The Field At An Electric Dipole :

Electric field due to a dipole at any point can be found out from coulomb's law and principle of superposition.

The two cases are -

(1) ## Electric Field due to a dipole, at any point P on the Axial line
(FOR the point on the axis)

Consider an electric dipole AB of length 2a. At a point 'P' on the axis of dipole electric field is to be found.
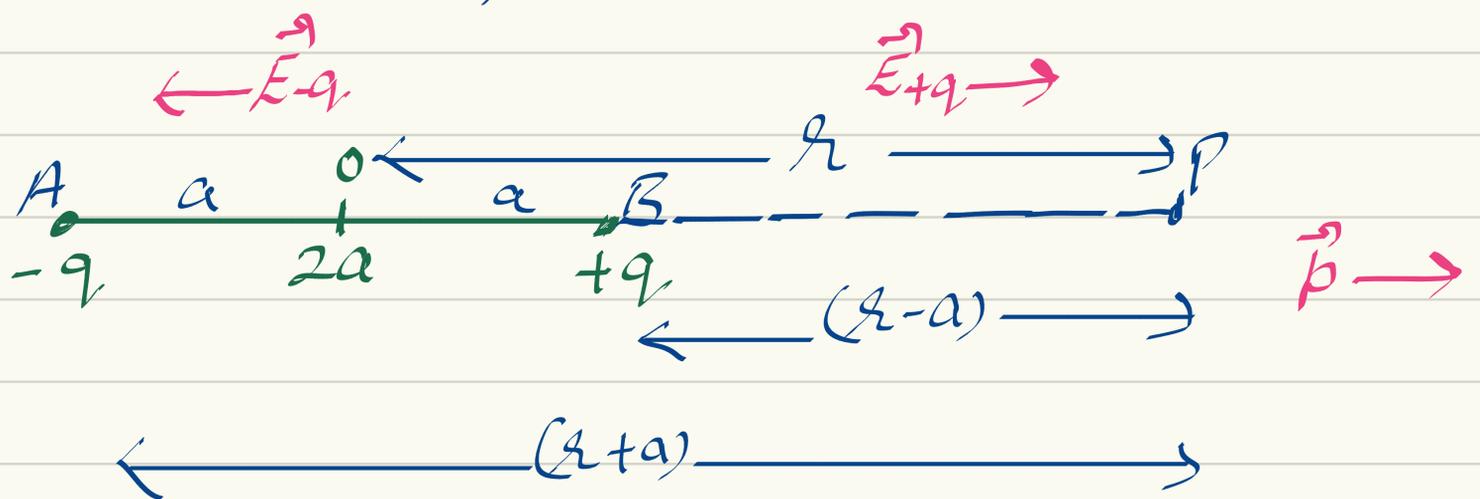
In fig — $OP = R$, $AB = 2a$
$$AP = (R+a)$$
$$BP = (R-a)$$

Field at 'P' due to $-q$ charge

$$\vec{E}_{-q} = -k\frac{q}{AP^2}\,\hat{p}$$

[$-ve$ sign shows the opposite dir^n of $\vec{E}_{-q}$ and $\vec{p}$]

So $\vec{E}_{-q} = -k\frac{q}{(R+a)^2}\,\hat{p}$ —— (1)

and $\vec{E}_{+q} = k \dfrac{q}{BP^2} \hat{p}$ [$\vec{E}$ & $\vec{p}$ have same direction]

$$E_{+q} = k \dfrac{q}{(8-a)^2} \hat{p} \quad -(2)$$

Now the net field on 'P'

$$\vec{E}_{axial} = \vec{E}_{net} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \left[k \dfrac{q}{(8-a)^2} - k \dfrac{q}{(8+a)^2} \right] \hat{p}$$

$$= kq \left[\dfrac{1}{(8-a)^2} - \dfrac{1}{(8+a)^2} \right] \hat{p}$$

$$= kq \left[\dfrac{(8+a)^2 - (8-a)^2}{(8-a)^2(8+a)^2} \right] \hat{p}$$

$$= kq \left[\dfrac{8^2 + a^2 - 2\,8a - 8^2 - a^2 + 2\,8a}{(8^2 - a^2)^2} \right] \hat{p}$$

$$= kq \left[\dfrac{4\,8a}{(8^2 - a^2)^2} \right] \hat{p}$$

$$\boxed{\vec{E}_{ax} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2\vec{p}\,8}{(8^2 - a^2)^2}}$$

For $8 \gg a$

$$\vec{E}_{ax} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2p\,8}{(8^2)^2}$$

$$\boxed{\vec{E}_{ax} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{2\vec{p}}{8^3}} \qquad * \quad [\theta \ b/w \ \vec{E} \ \& \ \vec{p} = 0°]$$

i.e $\quad E \propto \dfrac{1}{R^3} \quad$ [for dipole]

and $\quad E \propto \dfrac{1}{R^2} \quad$ [for point charge]

~~&~~ Direction of $\vec{E}_{axial}$ is same as of $\vec{P}$.
i.e. angle between $\vec{E}_{axial}$ and $\vec{P}$ is zero.

