Chapter- 3

Motion in a Plane (2D)

Scalars and Vectors

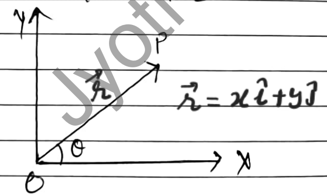
→ The physical quantities which have only magnitudes, no direction are called scalar quantities or scalars.
e.g. distance, speed, work, energy etc.

→ The physical quantities which have both magnitudes and directions are called vector quantities or vectors.
e.g. velocity, force, momentum, torque etc.

Representation of a vectors - By drawing a line arrow over the symbol of a quantity.
e.g. Force → $\vec{F}$
displacement O to A → $\vec{OA}$

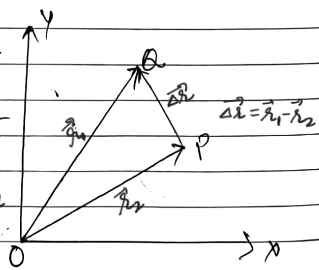
Position and displacement Vectors

Position Vector - $\vec{OP}$ is the position vector. It tells the direction w.r.t. to origin.



$\vec{r} = x\hat{i} + y\hat{j}$

Displacement Vector - $\vec{PQ}$ is the displacement vector.
It is the change in position vector.



$\vec{\Delta r} = \vec{r_1} - \vec{r_2}$

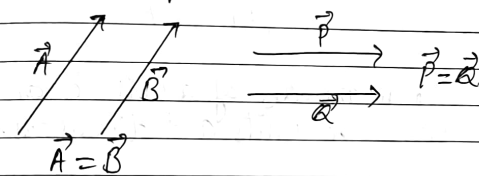
Polar and Axial Vector

Polar → straight vector, e.g. → $\vec{F}$, $\vec{v}$

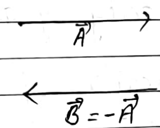
Axial → Rotating vector, e.g. → $\vec{\omega}$, $\vec{\tau}$

Some Definitions.

1. **Equal Vectors** - Vectors having same magnitude and same direction are called equal vectors.



2. **Negative Vector** - A vector having the same magnitude but opposite in direction of a given vector is called negative of that vector.



3. **Collinear Vectors** - Vectors which are along the same line or along parallel lines are called collinear vectors.

✱ For collinear vector θ = 0° (like or ∥ vectors)
θ = 180° (Unlike or anti∥)

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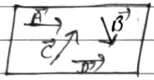
Coplanar Vectors - The vectors which act in the same plane.



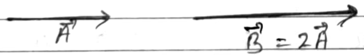
Unit Vector - A vector divided by its magnitude is called unit vector.

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}}$$

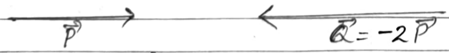
* The magnitude of a unit vector is unity. It has no units and dimensions.

Zero Vector - A vector that has zero magnitude and arbitrary direction is called zero or null vector.
It is represented by $\vec{0}$
e.g. velocity of an object which is at rest.

Multiplication of a Vector by a real number -



Here $\vec{B} = \lambda\vec{A}$ and $\lambda = 2$



here $\vec{Q} = \lambda\vec{P}$ and $\lambda = -2$

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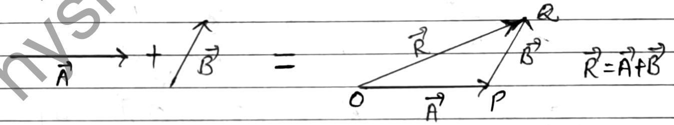
Addition of Vectors

1. **Triangle Law of vector addition (for 2 vectors)**

If two vectors are represented as two sides of the triangle with the order of magnitude and direction then the third side of the triangle represents the resultant vector.

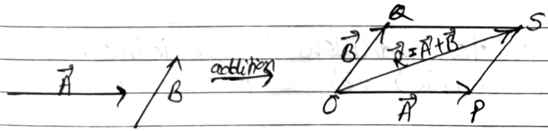


According to triangle law of vector addition

$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$

2. **Parallelogram Law of Vector Addition** (for two vectors)

If two vectors are represented by two adjacent sides of a parallelogram then the diagonal of the parallelogram passing through the common point of the vectors represents the resultant vector.



OR

If two vectors $\vec{A}$ and $\vec{B}$ represent two sides of a parallelogram, then their sum $\vec{A} + \vec{B}$ = the diagonal of the parallelogram through their common point.

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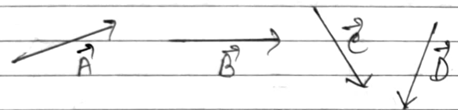
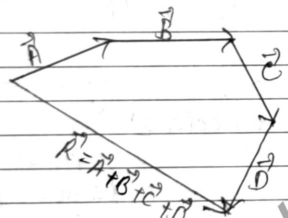
Polygon Law of Vector addition -
(For a number of vectors)

If a number of vectors represented by the sides of an open polygon taken in the same order, then their resultant is represented by the closing side of the polygon taken in opposite direction.



addition of $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ by polygon law -



* Vector addition is commutative.
i.e $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

* Vector addition is associative.
i.e. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

* By applying triangle law of vector addition to different triangles of the polygon, polygon law can be verified.

