Topic ALTERNATING CURRENT Date

Chapter-7

Example 7.1

Given,

$P = 100\,W$, $V = 220\,V$

(a) Resistance $R = \frac{V^2}{P}$

$= \frac{(220)^2}{100} = \frac{48400}{100}$

or $R = 484\,\Omega$ Ans

(b) The peak voltage

$V_m = \sqrt{2}\,V_{rms}$ $[\because V = V_{rms}]$

$= 1.414 \times 220$

$= 311$ volt Ans

(c) $I_{rms} = ?$

by $P = VI$

$I_{rms} = \frac{P}{V_{rms}} = \frac{100}{220} = 0.454$

or $I_{rms} = 0.454\,A$ Ans

Teacher's Sign ....

Example 7.2

Given,

Inductance $L = 25\ mH = 25\times10^{-3} H$

$V_{rms} = 220 V$, frequency $\nu = 50 Hz$

We know

Inductive reactance $X_L = \omega L$

$= 2\pi\nu L$

$= 2\times3.14\times50\times25\times10^{-3}$

$= 3.14\times25\times10^{-1}$

$X_L = 7.85\ \Omega$ Ans

Now $I_{rms} = \dfrac{V_{rms}}{X_L}$

$= \dfrac{220}{7.85}$

or $I_{rms} = 28 A$ Ans

Example 7.3

When the lamp is connected to a capacitor and the capacitor is connected with DC source no current flows in the circuit because

capacitive reactance $X_C = \dfrac{1}{2\pi\nu C}$

for dc frequency $\nu = 0$

then $X_C = \infty$ i.e current $I = 0$

Therefore no current flows in the circuit even ~~to~~ capacitance of the capacitor is reduced. Hence lamp will not glow.

Example 7.4

Given

$C = 15\ \mu F = 15 \times 10^{-6}\ F$, $V = 220$ Volt

frequency $\nu = 50\ Hz$, $X_C = ?$

We know

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$X_C = \frac{1}{2 \times 3.14 \times 50 \times 15 \times 10^{-6}}$$

$$= \frac{10^{6}}{314 \times 15} = \frac{10^{5}}{471}$$

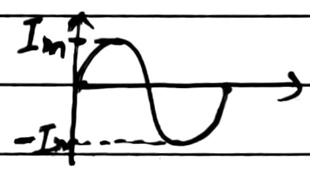
$$= 212\ \Omega$$ Ans

Now

$$I_{rms} = \frac{V_{rms}}{X_C}$$

$$= \frac{220}{212} = 1.04\ A$$ Ans

$$I_{peak} = I_m = \sqrt{2}\, I_{rms}$$



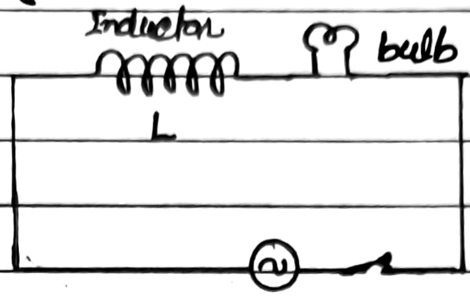
$$= 1.41 \times 1.04$$

$$= 1.47\ A$$ Ans

i.e the current oscillates between $+1.47\ A$ and $-1.47\ A$.

If frequency ν is doubled the capacitive reactance X_C is halved as $X_C = \frac{1}{2\pi\nu C}$ and current I is doubled. $\left[\because I = \frac{V}{X_C}\right]$

Example 7.5



When an iron rod is inserted into the interior of the inductor, inductance L of the coil increases. By $X_L = \omega L$, the ~~see~~ inductive reactance X_L also increases. Therefore as a result current $I = \frac{V_{rms}}{X_L}$ decreases. Hence the brightness of the bulb decreases. i.e the glow of light bulb decreases. Option (b) is correct.

* If AC source is replaced by DC source, the bulb glows more and there will be no effect on inserting iron rod.