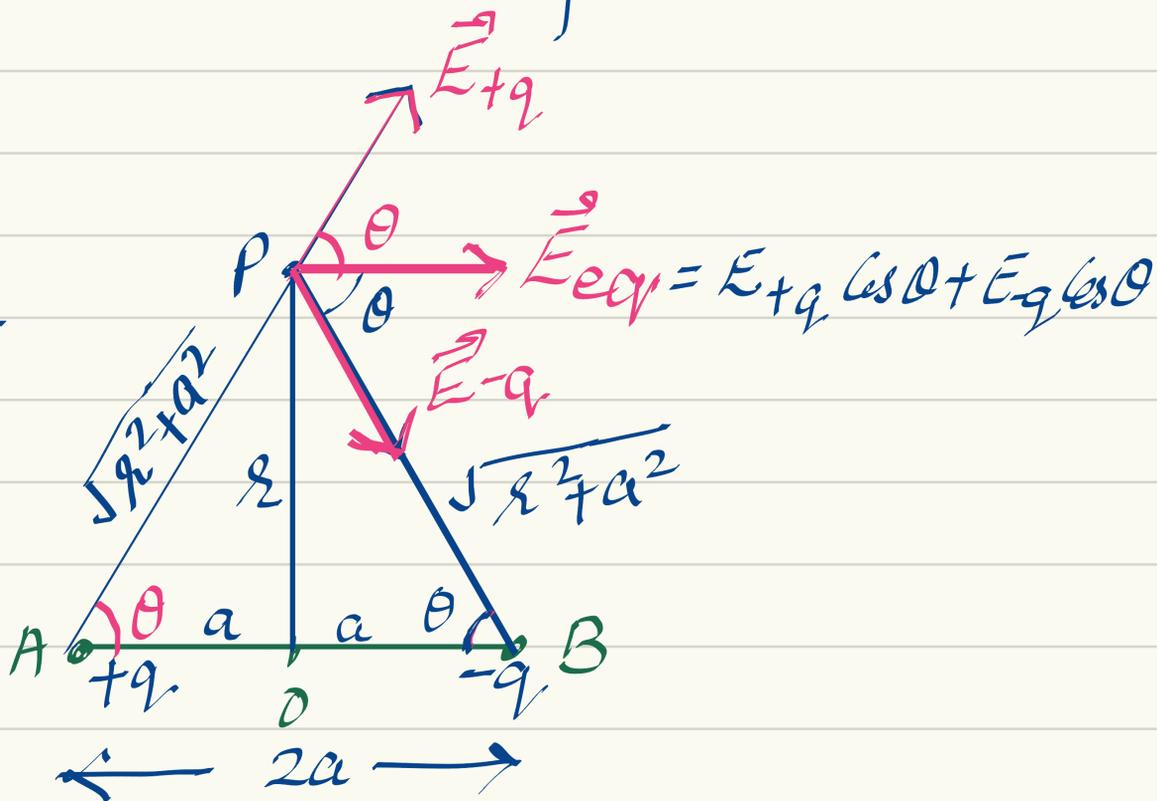(II) <u>Electric field due to a dipole on the Equatorial Plane</u>

Let 'P' is the point on the equatorial plane of a dipole where electric field is to be determined.

In fig

$AP = PB = \sqrt{R^2 + a^2}$

$OP = R$

$\cos\theta = \dfrac{a}{\sqrt{R^2 + a^2}}$

Magnitudes of the electric fields due to the charges $+q$ and $-q$ are equal.

$$E_{+q} = E_{-q} = \dfrac{k\,q}{(\sqrt{R^2 + a^2})^2}$$

$$= k\dfrac{q}{(R^2 + a^2)}$$

clearly the components of E_{+q} and E_{-q}, normal to the dipole cancel away.

The components along the dipole axis
add up.

So $\quad E_{eq} = E_{net} = E_{+q}\cos\theta + E_{-q}\cos\theta$

$$= 2k\frac{q}{(r^2+a^2)}\cos\theta \quad [\because E_{+q}=E_{-q}]$$

$$= \frac{2kq}{(r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}} \quad \left[\because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}\right]$$

$$E_{eq} = k\frac{p}{(r^2+a^2)^{3/2}}$$

In vector form

$$\vec{E_{eq}} = -\frac{k\vec{p}}{(r^2+a^2)^{3/2}}$$

−ve sign shows the opposite direction of
$\vec{E_{eq}}$ and $\vec{p}$.

For $r \gg a$,

$$\vec{E_{eq}} = -\frac{k\vec{p}}{r^3}$$

or $\quad \boxed{\vec{E_{eq}} = -\frac{1}{4\pi\epsilon_0}\cdot\frac{\vec{p}}{r^3}}$

* Angle b/w $\vec{E_{equatorial}}$ and $\vec{p}$ is 180°.

* $\vec{E_{ax}}$ and $\vec{E_{eq}}$ depend on product of
q and a ie on qa but independent
of q and a separately.

* $\boxed{E_{axial} = 2\times E_{equatorial}}$ $\quad \boxed{\vec{E_{ax}} = -2\vec{E_{eq}}}$

* Magnitude and direction of the dipole field depends on distance r and angle b/w $\vec{r}$ and $\vec{p}$.

* <u>Point dipole</u> : An ideal dipole in which size $2a \to 0$ and charge $q \to \infty$ in such a way that dipole moment $p = q \times 2a$ has a finite value. Such a dipole is called point dipole.
  (A dipole of negligibly small size is called point dipole or an ideal dipole.)
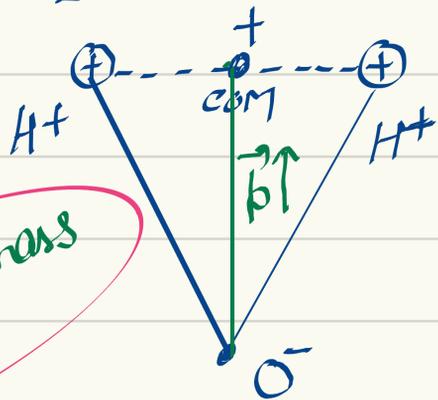
<u>Physical Significance of dipoles</u> :
Polar and Non polar molecules —

Polar Molecules $\downarrow$	Non polar molecules $\downarrow$
COM of +ve and −ve charge does not concide. e.g. H_2O	COM of +ve and −ve charge concides. e.g. CO_2, CH_4, H_2, N_2, O_2 etc.

COM → centre of mass

* Polar molecules have permanent dipole moment.

