Motion In A Plane   Ch. 3
NCERT Exercise

3.1 <u>Scalar quantities</u> – Volume, mass, speed, density, no. of moles and angular frequency.

<u>Vector quantities</u> – Acceleration, velocity, displacement and angular velocity.

3.2 Work and current are two scalar quantities.

3.3 Impulse is vector quantity.

3.4 (a) No, because only same dimensions terms can be added.

(b) No, because scalar can not be added to a vector.

(c) Yes, multiplication of scalar and vector gives n times of vector [if n is scalar] in same direction.

e.g $\xrightarrow{\quad \vec{A} \quad}$ $\xrightarrow{\qquad 2\vec{A} \qquad}$

(d) Yes, any two scalars can be multiplied.

(e) No, only same dimensions vector can be added.

(f) Yes, because both have same dimensions.

3.5 (a) True

(b) False

(c) False

(d) True

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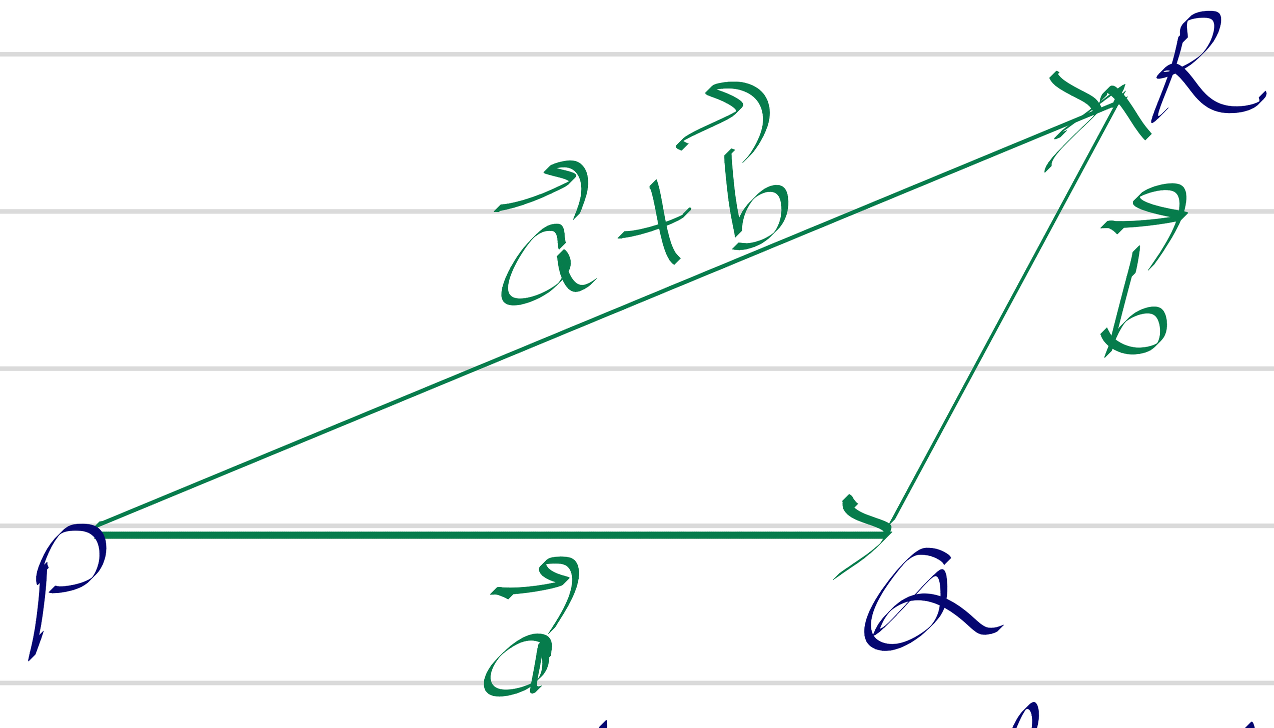
3.6 (a) To prove $|\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}|$

Ans.- Let $\vec{a}$ and $\vec{b}$ are two vectors and $\vec{a}+\vec{b}$ is the resultant as shown in fig

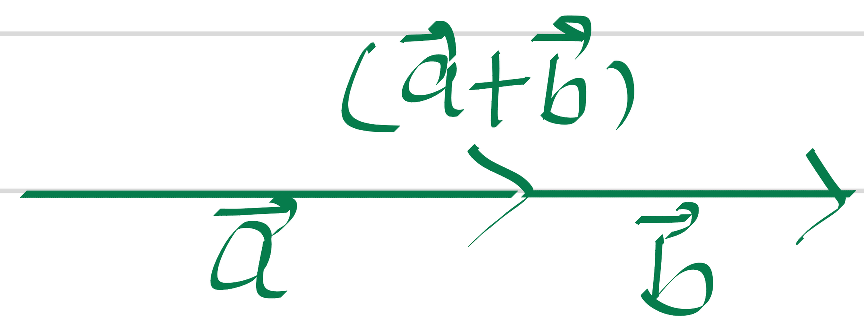


We know in a triangle length of one side is always less than the sum of the length of other two sides. Hence

$$|\vec{a}+\vec{b}| < |\vec{a}|+|\vec{b}| \quad \text{— (i)}$$

If the two vectors $\vec{a}$ and $\vec{b}$ are in straight line and in the same direction, then