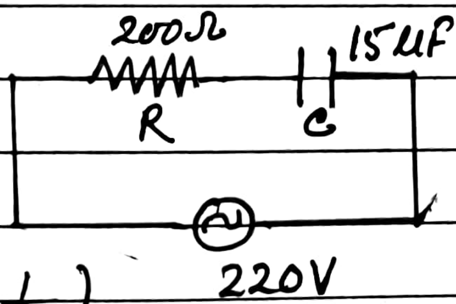
Example 7.6

Given,

$R = 200\Omega$, $C = 15.0\mu F = 15 \times 10^{-6} F$

$V = 220$ Volt , $\nu = 50 Hz$

(a) $I = ?$



We know

$Z = \sqrt{R^2 + X_C^2}$

$= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ $\left[\because X_C = \frac{1}{\omega C}\right]$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2}$$

$$= \sqrt{(200)^2 + \frac{1}{(2\times3.14\times50\times15\times10^{-6})^2}}$$

$$= \sqrt{4\times10^4 + (212)^2}$$

$$= \sqrt{4\times10^4 + 44944}$$

$$= \sqrt{4\times10^4 + 4.5\times10^4}$$

$$= 10^2\sqrt{4+4.8}$$

$$= \sqrt{8.5}\times100$$

$$= 2.915\times100$$

$$Z = 291.5\Omega$$

so $I = \frac{V}{Z}$

$$= \frac{220}{291.5} = 0.755\ A$$

$$I = 0.755\ A$$

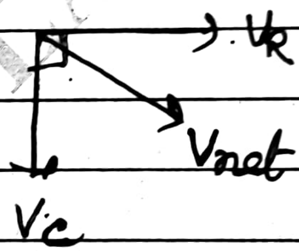
(b) Since current is same throught the circuit, we have

$$V_R = IR = 0.755\times200 = 151V.$$

$V_C = I X_C$
$= 0.755 \times 212.3$
$V_C = 160.3\ V$ Ans

here $V_R + V_C = 151 + 160.3$
$= 311.3\ V$

which is more than source voltage of 220 V. i.e $V_R + V_C > 220\ V$
because V_R and V_C are not in same phase. There is a phase difference of $\frac{\pi}{2}$ between V_R and V_C.



Therefore

$V_{R+C} = \sqrt{V_R^2 + V_C^2}$
$= \sqrt{(151)^2 + (160.3)^2}$
$= \sqrt{22801 + 25696}$
$= \sqrt{48497}$
$V_{R+C} = 220.2$ Volt
$\approx 220\ V$

Thus the phase different between the two voltage should be considered properly to get or verify $V_R + V_C = V$

Example 7.7

(a) Power of an AC circuit is given by

$$P = VI\cos\phi, \qquad \cos\phi \rightarrow \text{Power factor}$$

For a given value of P and V

$$I \propto \frac{1}{\cos\phi}$$

i.e. if $\cos\phi$ is small, current I increases.
By $P = I^2R$, power loss also increases.
So it is clear that low power factor implies large power loss.

(b) We know

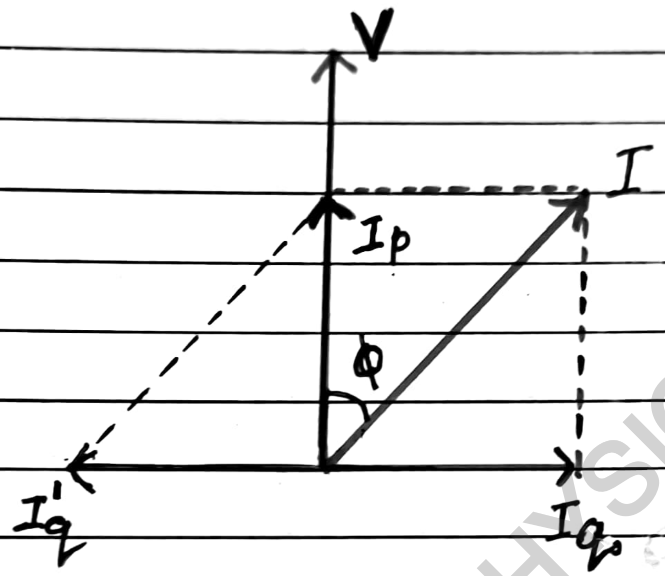
$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$$

$$= \frac{R}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \qquad [\because Z \text{ is impedance}]$$

As the value of c changes value of z also changes.
for $\cos\phi = 1$, $\omega L = \frac{1}{\omega C}$

i.e power factor $\cos\phi$ can be improved with the help of appropriate value of capacitance C.

II. Method



We resolve I into two components. I_p along V and I_q perpendicular to V. As I_q is perpe-ndicular to V, there is no power loss.

$$P = VI\cos\phi$$

for $\phi = 90$, $P = 0$

To improve the power factor I_q must be ~~neut~~ neutralise by leading current I_q'. This can be done by connecting a capacitor of appropriate capacitance in parallel.