* Under influence of uniform electric field it becomes polar molecules.

$$\vec{P} = 0$$

$\overrightarrow{-P+}$

$\underrightarrow{-E}$

Various materials show interesting properties and important application in the present or absence of electric field.
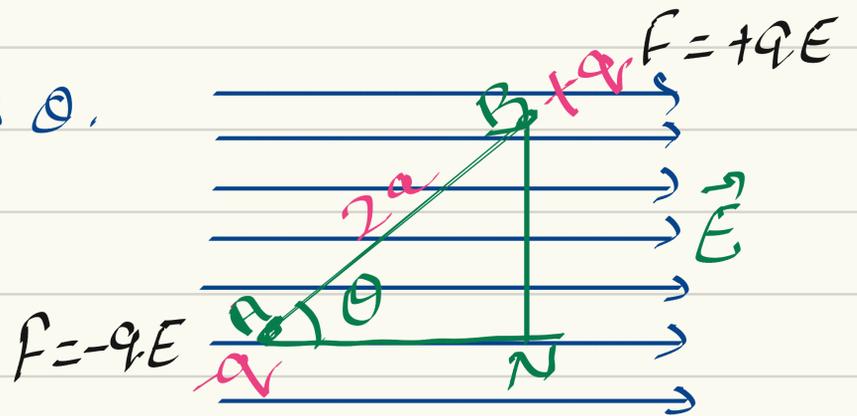
Dipole in Uniform External Field; Torque on Dipole

Consider a dipole AB placed in uniform electric field as shown in fig.

Let angle b/w $\vec{p}$ and $\vec{E}$ is θ.

here

Net force on the dipole

$F_{net} = qE - qE$

$= 0$

$F = +qE$

$F = -qE$

But a couple of force acting on dipole develops a torque -

$$\tau = |\vec{F}| \times \perp \text{ distance } (BN)$$

$$\tau = qE \times 2a \sin\theta \qquad \left[\because \sin\theta = \frac{BN}{2a}\right]$$

$$= (2aq) E \sin\theta$$

$$\boxed{\tau = pE\sin\theta}$$

OR $\qquad \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

The direct. of torque $\vec{\tau}$ can be given by right hand thumb rule.

Special cases

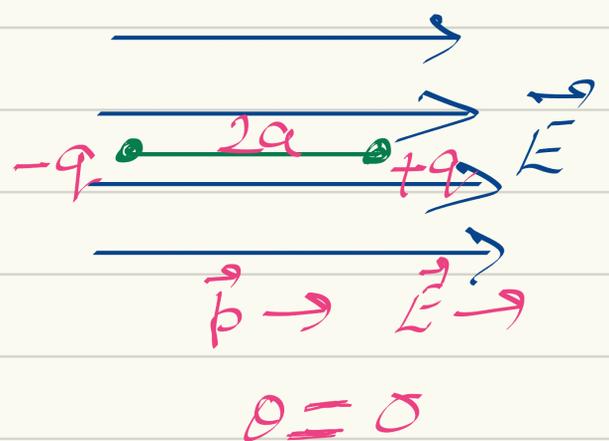(1) If $\theta = 0°$

by $\tau = pE\sin\theta$

$$\boxed{\tau = 0}$$

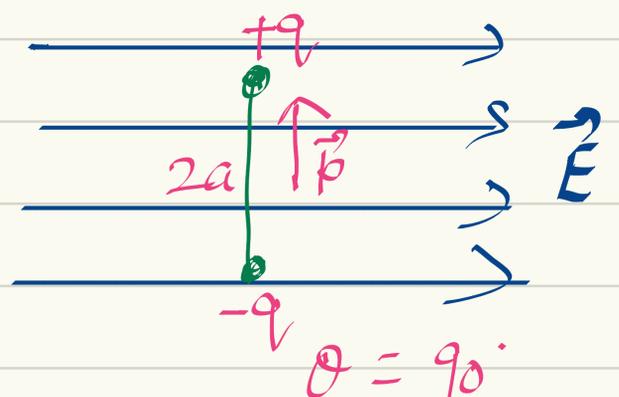No torque is acting when $\vec{p}$ and $\vec{E}$ are parallel.
(∗ Stable equilibrium)

$-q \quad 2a \quad +q$

$\vec{p} \rightarrow \quad \vec{E} \rightarrow$

$\theta = 0$

(11) If $\theta = 90°$

$\tau = pE\sin 90°$

OR $\boxed{\tau = pE}$ [Max. torque]

τ_{max}

$+q$

$2a \quad \vec{p}$

$-q$

$\vec{E}$

$\theta = 90°$

(III) If $\theta = 180°$

$\tau = pE \sin 180$

or $\boxed{\tau = 0}$
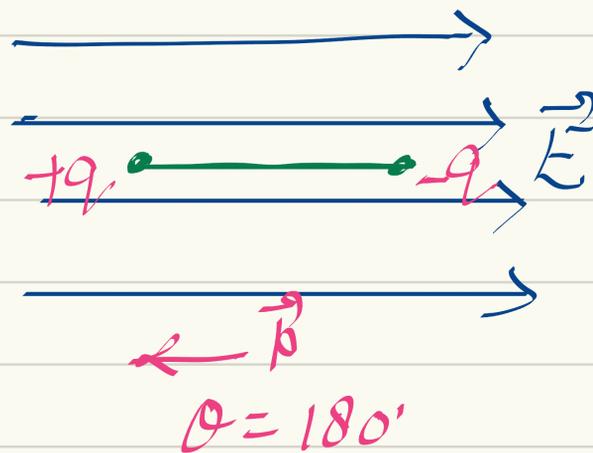
No torque is acting when $\vec{P}$ and $\vec{E}$ are antiparallel.
(Unstable equilibrium)

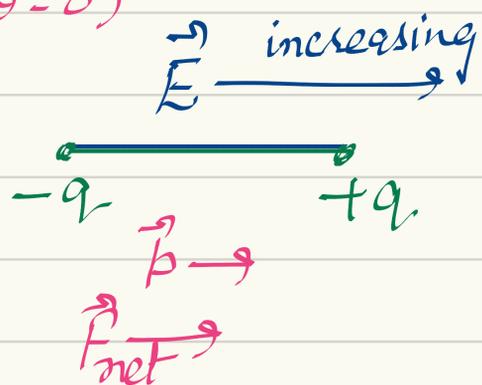## Dipole in Non-Uniform Electric field :

When field is not uniform, the net force will not
be zero.

Let when $\vec{P}$ is parallel or antiparallel to $\vec{E}$
  * Net force is not zero
  * Net torque is zero.