$$|\vec{a}+\vec{b}| = |\vec{a}|+|\vec{b}| \quad \text{— (ii)}$$

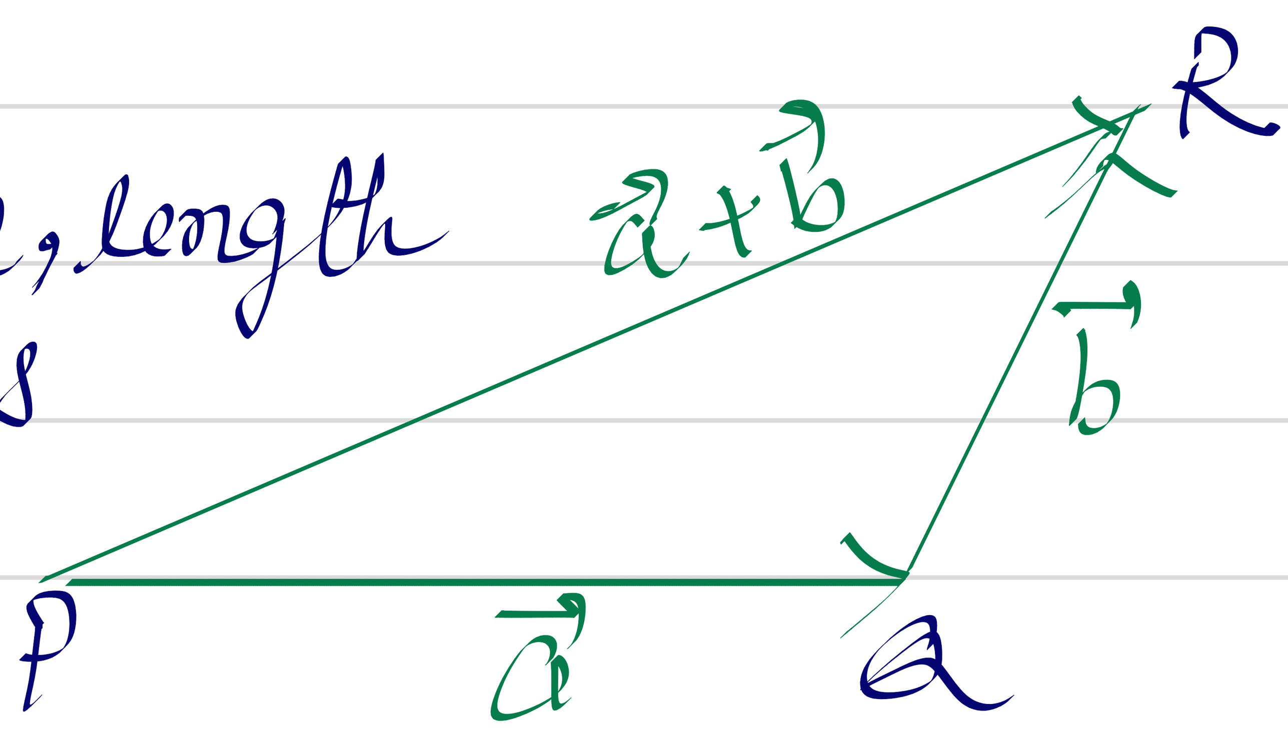


From (i) and (ii)

$$|\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}| \qquad \text{Proved}$$

(b) To prove $|\vec{a}+\vec{b}| \geq ||\vec{a}|-|\vec{b}||$

Ans. We know in a triangle, length difference of any two sides always less than the length of third side.



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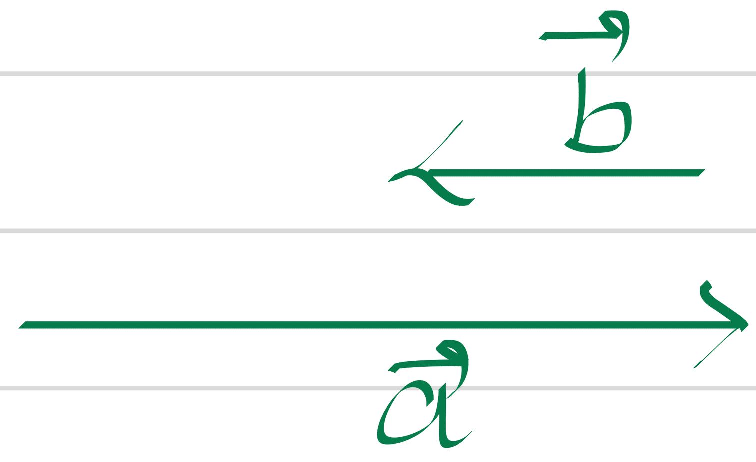
Hence

$$|\vec{a}+\vec{b}| > ||\vec{a}|-|\vec{b}|| \quad \text{——(i)}$$

If the two vectors are collinear vectors and in opposite direction, then



$$|\vec{a}+\vec{b}| = ||\vec{a}|-|\vec{b}|| \quad \text{——(ii)}$$

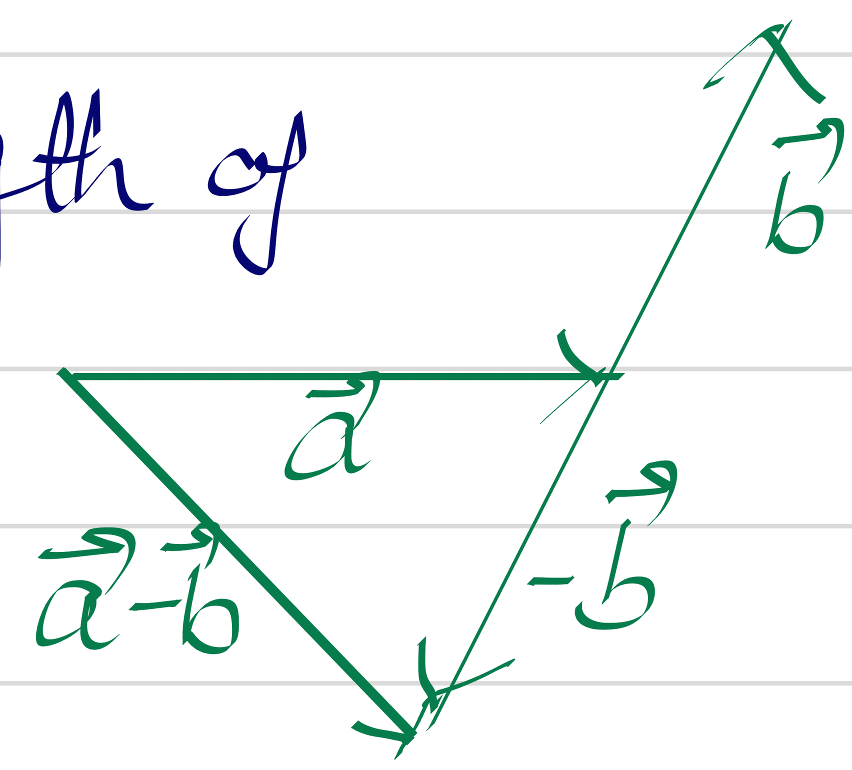
From (i) and (ii)

$$|\vec{a}+\vec{b}| \geq ||\vec{a}|-|\vec{b}||$$

Proved

(e) To prove $|\vec{a}-\vec{b}| \leq |\vec{a}| + |\vec{b}|$

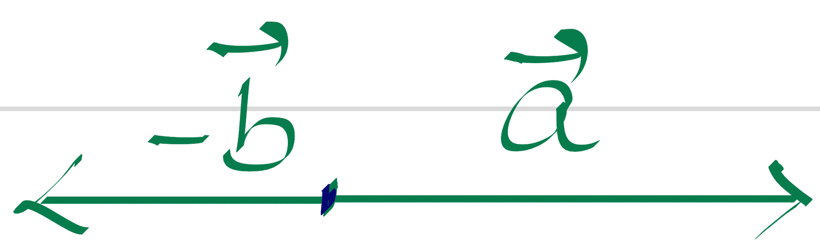
Ans. We know in a triangle, length of two sides is always greater than the length of third side. Hence



$$|\vec{a}-\vec{b}| < |\vec{a}| + |(-\vec{b})|$$

or $|\vec{a}-\vec{b}| < |\vec{a}| + |\vec{b}|$ ——(i) $[|-\vec{b}| = |\vec{b}|]$

If $\vec{a}$ and $-\vec{b}$ are in straight line and in the opposite direction, then $\theta = 180°$



