NCERT PHYSICS SOLUTIONS

CHAPTER : MOTION IN A PLANE Ch. 3 CLASS : XI

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Example 4.1

Rain is falling – – – – his umbrella?

Solution:

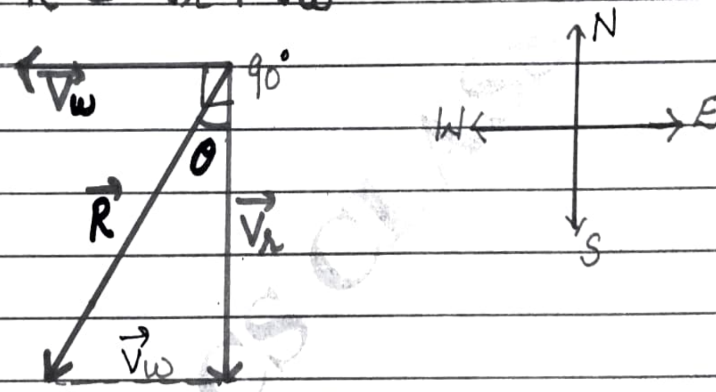
Given,

velocity of rain $\vec{V_r} = 35$ m/s (vertically downward)

velocity of wind $\vec{V_w} = 12$ m/s (east to west)

Resultant of $\vec{V_r}$ and $\vec{V_w}$,

$$\vec{R} = \vec{V_r} + \vec{V_w}$$



The magnitude of $\vec{R}$

$$R = \sqrt{V_r^2 + V_w^2 + 2V_rV_w \cos 90^\circ}$$

[∵ angle b/w $\vec{V_w}$ and $\vec{V_r} = 90^\circ$]

or $R = \sqrt{V_r^2 + V_w^2}$ [∵ $\cos 90^\circ = 0$]

or $R = \sqrt{35^2 + 12^2}$

or $R = \sqrt{1225 + 144}$

$= \sqrt{1369}$

$= 37$ m/s

or, the resultant velocity of rain due to wind $R = 37$ m/s

Now the direction $\vec{R}$ with $\vec{V_r}$

$$\tan\theta = \frac{V_w}{V_r}$$

$$= \frac{12}{35}$$

or $\tan\theta = 0.343$

or $\theta = \tan^{-1}(0.343)$

or $\theta = 19^\circ$

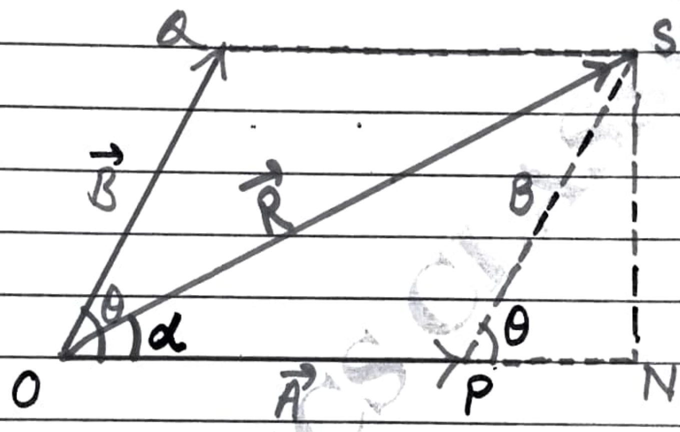
Therefore the boy should hold his umbrella at an angle about 19° with the vertical towards the east.

Example 4.2

Find the magnitude - - - - - - between them.