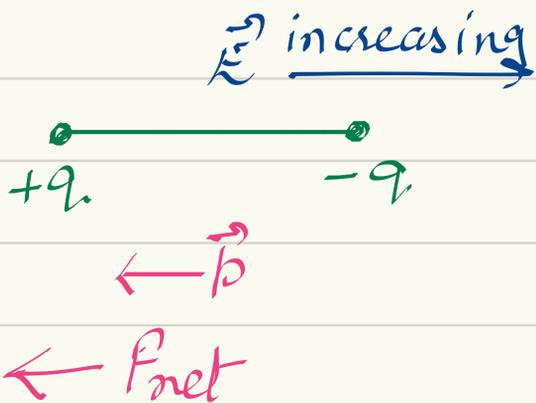(I) • $\vec{P} \parallel \vec{E}$ $(\theta = 0°)$

$\vec{E}$ $\xrightarrow{\text{increasing}}$

$-q$ $\quad\quad +q$
$\vec{P} \rightarrow$
$\vec{F}_{net} \rightarrow$

Net force $F_{net} \neq 0$
(In the dir^n of increasing field)

Net torque is zero
(Dipole will not rotate)

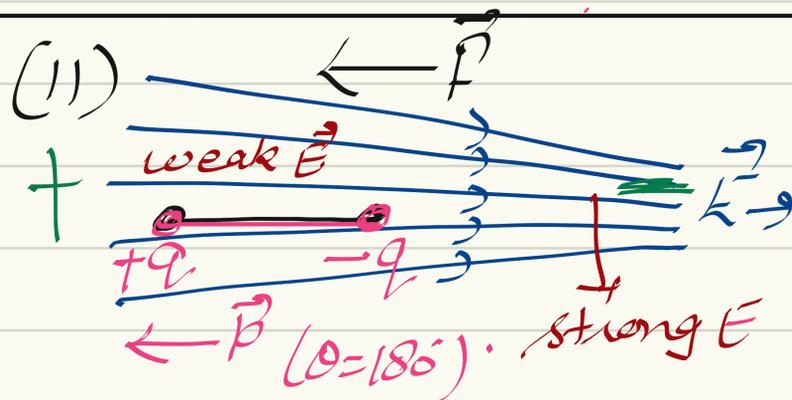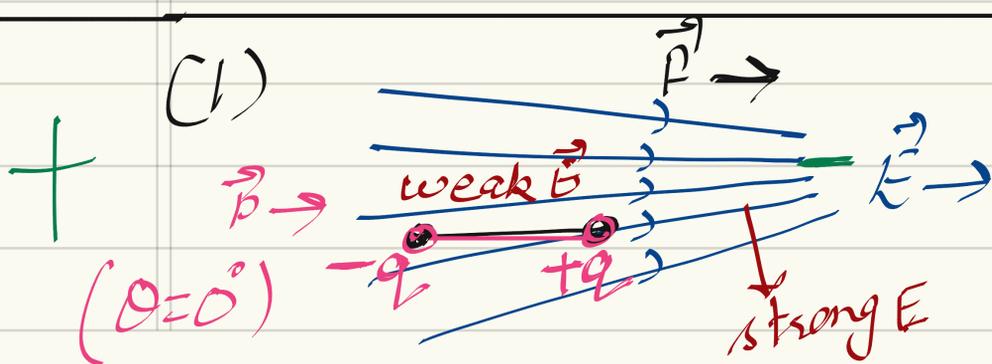(II) • $\vec{P}$ anti$\parallel$ $\vec{E}$ $(\theta = 180°)$

$\vec{E}$ $\xrightarrow{\text{increasing}}$

$+q$ $\quad\quad -q$
$\leftarrow \vec{P}$
$\leftarrow F_{net}$

Net force $F_{net} \neq 0$
(In the dir^n of decreasing field)

Net torque is zero
(Dipole will not rotate)

(I) $\vec{P} \rightarrow$

$\vec{P} \rightarrow$ weak $\vec{E}$ $-q$ $\quad +q$ $\vec{E} \rightarrow$
$(\theta = 0°)$ $\downarrow$ strong E

(II) $\leftarrow \vec{P}$

weak $\vec{E}$ $+q$ $\quad -q$ $\vec{E} \rightarrow$
$\leftarrow \vec{P}$ $(\theta = 180°)$ $\downarrow$ strong E

Continuous Charge Distribution

At macroscopic level charge is considered as continuous.

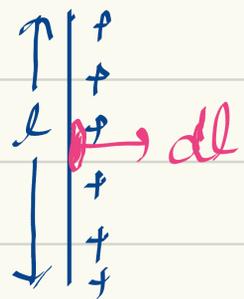Types of continuous charge distribution are

1. **Linear Charge Distribution**

The linear charge density λ is defined as

$$\lambda = \frac{\Delta Q}{\Delta l} \qquad [\text{charge per unit length}]$$

for small line element

$$\lambda = \frac{dq}{dl}$$

SI unit - $C\text{-}m^{-1}$
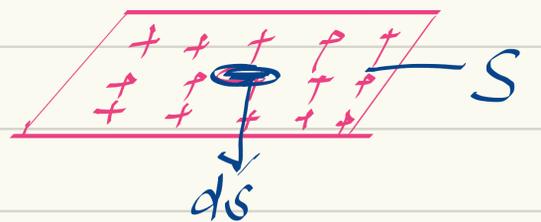
2. **Surface charge Distribution**

The surface charge density σ is defined as

$$\sigma = \frac{\Delta Q}{\Delta S} = \frac{dq}{dS} \qquad [\text{charge per unit area}]$$
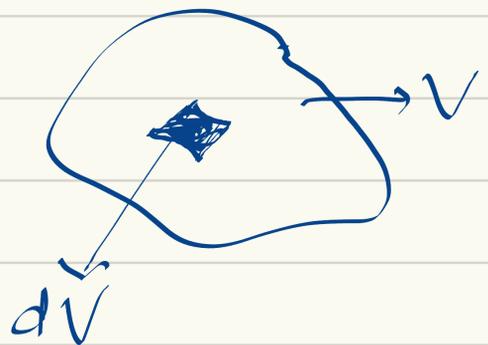
SI unit - $C\text{-}m^{-2}$

3. **Volume charge Distribution**

Volume charge density ρ is defined as

$$\rho = \frac{\Delta q}{\Delta V} = \frac{dq}{dV} \qquad [\text{charge per unit volume}]$$