$$|\vec{a}-\vec{b}| = |\vec{a}| + |-\vec{b}|$$

$$|\vec{a}-\vec{b}| = |a| + |\vec{b}| \quad \text{——(ii)}$$

From (i) and (ii)

$$|\vec{a}-\vec{b}| \leq |\vec{a}| + |\vec{b}| \qquad \text{Proved}$$

$R^2 = a^2 + (-b)^2 + 2a(-b)\cos 180°$

$= a^2 + b^2 + 2ab = (a+b)^2$

$R = a + b = |a| + |b|$

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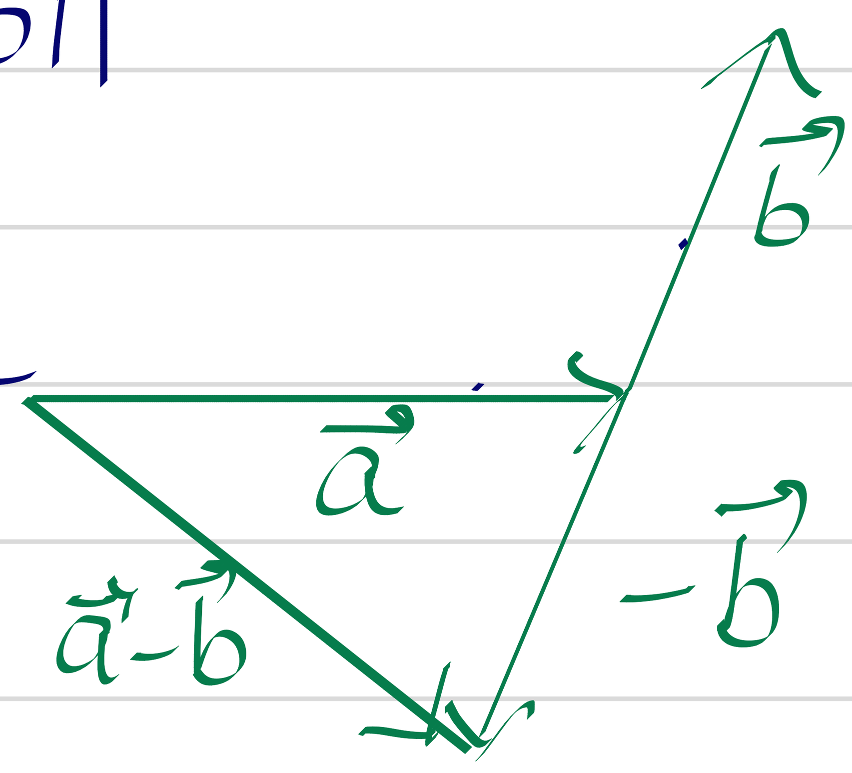
(d) To prove $|\vec{a}-\vec{b}| \geq ||\vec{a}|-|\vec{b}||$

Ans. We know that in a triangle length of one side is always greater than length of difference of the two other sides.

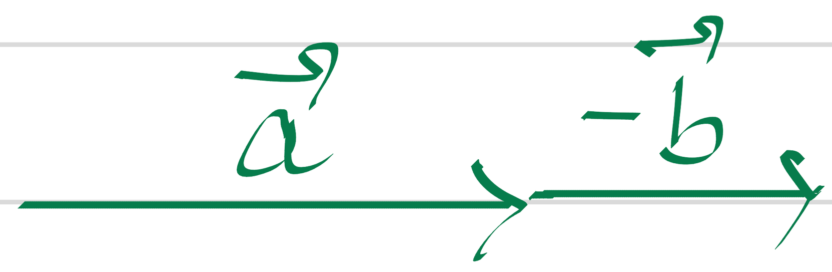


Hence,

$$|\vec{a}-\vec{b}| > ||\vec{a}|-|-\vec{b}||$$

or $|\vec{a}-\vec{b}| > ||a|-|b||$ ——— (I) $[|-b| = |\vec{b}|]$

If $\vec{a}$ and $-\vec{b}$ vectors are in straight line and in the same direction, then



$$|\vec{a}-\vec{b}| = |\vec{a}| - |-\vec{b}|$$

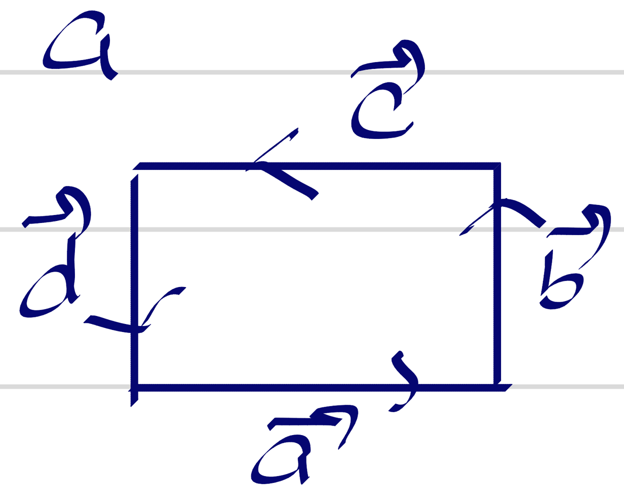
$$|\vec{a}-\vec{b}| = ||\vec{a}|-|\vec{b}||$$ ——— (II)

From (I) and (II)

$$|\vec{a}-\vec{b}| \geq ||a|-|b||$$

Proved

3.7. (a) False because if all vectors form a quadrilateral then $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$. So it is not necessary that if all vectors are null vectors only then their vector sum will be zero.



(b) Correct, because, if

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$$

$$\vec{a}+\vec{c} = -(\vec{b}+\vec{d})$$

or $|\vec{a}+\vec{c}| = |\vec{b}+\vec{d}|$

i.e magnitude of $(\vec{a}+\vec{c})$ = magnitude of $(\vec{b}+\vec{d})$

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(c) correct, because if

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$$

$$\vec{a} = -(\vec{b}+\vec{c}+\vec{d})$$

or $|\vec{a}| = |-(\vec{b}+\vec{c}+\vec{d})|$

or mag of $\vec{a}$ = mag. of $(\vec{b}+\vec{c}+\vec{d})$

So mag. of $\vec{a}$ can never be greater than mag. of $(\vec{b}+\vec{c}+\vec{d})$

(d) correct,

for $\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$

Vector $\vec{a}$, $(\vec{b}+\vec{c})$ and $\vec{d}$ must lie in the same plane if $\vec{a}$ and $\vec{d}$ are not collinear.

If $\vec{a}$ and $\vec{d}$ are collinear, than $(\vec{b}+\vec{c})$ is in the line of $\vec{a}$ and $\vec{d}$ to be

$$\vec{a}+\vec{b}+\vec{c}+\vec{d}=0$$

3.8 Given, radius of circular ice ground = 200 m

so diameter PQ = 400 m

Displacement of all three girls are equal

$= \vec{PQ} = 400$ m

For girl B, displacement = actual path length.