Then

$$P = VI_p$$

When capacitor is connected to AC source
by $X_C = \frac{1}{\omega C}$ [current flows and lamp glows]

On reducing the capacitance C, X_C increases.
Therefore lamp will glow less.

Example 7.8

Given, $V_0 = 283\ V$

$\nu = 50\ Hz$

$R = 3\ \Omega$

$L = 25.48\ mH = 25.48 \times 10^{-3}\ H$

$C = 796\ \mu F = 796 \times 10^{-6}\ F$

(a) To find the impedance Z, we first calculate X_L and X_C

$$X_L = 2\pi\nu L$$
$$= \underline{2} \times 3.14 \times \underline{50} \times 25.48 \times 10^{-3}\ H$$
$$= 100 \times 3.14 \times 25.48 \times 10^{-3}$$
$$= 314 \times 25.48 \times 10^{-3}$$
$$= 8000.72 \times 10^{-3}$$
$$X_L \simeq 8\ \Omega$$

Now

$$X_C = \frac{1}{2\pi\nu C}$$
$$= \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}}$$
$$= \frac{10^6}{314 \times 796}$$
$$\simeq 4\ \Omega$$

Rough

796 ≃ 800

$$\frac{\cancel{1000}^{1.25} \times 10^3}{\cancel{800} \times 314}$$

$$= \frac{1250}{314} \simeq 3.98$$

Therefore

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{3^2 + (8-4)^2}$$
$$= \sqrt{9 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$

Or $Z = 5\ \Omega$ Ans

(b) Phase difference, $\phi = \tan^{-1} \frac{X_C - X_L}{R}$

$$\phi = \tan^{-1}\left(\frac{4-8}{3}\right)$$

$$= \tan^{-1}\left(-\frac{4}{3}\right) = \tan^{-1}(-1.33)$$

$$\phi \simeq -53.1^\circ$$

$X_C < X_L$, ϕ is '-ve' and circuit is predominatly inductive. i.e current lags the voltage across the source.

(c)

The power dissipated in the circuit

$$P = I^2 R$$

here $I_0 = \frac{V_0}{Z} = \frac{283}{5}$ so $I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{283}{5}\right)$

$$= \frac{1}{\sqrt{2}}\left(\frac{283}{5}\right) \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{283\sqrt{2}}{10}$$

$$= 28.3 \times 1.414$$

$$= 40.016$$

$$\simeq 40 \text{ A}$$

Therefore $P = 40^2 \times 3$

$$= 1600 \times 3$$

$$= 4800 \text{ W}$$ Ans

(d)

Power factor $= \cos\phi = \cos(-53.1^\circ)$

$$= 0.6$$ Ans

Example 7.9

(a) Frequency at which the resonance occurs is

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{where } L = 25.48\times10^{-3}\,H, \quad C = 796\times10^{-6}\,F$$

$$= \frac{1}{\sqrt{25.48\times10^{-3}\times796\times10^{-6}}}$$

$$= \frac{1}{\sqrt{25.48\times796\times10^{-9}}}$$

$$\simeq 2.22\times10^{2}$$

or $\omega_0 \simeq 222\ \text{rad}\,s^{-1}$

Rough

$25.45 \simeq 25.5$

$796 = 800$

$$= \frac{1}{\sqrt{25.5\times800\times10^{-8}}}$$

$$= \frac{1}{5\times9\times10^{-4}}$$

$$= \frac{10^4}{45}$$

$$= \frac{\cancel{10}^{20}}{\cancel{45}_{9}}\times10^2 = 2.22\times10^2$$

Now

$$\nu_R = \frac{\omega}{2\pi} = \frac{222}{2\times3.14}$$

$$= \frac{222}{6.28}$$

$$\nu_R = 35.35\ Hz \quad \text{Ans}$$

(b) At resonant condition

$$Z = R = 3\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{283}{\sqrt{2}\times3} \qquad \left[\because V_{rms} = \frac{V_0}{\sqrt{2}}\right]$$

or $I_{rms} = 66.7\ A$

The power dissipated at resonace

$$P = I^2R = (66.7)^2\times3$$

$$= 4448.89\times3$$

$$= 13346.67$$

$$P \simeq 13.35\ KW$$

* i.e Power dissipation is more in resonance (than Q. 7.8)

Example 7.10

A metal detector works on the principle of resonance in AC circuit.

The doorway has a coil, connected with a capacitor, forming an LCR circuit. The circuit is tuned to be in resonance when no metal is present.

When a person with metal walks, impedance of the circuit changes so current also changes. This change is detected and triggers an alarm.