※ The notion of continuous charge distribution is similar to continuous mass distribution in mechanics. We ignore discrete molecular distribution of mass.
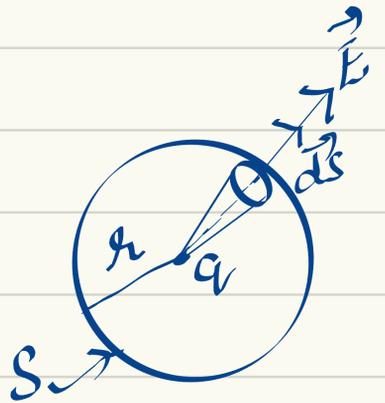
Gauss Law : Electric flux through a closed surface S is equal to $\frac{1}{\epsilon_0}$ times of charge enclosed by S.

$$\phi = \frac{q}{\epsilon_0}$$

where q = total charge enclosed

Proof :
Consider the total flux through a sphere of radius r which encloses a point charge q at its centre.
The flux through a small area element $\vec{dS}$

$$\phi = \int d\phi = \int \vec{E} \cdot \vec{dS}$$

or $\quad \phi = \int E \, dS \cos 0' \qquad [\theta = 0']$

$$= E \int dS$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \int dS \qquad \left[E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \times 4\pi r^2 \qquad \left\{ \because \int dS = S \atop = 4\pi r^2 \right\}$$

or $\qquad \boxed{\phi = \dfrac{q}{\epsilon_0}}$

Proved

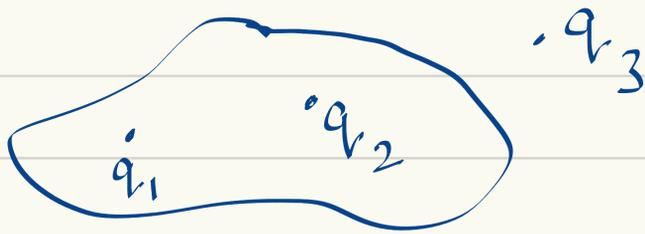* If no charge encloses the total flux is zero.

* Gauss law is true for any closed surface, no matter what is it shape or size.

* In Gauss law q represents sum of all charges enclosed by the surface S.

✳ If some charges are inside and outside the surface then the flux φ will be due to all the charges but q however represents only the total charge inside S.

e.g.

In Gauss law, $\phi = \dfrac{q}{\epsilon_0}$

the term $\phi \, (= \int \vec{E} \cdot \vec{ds})$ is due to q_1, q_2 and q_3 but the term $q = q_1 + q_2$ only.

✳ Gaussian Surface - Gaussian surface is an imaginary surface around a symmetric charge distribution and electric field intensity is same at all the points of surface.

✳ Gauss law is based on the inverse square dependence on distance contained in coulomb's law. violation of Gauss's law will indicate departure from inverse square law.
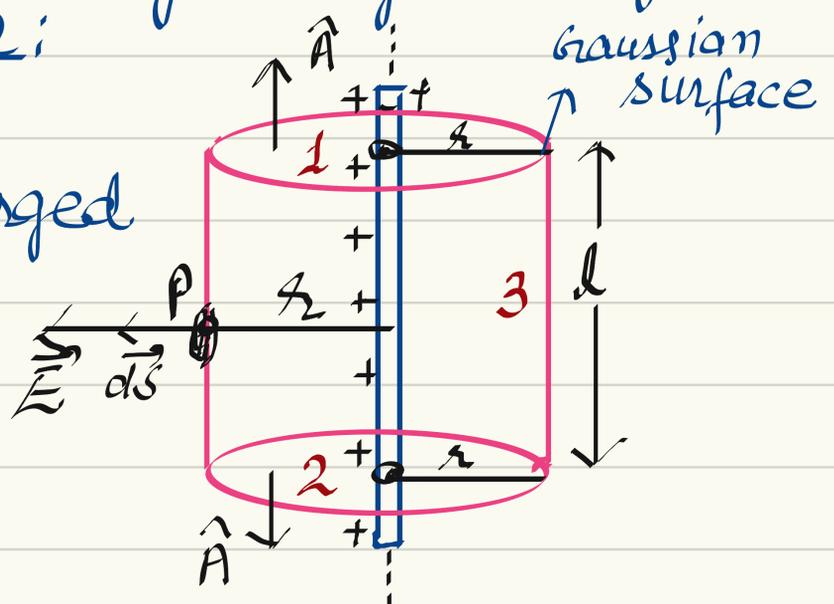
Applications of Gauss's law

For some symmetric charge configuration it is possible to obtain the electric field in a simple way using the Gauss law.

1. ## Field due to an infinitely long straight uniformly charged wire:

Consider a uniformly charged thin straight wire of charge density λ.
Let λ > 0, so dis^n of $\vec{E}$ is radially outward.

Let 'P' is the point where electric field is to be determined.

Draw the cylindrical gaussian surface passing through 'P' as show in fig.

Flux through the gaussian surface.

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$= 0 + 0 + \int E dS \cos 0°$$

$\begin{cases} \therefore \text{ for } \phi_1 \text{ and } \phi_2 \\ \quad \theta = 90° \end{cases}$

So,

$$\phi = E \int dS$$

$$\phi = E \times 2\pi r l$$

$\begin{bmatrix} \therefore \int ds = \text{surface area of} \\ \quad\quad\quad \text{curved surface} \end{bmatrix}$

by Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

hence $\quad \dfrac{q}{\epsilon_0} = E \times 2\pi r l$

or $\quad\quad E = \dfrac{1}{2\pi \epsilon_0} \cdot \dfrac{q}{r l}$

$$\boxed{E = \frac{q}{2\pi \epsilon_0 \, r l}}$$

put $\dfrac{q}{l} = \lambda$

$$\boxed{E = \frac{\lambda}{2\pi \epsilon_0 \, r}}$$

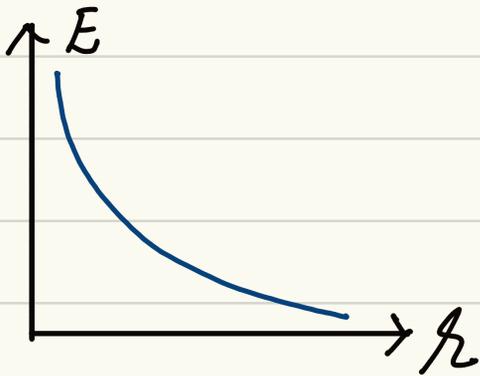Vector form - $\boxed{\vec{E} = \frac{\lambda}{2\pi \epsilon_0 r} \cdot \hat{n}}$

$\hat{n}$ is the unit vector in the dir^n of electric field.