Solution:

Let OP and OQ represent the two vectors $\vec{A}$ and B making an angle θ as shown in fig



$OP = A$
$OQ = B$
$OS = R$
$SN = B\sin\theta$
$PN = B\cos\theta$

By using parallelogram's law of vector addition

$$\vec{OS} = \vec{R} = \vec{A} + \vec{B}$$

Magnitude of $\vec{R}$:

In ΔOSN

$$OS^2 = ON^2 + SN^2$$

or $OS^2 = (OP + PN)^2 + SN^2$

or $R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or $R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

or $R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

or $R^2 = A^2 + B^2 + 2AB\cos\theta$ $[\because \cos^2\theta + \sin^2\theta = 1]$

or

$$\boxed{R = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$ → Law of Cosines

Direction of $\vec{R}$:

In ΔOSN

$$\tan\alpha = \frac{SN}{ON}$$

$$= \frac{SN}{OP + PN}$$

or $$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

or $$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

The value of α gives the direction of resultant vector $\vec{R}$ from vector $\vec{A}$.

Example 4.3

A motorboat is ---------- velocity of the boat.

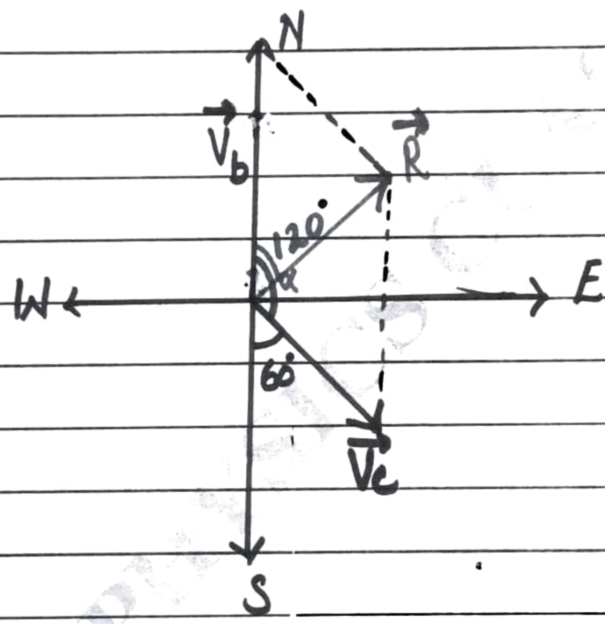
Solution:

Given,

velocity of water current $\vec{V_c} = 10$ Km/H

velocity of the motorboat $\vec{V_b} = 25$ km/H

from figure it is clear that angle between $\vec{V_c}$ and $\vec{V_b}$ is $120°$



The resultant velocity V_R

$$= \sqrt{V_b^2 + V_c^2 + 2V_bV_c \cos 120°}$$

or $$V_R = \sqrt{25^2 + 10^2 + 2 \times 25 \times 10 \left(-\frac{1}{2}\right)}$$

$$= \sqrt{625 + 100 - 250}$$

$$= \sqrt{475}$$

$$= 21.79 \doteq 21.8 \text{ Km/H}$$

or resultant velocity of boat is 21.8 Km/H Ans

direction of boat

$$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta} = \frac{V_c \sin\theta}{V_b + V_c\cos\theta}$$

$$= \frac{10\sin 120^\circ}{25+10\cos 120^\circ}$$

$$= \frac{10\times(\sqrt{3}/2)}{25+10\left(-\frac{1}{2}\right)}$$

$\sin 120^\circ = \frac{\sqrt{3}}{2}$

$\cos 120^\circ = -\frac{1}{2}$

$$= \frac{10\sqrt{3}}{2\times 20}$$

$$= \frac{\sqrt{3}}{4} = \frac{1.732}{4}$$

or $\tan\alpha = 0.433$

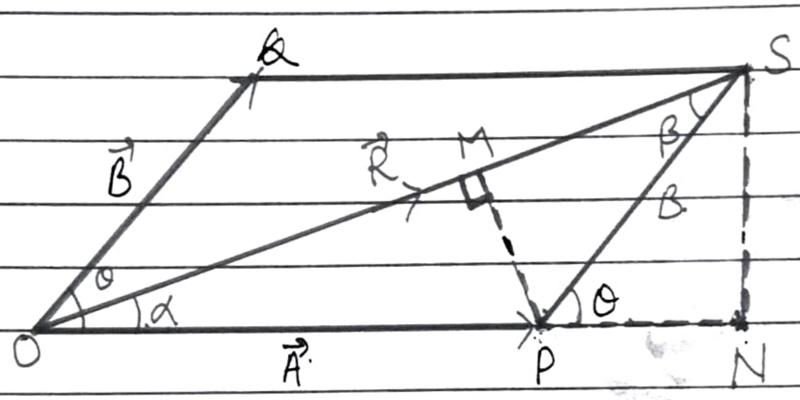
or $\alpha = \tan^{-1}(0.433)$

$\alpha = 23.4^\circ$

i.e the direction boat is 23.4° east of north.