(Since displacement is the shortest path)

3.9 (a) Initial and final position of the cyclist is same. Therefore

Net displacement = 0

(b) Since displacement is zero therefore

Average velocity = 0 $\left\{ \text{Av. velocity} = \dfrac{\text{Displacement}}{\text{time}} \right.$

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(c) Average speed $= \dfrac{\text{Total distance}}{\text{total time}}$

$$= \frac{OP + PQ + QO}{10 \text{ min}}$$

$$= \frac{1\text{ Km} + \frac{1}{4}2\pi r + 1\text{ Km}}{1\cancel{0}/6\cancel{0} \text{ hrs}}$$

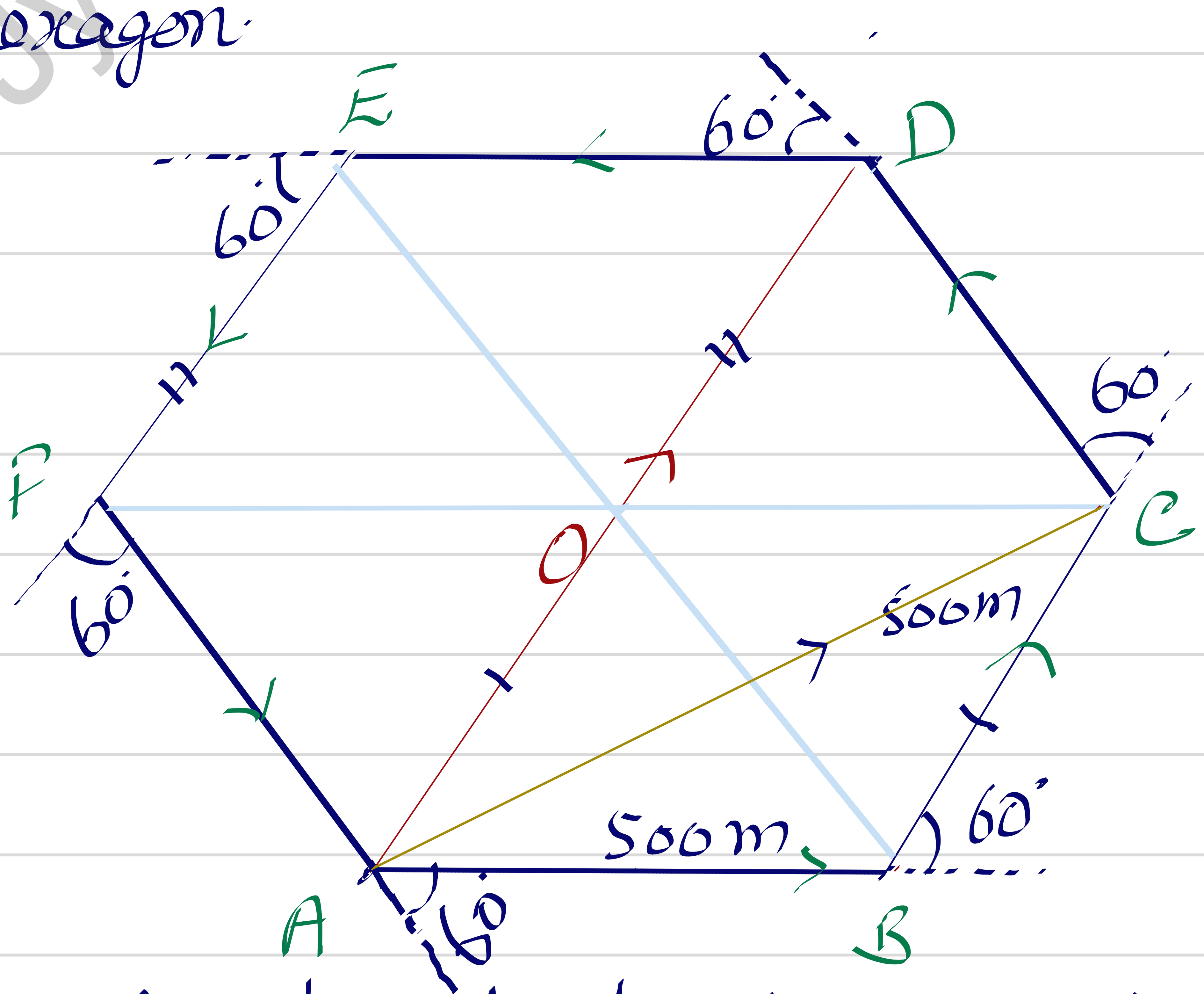
$$= \left(2 + \frac{\pi \times 1}{2}\right)6$$

$$= (2 + 1.57) \times 6 \qquad [\pi = 3.14]$$

$$= 3.57 \times 6$$

$$= 21.42 \text{ Km/h}$$ Ans

3.10 The track is show in the fig. which is a regular hexagon.



Let the motorist starts from point A.

(i) He takes 3rd turn at D

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Magnitude of displacement $AD = OA + OD$
$= 500 + 500$
$= 1000\ m$

Path length of $AD = AB + BC + CD$
$= 500 + 500 + 500$
$= 1500\ m$

(ii) Motorist takes 6th turn at 'A'
He reaches at starting point 'A'. So
Displacement $= 0$
Path length $= AB + BC + CD + DE + EF$
$= 6 \times 500$
$= 3000\ m$

(iii) He takes 8th turn at 'C'
magnitude of displacement $= |\vec{AC}| = |\vec{AB} + \vec{BC}|$

$$AC^2 = AB^2 + BC^2 + 2 \cdot AB \cdot BC \cos 60^\circ$$
$$= 500^2 + 500^2 + \cancel{2} \times 500 \times 500 \times \frac{1}{\cancel{2}}$$
$$= 100^2 [5^2 + 5^2 + 5^2]$$
$$= 100^2 [25 + 25 + 25]$$
$$AC^2 = 100^2 \times 75$$
$$AC = 100 \times \sqrt{75}$$
$$AC = 100 \times 5\sqrt{3}$$
$$= 100 \times 5 \times 1.732$$
$$= 100 \times 8.660$$
$$= 866\ m$$

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Path length $= AB + BC + 3000$
$= 1000 + 3000$
$= 4000\,m$

3.12 Given,

$H = 25\,m$
$u = 40\,m/s$
$R = ?$
$g = 9.8\,m/s^2$

We know,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

$$25 = \frac{40^2 \times \sin^2\theta}{2 \times 9.8}$$

$$\sin^2\theta = \frac{\cancel{25}^{5} \times \cancel{2} \times 9.8}{\cancel{40}_{\cancel{20}\,4} \times \cancel{40}_{8}} = \frac{\cancel{9.8}^{4.9}}{\cancel{4}_{2} \times 8}$$

$$\sin^2\theta = \frac{4.9}{16} = \frac{49}{160}$$

$$\sin\theta = \sqrt{\frac{4.9}{16}} = \sqrt{\frac{49}{16 \times 10}} = \frac{7}{4\sqrt{10}}$$