∗ E is outward if λ is +ve and inward if λ is -ve.

* For line charge $\boxed{E \propto \dfrac{1}{r}}$

* Graph-

* K.E of an electron revolving a line charge

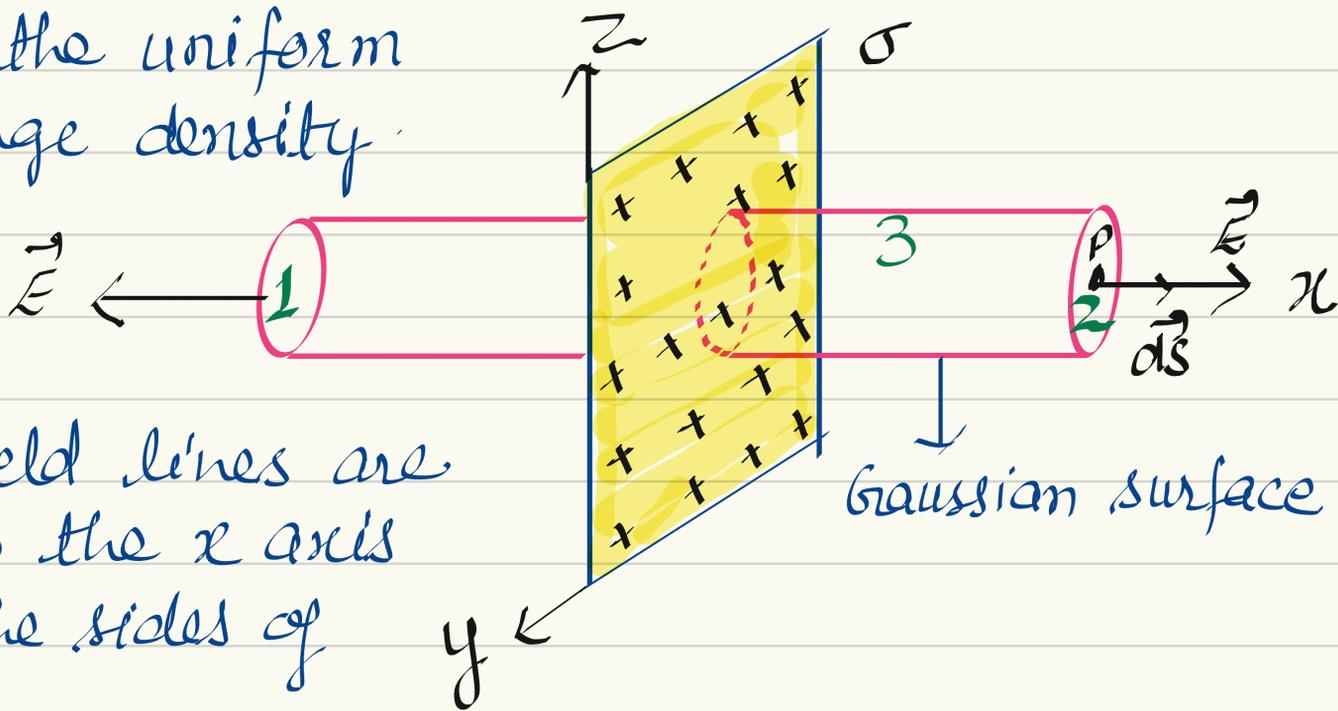$$F_e = F_c$$
$$eE = \frac{mv^2}{r}$$
$$\frac{e\lambda}{2\pi e_0 \cancel{r}} = \frac{mv^2}{\cancel{r}}$$

$$mv^2 = \frac{e\lambda}{2\pi e_0}$$
$$\boxed{K.E = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi e_0}}$$

2. Field due to a uniformly charged infinite plane

Let σ bet the uniform surface charge density.

Gaussian surface

Electric field lines are parallel to the x axis on both the sides of sheet.

Now net flux
$$\phi = \phi_1 + \phi_2 + \phi_3$$

Only the faces 1 and 2 contribute to flux as $\theta = 0°$, and $\phi_3 = 0$ as $\theta = 90°$
so
$$\phi = \int EdS + \int EdS$$
$$= 2E \int dS$$
$$\phi = 2EA \qquad \left[\because \int dS = A \right]$$

by Gauss's law
$$\phi = \frac{q}{e_0}$$

So $\dfrac{q}{e_0} = 2EA$

$$\text{but} \quad q = \sigma A$$

$$\frac{\sigma A}{\epsilon_0} = 2EA$$

$$\text{or} \quad \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

vector form

$$\boxed{E = \frac{\sigma}{2\epsilon_0} \hat{n}}$$

Where $\hat{n}$ is the unit vector normal to the plane and going away from it.