Law of Sines:

We draw a perpendicular PM on OS.



In Δ PSN

$$SN = B\sin\theta$$

In Δ OSN

$$SN = R\sin\alpha$$

so, $B\sin\theta = R\sin\alpha$

or $\dfrac{B}{\sin\alpha} = \dfrac{R}{\sin\theta}$ —(1)

Now in Δ OMP

$$PM = A\sin\alpha$$

and in Δ SMP

$$PM = B\sin\beta$$

so, $A\sin\alpha = B\sin\beta$

or $\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha}$ —(2)

from (1) & (2)

$$\boxed{\dfrac{A}{\sin\beta} = \dfrac{B}{\sin\alpha} = \dfrac{R}{\sin\theta}} \rightarrow \text{Sine Rule}$$

Example 4.4

The position of a particle - - - - - -
- - - - - direction of v(t) at t = 1.0 s

Solution:

Given,

The position of a position is

$$\vec{r} = 3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k}$$

where t is in sec and r is in m.

(a) $$\vec{v}(t) = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}(3.0t\hat{i} + 2.0t^2\hat{j} + 5.0\hat{k})$$

$$\vec{v}(t) = 3.0\hat{i} + 4.0t\hat{j}$$

and $$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.0\hat{i} + 4.0t\hat{j})$$

$$= 4.0\hat{j}$$

or $a = 4.0\ ms^{-2}$ along y-direction

At t = 1.0 s,

$$\vec{v} = 3.0\hat{i} + 4.0 \times 1.0\hat{j}$$

$$\vec{v} = 3.0\hat{i} + 4\hat{j}$$

Magnitude of $v = \sqrt{(3^2 + 4^2)}$

$= 5.0\ m/s$

and direction,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}(1.33)$$

or $\theta = 53^\circ$ with x axis Ans

Example 4.5

A particle starts from — — — —

— — — — — particle at this time?

Solution:

(a) The position of a particle is given by.

$$\vec{r}(t) = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

here $\vec{u} = 5.0\hat{i}$ m/s , $\vec{a} = (3.0\hat{i} + 2.0\hat{j})$ m/s^2

so $\vec{r}(t) = 5.0\hat{i}t + \frac{1}{2}(3.0\hat{i} + 2.0\hat{j})t^2$

or $\vec{r}(t) = 5.0\hat{i}t + 1.5\hat{i}t^2 + 1.0\hat{j}t^2$

or $\vec{r}(t) = (5.0t + 1.5t^2)\hat{i} + 1.0t^2\hat{j}$

therefore on comparing with $r(t) = x(t)\hat{i} + y(t)\hat{j}$

we get.

$$x(t) = (5.0t + 1.5t^2)$$

and $y(t) = 1.0t^2$

given $x(t) = 84$m

so. $84 = 5.0t + 1.5t^2$

or $1.5t^2 + 5.0t - 84 = 0$

or $1.5t^2 + 14t - 9t - 84 = 0$

or $t(1.5t + 14) - 6(1.5+14) = 0$

or $(t-6)(1.5t+14) = 0$

or $t = 6$ s (ignore -ve value of t)

at $t = 6$ sec, $y(t) = 1.0(6)^2 = 36$ m

(b) Now $\vec{V} = \frac{d\vec{r}}{dt} = \frac{d[(5.0t + 1.5t^2)\hat{i} + 1.0t^2\hat{j}]}{dt}$