Now

$$\cos^2\theta = 1 - \sin^2\theta$$
$$= 1 - \frac{49}{160} = \frac{111}{160}$$

$$\cos^2\theta = \frac{111}{160}$$

$$\cos\theta = \frac{1}{4}\sqrt{\frac{111}{10}}$$

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$$\text{Now} \quad R = \frac{u^2 \sin 2\theta}{g}$$

$$= \frac{40^2 (2 \sin\theta \cos\theta)}{9.8}$$

$$= \frac{\cancel{40}^{10} \times \cancel{40}^{10} \times 2 \times \frac{7}{\cancel{4}\sqrt{10}} \times \frac{1}{\cancel{4}}\sqrt{\frac{111}{10}}}{9.8}$$

$$= \frac{\cancel{100}^{10} \times 2 \times 7 \sqrt{111}}{\cancel{10} \times 9.8}$$

$$= \frac{\cancel{140}^{20} \times \sqrt{111}}{\cancel{9.8}_{1.4}} = \frac{10 \times \sqrt{111}}{0.7}$$

$$= \frac{100 \times 10.53}{0.7}$$

$$= \frac{1053}{7}$$

$$R = 150.4 \text{ m}$$

Ans

$\sqrt{111} = 10.53$: 1 | 111; 1; 25 | 0 1100; 1025; 2103 | 7500; 6309

3.13. Given

$$R_{max} = \frac{u^2}{g}$$

$$100 = \frac{u^2}{g}$$

$$\text{or} \quad \frac{u^2}{g} = 100 \quad -(1)$$

$$\left[R = \frac{u^2 \sin 2\theta}{g}, \ \theta = 45^\circ \right.$$

$$\left. R_{max} = \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g} \right.$$

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$$H_{max} = \frac{u^2}{2g}$$

$$\left[H = \frac{u^2 \sin^2\theta}{2g}, \ \theta = 90^\circ \qquad H = \frac{u^2}{2g} \right.$$

$$= \frac{1}{2}\left(\frac{u^2}{g}\right)$$

$$= \frac{1}{2} \times 100 \qquad [\text{from eq}^n (1)]$$

$$H_{max} = 50 \text{ m}$$

3.14. Given,

$$r = 80 \text{ cm}$$
$$= 0.8 \text{ m}$$

$$\text{frequency } n = \frac{\text{no. of revolutions}}{\text{Time}}$$

$$n = \frac{14}{25} \text{ rev}$$

$$\text{acc}^n \ a = r\omega^2$$

$$= r(2\pi n)^2 \qquad [\omega = 2\pi n]$$

$$= 4\pi^2 r n^2$$

$$= 4 \times \frac{22}{\cancel{7}} \times \frac{22}{\cancel{7}} \times 0.8 \times \frac{\cancel{14}^{\,2}}{25} \times \frac{\cancel{14}^{\,2}}{25}$$

$$= \frac{16 \times 484 \times 0.8}{625}$$

$$= \frac{7744 \times 0.8}{625} = \frac{6195.2}{625}$$

$$a = 9.91 \text{ m/s}^2$$

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3.15 Given,

$$r = 1\text{ Km} = 10^3\text{ m}$$

$$v = 900\text{ Km h}^{-1}$$

$$= 900 \times \frac{5}{18} = 250\text{ m/s}$$

centripetal acc^n $a = \dfrac{v^2}{r}$

$$= \frac{(250)^2}{10^3}$$

$$= \frac{250 \times 250}{1000}$$

$$= \frac{625}{10} = 62.5\text{ m/s}^2$$

$$a = 62.5\text{ m/s}^2$$

compare with 'g'

$$\frac{a}{g} = \frac{62.5}{9.8}$$

$$\frac{a}{g} = 6.38$$

$$a = 6.38\,g$$

Ans

3.16. (a) False. Net acceleration is not always directed towards centre of the circle. It happens only in the case of uniform circular motion.

(b) True. At a point on a circular path a particle appears to move tangentially to the circle. Hence the velocity vector is always along the tangent at a point.

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(e) True. In UCM the direction of the acceleration vector is towards centre of the circle. Due to symmetry acceleration vectors pointing in all directions around the circle cancel out each other. Therefore average acceleration over one cycle is zero (null vector)

3.17 Given,

$$\vec{r} = (3t\hat{i} - 2t^2\hat{j} + 4\hat{k})\, m$$

(a) velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3t\hat{i} - 2t^2\hat{j} + 4\hat{k})$$