* E is away from the plate if $\sigma > 0$ [+ve]
* E is towards the plate if $\sigma < 0$ [-ve]
* E is independent of x
* Graph -

3. **Field due to a uniformly charged thin spherical shell** : (for conducting or non-conducting)
Let σ be the uniform surface charge density of a thin spherical shell of radius R.
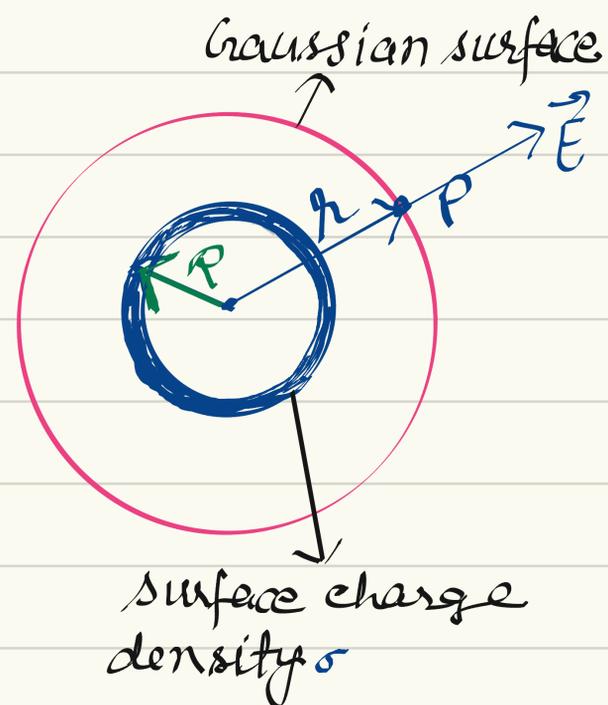
(1) **Field outside the shell** : ($r > R$)
Consider a point P outside the shell with radius vector $\vec{r}$.
At every point of Gaussian surface $\vec{E}$ and $\vec{\Delta s}$ are parallel ($\theta = 0°$)
Now net flux

$$\phi = \int E \, ds \cos 0°$$

Gaussian surface

surface charge density σ

or $\qquad \phi = E \int ds$

$\qquad\qquad = E \times 4\pi R^2 \qquad\qquad [\because \int ds = 4\pi R^2]$

by Gauss's law

$\qquad \phi = \dfrac{q}{\epsilon_0}$

so $\qquad \dfrac{q}{\epsilon_0} = E \times 4\pi R^2$

or $\qquad \boxed{E = \dfrac{1}{4\pi\epsilon_0} \dfrac{q}{R^2}}$

put $\quad q = \sigma A = \sigma \times 4\pi R^2 \qquad\qquad$ [$q \to$ total charge
we get $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ on shell]

$\qquad\qquad E = \dfrac{1}{4\pi\epsilon_0} \cdot \dfrac{\sigma \times 4\pi R^2}{R^2}$

or

$\qquad\qquad \boxed{E = \dfrac{\sigma R^2}{\epsilon_0 R^2}}$

Vector form

$\qquad\qquad \boxed{E = \dfrac{q}{4\pi\epsilon_0 R^2} \hat{R}}$

* E is directed outward if $q > 0$ and inward
if $q < 0$.

* Field outside the shell is as if entire charge
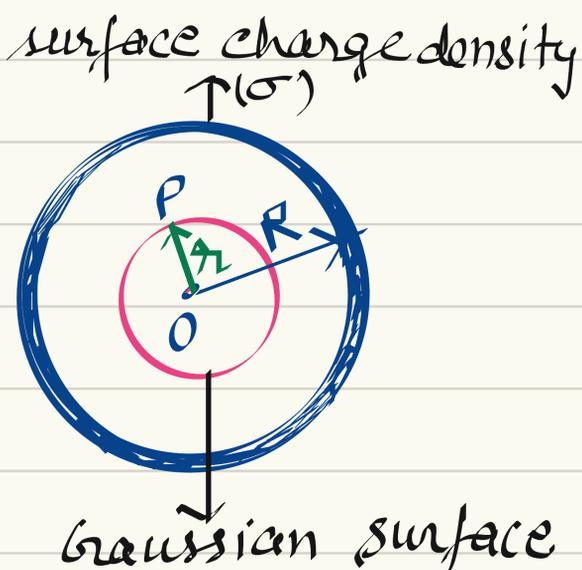of the shell is situated at its centre.

(11) <u>Field Inside the shell</u> : $(R < R)$ surface charge density
Let the point 'P' is inside the shell $\qquad\qquad\qquad$ $\uparrow(\sigma)$
at a distance R from O.
Gaussian surface is passing
through P.
Net flux through the gaussian
surface $\phi = E \int ds$

Gaussian surface

or $\quad \phi = E \times 4\pi r^2$
but inside the shell $q = 0$
so by Gauss's law
$$\phi = \frac{q}{\epsilon_0} = 0$$
or $\quad E \times 4\pi r^2 = 0$
or $\quad \boxed{E = 0}$