$$\vec{V} = (5.0 + 3.0t)\hat{i} + 2.0t\hat{j}$$

Or $\vec{v} = (5.0 + 3.0t)\hat{i} + 2.0t\hat{j}$

at $t = 6$ sec

$\vec{V} = (5.0 + 3.0 \times 6)\hat{i} + 2.0 \times 6\hat{j}$

$= (5.0 + 18.0)\hat{i} + 12.0\hat{j}$

or $\vec{V} = 23\hat{i} + 12\hat{j}$

Now magnitud of V

$V = \sqrt{23^2 + 12^2}$

$= \sqrt{529 + 144}$

$= \sqrt{673}$

$= 25.94$

$\simeq 26$ m/s

Thus the speed of the particle at $t = 6$ sec is 26 m/s

Example 4.6

Rain is falling - - - - -hold her umbrella?

Solution :

Given,

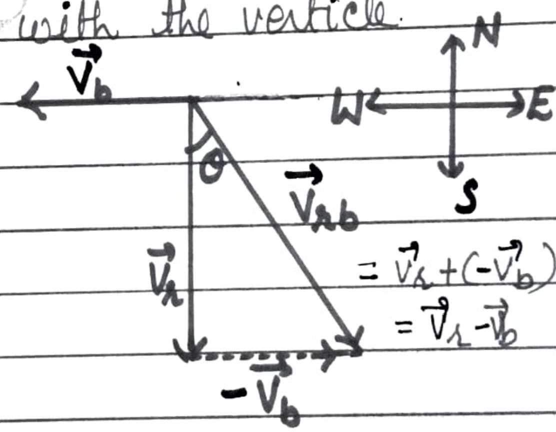
Velocity of rain $\vec{V_r}$ = 35 m/s (vertically downward)

Velocity of bicycle $\vec{V_b}$ = 12 m/s (east to west)

In order to protect from rain the woman must hold her umbrelly in the direction of the relative velocity $\vec{V_{rb}}$ of the rain with respect to the woman

$$\vec{V_{rb}} = \vec{V_r} - \vec{V_b}$$

This relative velocity vector is shown in fig. and makes an angle θ with the verticle



It is given by

$$\tan\theta = \frac{V_b}{V_r}$$

$$= \frac{12}{35}$$

or $\tan\theta = 0.343$

or $\theta = \tan^{-1}(0.343)$

or $\theta = 19^\circ$

Therefore the woman should hold her umbrella at an angle of about 19° with the verticle towards the west.

Example 4.7

Galileo, in his book - - - - - - the statement.

Solution:

For a projectile thrown at an angle θ is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

To prove: R at $\theta = 45° + \alpha$ and R at $\theta = 45° - \alpha$ are same. i.e

$$R_{45°+\alpha} = R_{45°-\alpha}$$

Now

$$R_{45°+\alpha} = \frac{u^2 \sin 2(45°+\alpha)}{g}$$

$$= \frac{u^2 \sin (90° + 2\alpha)}{g}$$

$$= \frac{u^2 \cos 2\alpha}{g} \quad -(1) \quad [\because \sin(90°+\theta) = \cos\theta]$$

and

$$R_{45°-\alpha} = \frac{u^2 \sin 2(45°-\alpha)}{g}$$

$$= \frac{u^2 \sin (90° - 2\alpha)}{g}$$

$$= \frac{u^2 \cos 2\alpha}{g} \quad -(2) \quad [\because \sin(90-\theta) = \cos\theta]$$

From eq^n (1) and (2)

$$R_{45°+\alpha} = R_{45°-\alpha}$$

Proved

Example 4.8

A hiker stands on - - - - - - - it hits the ground.