$$\vec{v} = 3\hat{i} - 4t\hat{j} + 0$$

or $\vec{v} = 3\hat{i} - 4t\hat{j}$ Ans

Now

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3\hat{i} - 4t\hat{j})$$

$$= 0 - 4\hat{j}$$

or $\vec{a} = -4\hat{j}$

(b) $\vec{v} = 3\hat{i} - 4t\hat{j}$

$t = 2\, sec$

$$\vec{v} = 3\hat{i} - 4\times 2\hat{j}$$

$$\vec{v} = 3\hat{i} - 8\hat{j}$$

$$|\vec{v}| = \sqrt{3^2 + (-8)^2}$$

$$= \sqrt{9 + 64}$$

$$|\vec{v}| = \sqrt{73}\ m/s$$

Magnitude of $\vec{v} = \sqrt{73}\ m/s$ Ans

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Direction,

$$\tan\alpha = \frac{v_y}{v_x}$$

$\vec{v} = 3\hat{i} - 8\hat{j}$

$\vec{v} = v_x\hat{i} + v_y\hat{j}$

$\tan\alpha = \frac{v_y}{v_x}$

$$\tan\alpha = \frac{-8}{3}$$

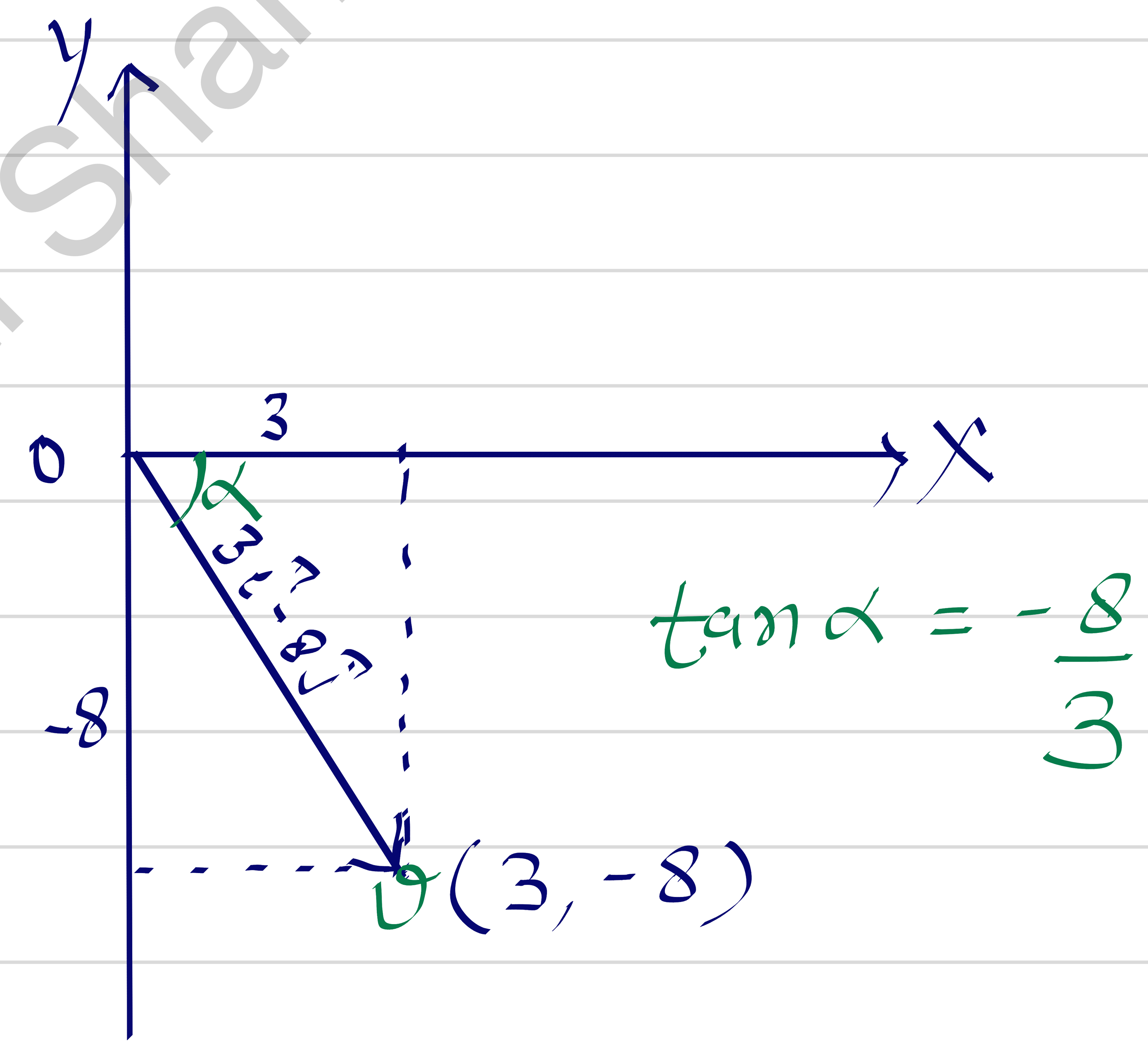
$$\alpha = \tan^{-1}\left(\frac{-8}{3}\right)$$

$$\alpha = -\tan^{-1}(2.667)$$

$$\alpha = -69.45'$$

Ans

-ve sign shows that dirⁿ of v is below x axis.



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3.18 Given,

$u_y = 10\hat{j}$ m/s , $u_x = 0$

$\vec{a} = 8\hat{i} + 2\hat{j}$ m/s^2

compare with $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$a_x = 8$ m/s^2

$a_y = 2$ m/s^2

$x = 16$ m

(a) We know

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$16 = 0 + \frac{1}{2} \times 8 \times t^2$$

$$t^2 = \frac{\cancel{16}^{2} \times 2}{\cancel{8}} = 4$$

$$\boxed{t = 2 \text{ sec}}$$ Ans

again

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = 10 \times 2 + \frac{1}{\cancel{2}} \times \cancel{2} \times (2)^2$$

$$y = 20 + 4$$

$$\boxed{y = 24 \text{ cm}}$$ Ans

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3.18 Given,

$u_y = 10\hat{j}$ m/s $\quad u_x = 0$

$\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\vec{u} = (0)\hat{i} + 10\hat{j}$ m/s

and $\vec{a} = \underline{8}\hat{i} + 2\hat{j}$ m/s^2

$\vec{a} = \underline{a_x}\hat{i} + \underline{a_y}\hat{j}$

(a) $x = 16$ m , $t = ?$

we have

$$x = x_0 + u_x t + \frac{1}{2} a_x t^2$$

$$16 = 0 + 0 \times t + \frac{1}{\cancel{2}} \times \overset{4}{\cancel{8}} \times t^2$$

$$16 = 4 \times t^2$$

$$t^2 = 4$$

$$\boxed{t = 2 \text{ sec}}$$ Ans

again,

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

$$= 0 + 10 \times 2 + \frac{1}{\cancel{2}} \times \cancel{2} \times 2^2$$

$$= 20 + 4$$

$$\boxed{y = 24 \text{ m}}$$ Ans

(b) For $t = 2$ sec , $v = ?$

$$v = u + at$$

$$= 10\hat{j} + (8\hat{i} + 2\hat{j}) \times 2$$

$$= 10\hat{j} + 16\hat{i} + 4\hat{j}$$

$$= 14\hat{j} + 16\hat{i}$$

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$$\vec{v} = 16\hat{i} + 14\hat{j}$$ Ans

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{16^2 + 14^2}$$

$$= \sqrt{256 + 196}$$

$$= \sqrt{452}$$

$$\boxed{v = 21.26 \text{ m/s}}$$ Ans

$$\begin{array}{r|l} & 21.26 \\ \hline 2 & \overline{4}\,\overline{52} \\ & 4 \\ \hline 41 & 52 \\ & 41 \\ \hline 422 & 1100 \\ & 844 \\ \hline 4246 & 25600 \end{array}$$