Field at the surface $(r=R)$
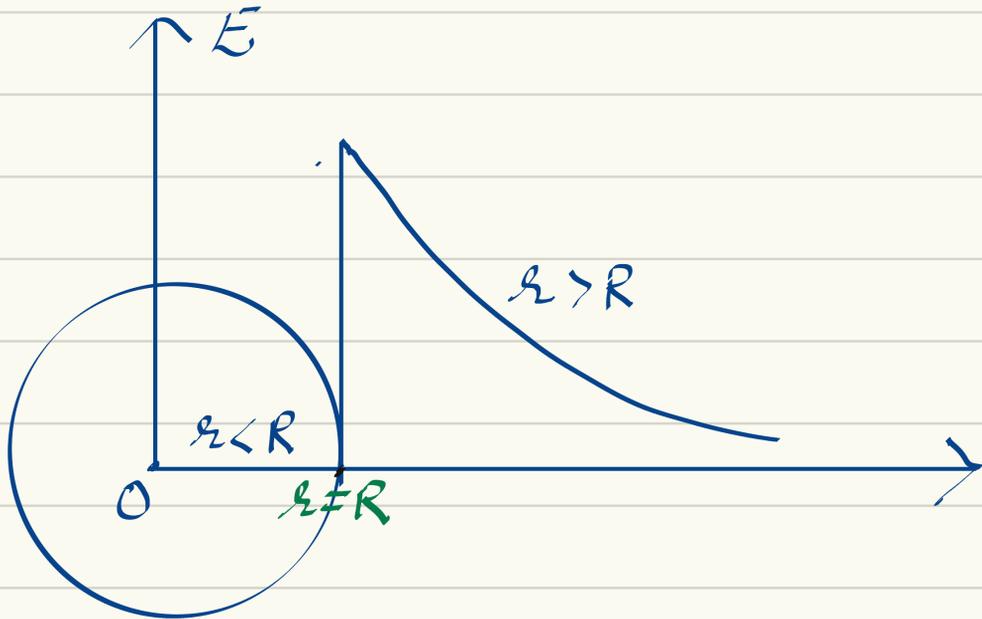By Gauss' law
$$\phi = \frac{q}{\epsilon_0} = E \times 4\pi R^2$$
or $\quad \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}}$
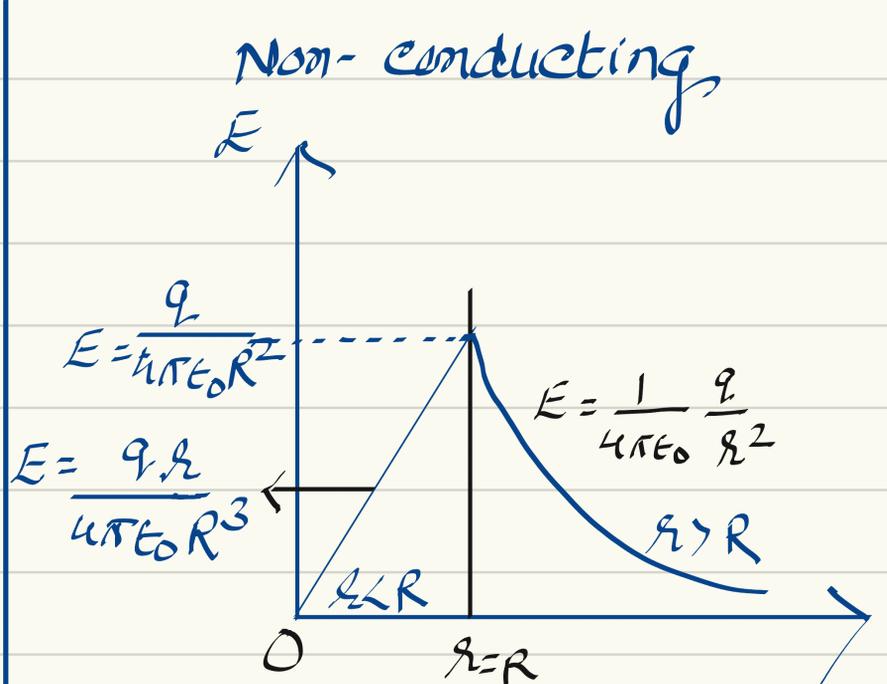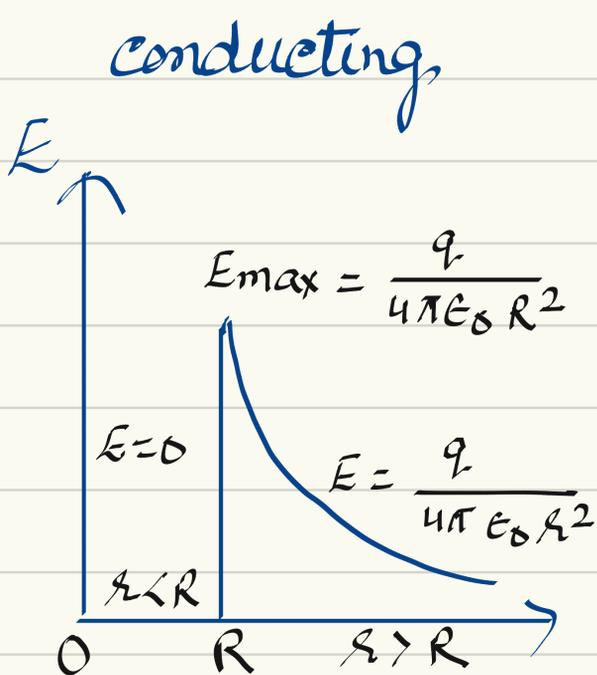Also by $E = \frac{\sigma R^2}{\epsilon_0 r^2}$
$\quad E = \frac{\sigma}{\epsilon_0} \quad [\because r = R]$

* Inside the spherical shell (for conducting and non conducting both) electric field is zero.
* Graph-

For solid sphere

conducting

$E_{max} = \frac{q}{4\pi\epsilon_0 R^2}$

$E = 0$

$E = \frac{q}{4\pi\epsilon_0 r^2}$

$r < R$

$O \quad R \quad r > R$

Non-conducting

$E = \frac{q}{4\pi\epsilon_0 R^2}$

$E = \frac{q r}{4\pi\epsilon_0 R^3}$

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$

$r < R$

$r > R$

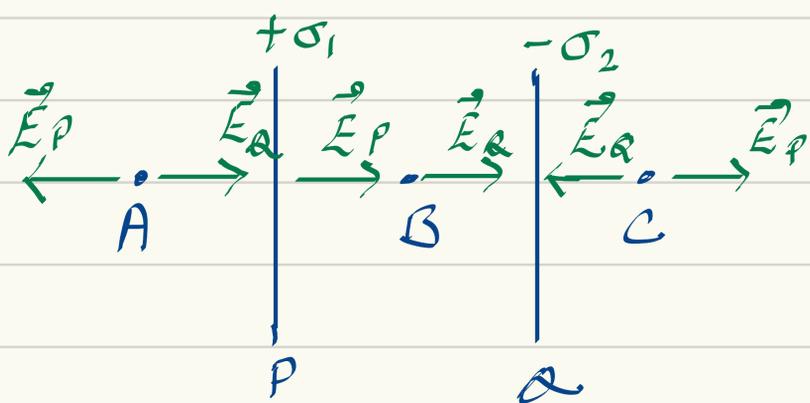$O \quad r = R$

Two infinite parallel sheets of charge

P & Q are uniformly charged sheets of charge densities $+\sigma_1$ and $-\sigma_2$ respectively.

Net field at 'A'

$$E_A = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2) \qquad [\because \sigma_1 > \sigma_2]$$

Net field at 'B'

$$E_B = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2) \quad [\text{towards right}]$$

Net field at 'C'

$$E_C = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2) \qquad [\because \sigma_1 > \sigma_2]$$

+

Polyurethane foam
Bear's fur, Hair
Glass
Mica
Nylon, Dry skin
Silk
Paper
Rubber
Copper
Gold
Polyester
Polystyrene
Acrylic
Polyethylene
Polypropylene
Polyimide (Kapton)
polytetrafluoroethylene

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