Solution :

Given,

Height of the cliff edge $h = 490\ m$

Initial velocity of the stone thrown horizontally

$$u_x = u = 15\ ms^{-1}$$

and acceleration due to gravity $g = 9.8\ ms^{-2}$

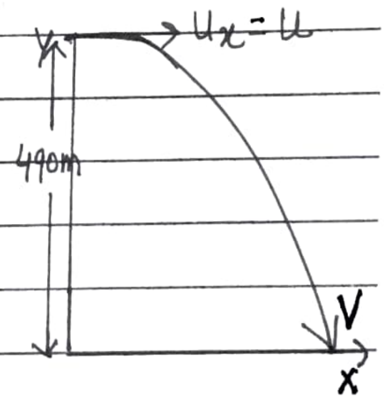
We have, time taken by the stone to reach ground

$$t = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2 \times 490}{9.8}}$$

$$= \sqrt{\frac{490}{4.9}} = \sqrt{100}$$

$$t = 10\ Sec \quad \text{Ans}$$



The velocity with which stone hits the ground,

$$V = \sqrt{V_x^2 + V_y^2}$$

here $V_x = u_x = 15\ ms^{-1}$

and $V_y = u_y + gt \Rightarrow V_y = 0 + 9.8 \times 10$

or $V_y = 98\ ms^{-1}$

so

$$V = \sqrt{15^2 + 98^2} = \sqrt{225 + 9604}$$

or $V = \sqrt{9829} = 99.14$

$$\approx 99\ ms^{-1} \quad \text{Ans}$$

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(C) The distance from the thrower to the point where the ball return to the same point = Horizontal range (R)

and

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{(28)^2 \sin 2\times30^\circ}{9.8}$$

$$= \frac{28\times28\times\sin 60^\circ}{9.8}$$

$$= \frac{28\times28\times\sqrt{3}}{9.8\times2} = \frac{2\times28\times\sqrt{3}}{0.7\times2} \quad \left[\sin 60^\circ = \frac{\sqrt{3}}{2}\right]$$

$$= 40\times1.732 \quad [\because \sqrt{3} = 1.732]$$

$$= 69.28$$

$$R \simeq 69 \text{ m}$$

Ans

Example 4.9

A cricket ball - - - - - - - to the same level.

Solution:

Given,

Speed at which ball is thrown $u = 28\ ms^{-1}$

angle $\theta = 30^\circ$

(a) Maximum height

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{(28)^2 \times (\sin 30)^2}{2 \times 9.8}$$

$$= \frac{28 \times 28 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 9.8} \quad \left[\because \sin 30^\circ = \frac{1}{2}\right]$$

$$= \frac{7 \times 14}{9.8}$$

$$= 10\ m \quad \text{Ans}$$

(b) Time taken by the ball to return to the same level = time of flight (T)

and

$$T = \frac{2u\sin\theta}{g}$$

$$= \frac{2 \times 28 \times \sin 30^\circ}{9.8} = \frac{2 \times 28 \times \frac{1}{2}}{9.8}$$

$$= \frac{28}{9.8} = \frac{14}{4.9} = \frac{2}{0.7} = 2.86\ sec \quad \text{Ans}$$

Example 4.10

An insect trapped - - - - - - - its magnitude?

Solution:

Given,

Radius $R = 12$ cm

Since insects completes 7 revolutions in 100 sec.

The time taken for 1 revolution $T = \frac{100}{7}$ sec

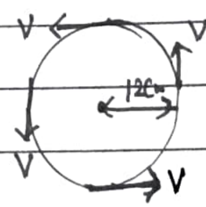
(a) Angular speed $\omega = \frac{2\pi}{T}$

$$= \frac{2\pi}{100/7} = \frac{2\pi \times 7}{100}$$

$$= \frac{2 \times 22 \times 7}{7 \times 100}$$

$$= \frac{44}{100}$$

$$= 0.44 \text{ rad/s}$$



The linear speed

$$V = R\omega$$

$$= 12 \times 0.44$$

or $V = 5.28$ cm/s

For circular motion the direction of velocity is tangential at every point. The direction of acceleration is toward centre. Since direction of velocity is changing continuously, acceleration here is not constant vector but its magnitude is constant.

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Magnitude of acceleration

$$a = R\omega^2$$

$$= 12 \times (0.44)^2$$

$$= 12 \times 0.1936$$

$$= 12 \times 0.19$$

or $a = 2.28 \text{ cm/s}^2$ Ans