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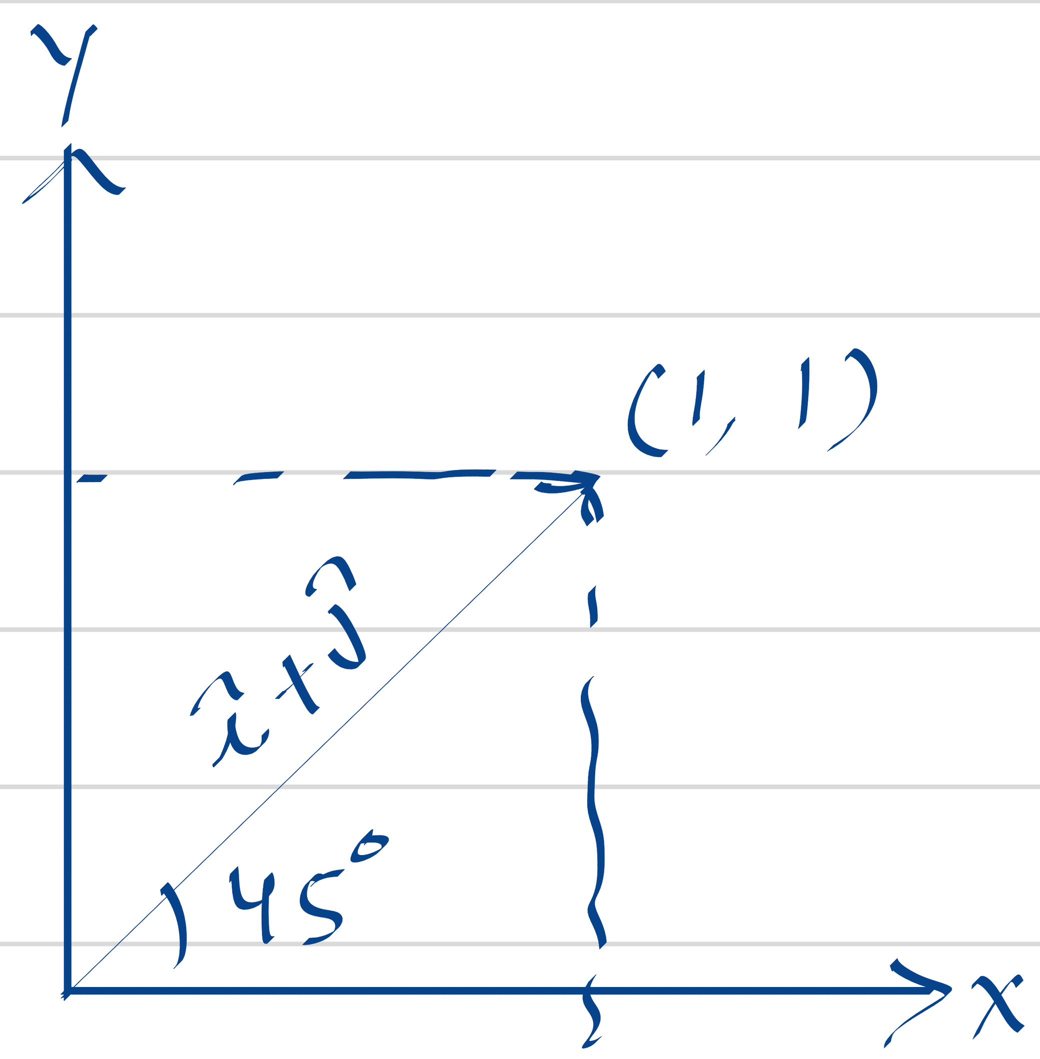
3.19 For $\hat{i} + \hat{j}$

$$|\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2}$$

$$= \sqrt{2} \text{ unit}$$ Ans



Direction

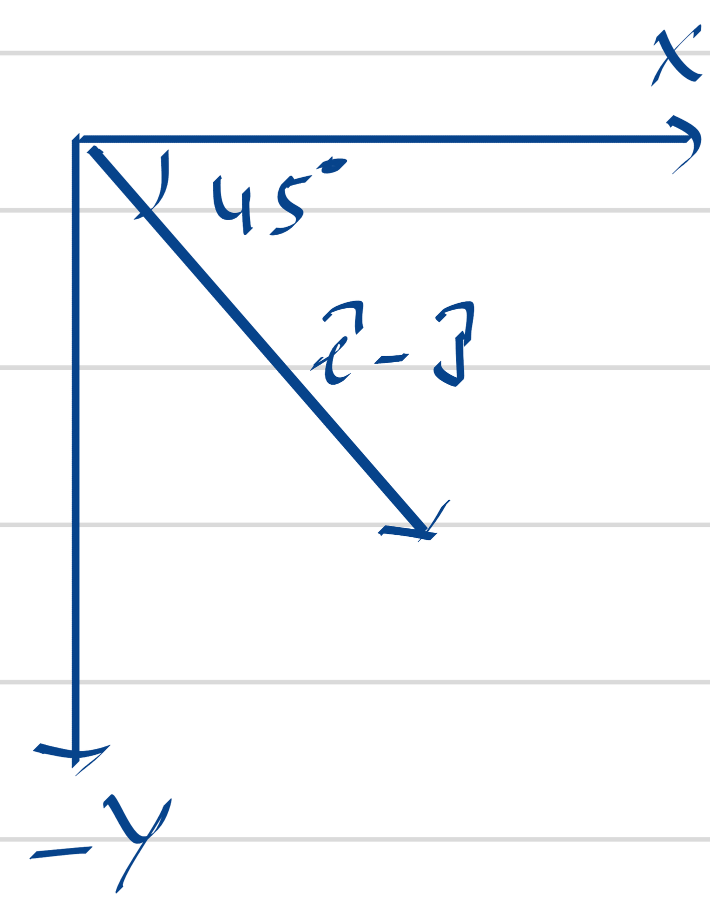
$$\tan\alpha = \frac{1}{1} = 1$$

$$\tan\alpha = \tan 45^\circ$$

$$\alpha = 45^\circ$$ Ans

For $\hat{i} - \hat{j}$

$$|\hat{i} - \hat{j}| = \sqrt{1^2 + (-1)^2} = \sqrt{2} \text{ unit}$$



Direction

$$\tan\alpha = \frac{-1}{1} = -1$$

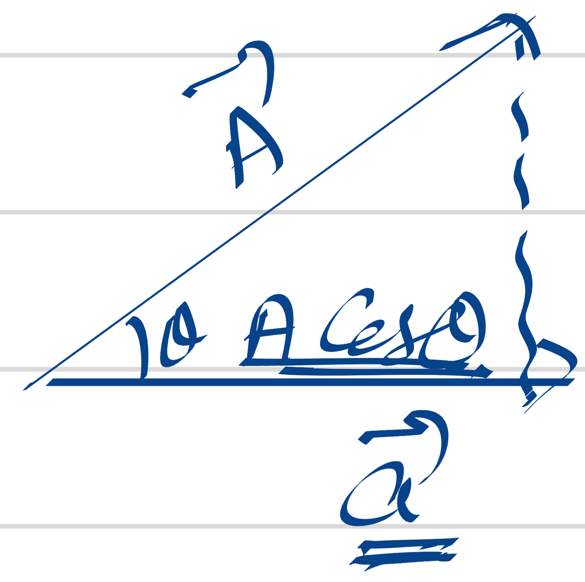
$$\tan\alpha = -1 = -\tan 45^\circ$$

$$\alpha = -45^\circ$$

Given $\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{a} = \hat{i} + \hat{j}$

$\vec{b} = \hat{i} - \hat{j}$



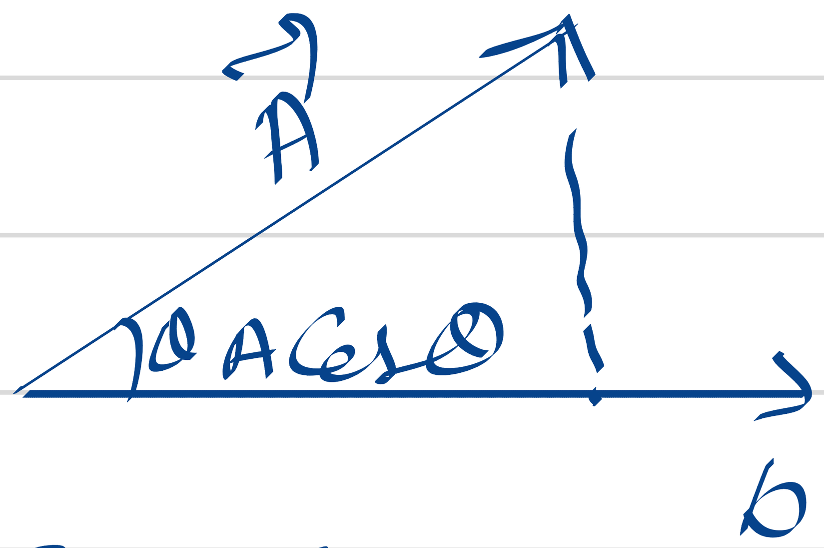
$$\vec{A}\cdot\vec{a} = A\,a\cos\theta$$
$$= \underline{(A\cos\theta)}\,a$$

$$A\cos\theta = \frac{\vec{A}\cdot\vec{a}}{a} = \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}+\hat{j})}{\sqrt{2}}$$
$$= \frac{2+3}{\sqrt{2}}$$

$$A\cos\theta = \frac{5}{\sqrt{2}}$$

i.e component of $\vec{A}$ along the dirn of $\hat{i}+\hat{j} = \frac{5}{\sqrt{2}}$ Ans

Similarly



$$\vec{A}\cdot\vec{b} = A\,b\cos\theta$$
$$\vec{A}\cdot\vec{b} = (A\cos\theta)\,b$$

$$A\cos\theta = \frac{\vec{A}\cdot\vec{b}}{b} = \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}-\hat{j})}{\sqrt{2}}$$

$$A\cos\theta = \frac{2-3}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

i.e the component of $\vec{A}$ along the dirn of $\hat{i}-\hat{j} = \frac{-1}{\sqrt{2}}$ Ans

3.20 Arbitrary motion → Variable acceleration

Only (b) and (e) are true. (a), (c) and (d) are invalid for arbitrary motion.

(True only for uniform acceleration)

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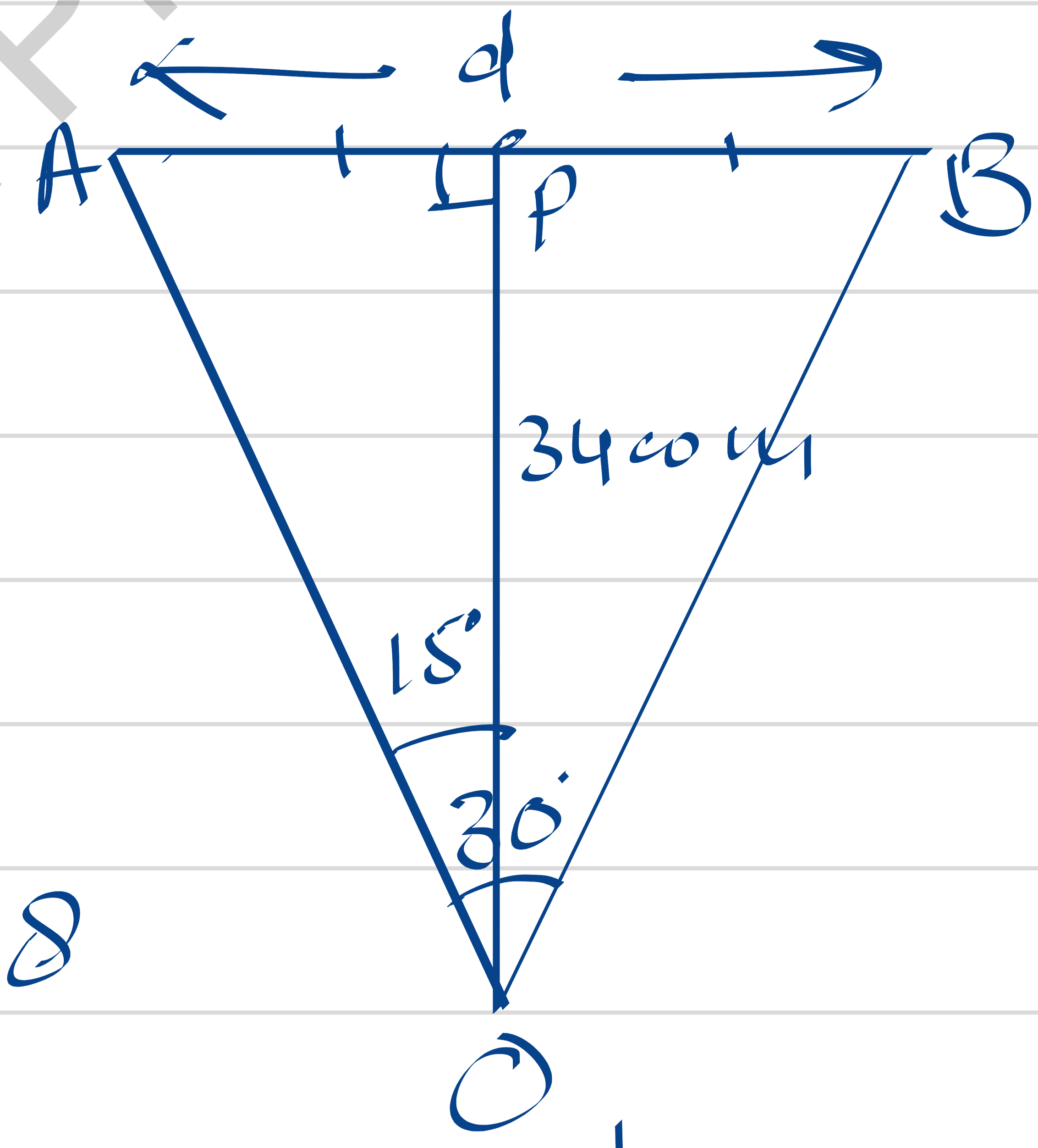
3.21 (a) False. e.g. → K.E is scalar quantity and not conserved in inelastic collisions.
(b) False. e.g. → Temperature can be -ve.
(c) False. e.g. → mass, volume etc. have dimensions.
(d) False. e.g. → Potential varies one point to another.
(e) True. e.g. Temperature in space is same for all observers.

3.22 Given $h = 3400\,m$
$t = 10\,sec$
$d = ?$



In ΔAOP

$$\tan 15 = \frac{AP}{OP}$$

$$AP = OP \tan 15$$
$$= 3400 \times 0.268$$
$$= 34 \times 26.8$$
$$AP = 911.2\ m$$

$$d = 2 \times AP$$
$$= 2 \times 911.2$$
$$d = 1822.4$$

Now $\text{speed} = \frac{d}{t} = \frac{1822.4}{10}$

$\text{speed} = 182.24\ m/s$

Speed of aircraft $= 182.2\ m/s$

Ans

$$\tan 15 = \tan(60-45)$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{1.732-1}{1.732+1}$$
$$= \frac{0.732}{2.732}$$
$$= 0.2679 = 0.268$$

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