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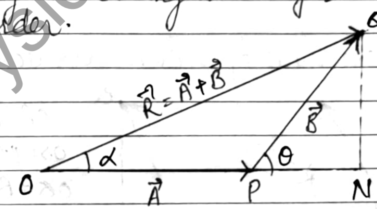
Analytical Method of Vector Addition

Triangle Law of Vector addition -

Let the two vectors $\vec{A}$ and $\vec{B}$ represent the two sides of a triangle. According to the triangle law, the resultant $\vec{R}$ is given by the closing side of the triangle in reverse order.



$\vec{OP} = \vec{A}$
$\vec{PQ} = \vec{B}$
$\vec{OQ} = \vec{R}$
$PN = B\cos\theta$
$QN = B\sin\theta$

In ΔPQN
$\sin\theta = \frac{QN}{PQ} = \frac{QN}{B}$
and
$\cos\theta = \frac{PN}{B}$

Magnitude of $\vec{R}$:

From right angled ΔQNO

$$OQ^2 = ON^2 + QN^2$$

$$R^2 = (OP + PN)^2 + QN^2$$

or $R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

or $R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

or $R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

or $R^2 = \underline{A^2 + B^2 + 2AB\cos\theta}$ $\quad \because \cos^2\theta + \sin^2\theta = 1$

or $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

So magnitude of resultant vector $\vec{R}$

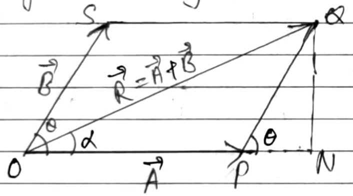
$$\boxed{R = \sqrt{(A^2 + B^2 + 2AB\cos\theta)}}$$

Direction of the resultant $\vec{R}$

$$\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP+PN}$$

or $$\boxed{\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}}$$

Parallelogram law of Vector addition



$\vec{OP} = \vec{A}$
$OS = PQ = B$
$OQ = R$
$PN = B\cos\theta$
$QN = B\sin\theta$

Let two vectors $\vec{A}$ and $\vec{B}$ are representing the two adjacent sides of a parallelogram. According to the law of $\|^{gm}$ of vector addition the diagonal $\vec{R}$ will be the resultant vector.

In right angled $\triangle QNO$

$$OQ^2 = ON^2 + QN^2$$
$$OQ^2 = (OP+PN)^2 + QN^2$$
$$R^2 = (A+B\cos\theta)^2 + (B\sin\theta)^2$$
or $$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$
or $$R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$
or $$R^2 = A^2 + B^2 + 2AB\cos\theta \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

or $$\boxed{R = \sqrt{A^2+B^2+2AB\cos\theta}}$$

Direction of $\vec{R}$

$$\tan\alpha = \frac{QN}{ON} = \frac{QN}{OP+PN}$$

or $$\boxed{\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}}$$

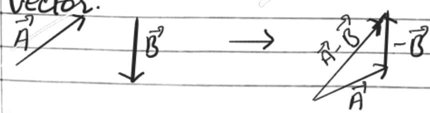
Special cases

(i) If $\theta = 0^\circ$, then
$$R^2 = A^2 + B^2 + 2AB\cos 0^\circ$$
or $$R^2 = A^2+B^2+2AB \qquad [\cos 0^\circ = 1]$$
or $$R^2 = (A+B)^2$$
or $$R = A+B \text{ and } \alpha = 0 \qquad [\because \tan 0^\circ = 0]$$

(ii) If $\theta = 180^\circ$
$$R = A - B \qquad [\because \cos 180^\circ = -1]$$
and dir^n will be along bigger vector.

(iii) If $\theta = 90^\circ$
$$R = \sqrt{A^2+B^2} \qquad [\because \cos 90^\circ = 0]$$
and the dir^n, $\tan\alpha = \dfrac{B}{A}$

Subtraction of vectors: It is simply the addition of a vector with negative of another vector.



$\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

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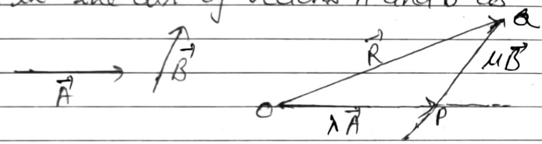
<u>Resolution of a vector</u> - The splitting of a vector into two or more vectors in such a way that their combined effect is same as that of the given vector is called resolution of a vector.
The splitted vectors are called component vectors.

e.g Suppose we want to resolve a vector $\vec{R}$ in the dir^n of vectors $\vec{A}$ and $\vec{B}$ as



from triangle law of vector addition

$$\vec{R} = \vec{OP} + \vec{PQ}$$

or $$\vec{R} = \lambda\vec{A} + \mu\vec{B}$$

* There is only one way in which a vector $\vec{R}$ can be resolved in the dir^n of $\vec{A}$ and $\vec{B}$.

<u>Rectangular Components of a Vector</u> - When a vector is resolved along two mutually perpendicular directions, the components so obtained is called rectangular components of the given vector.

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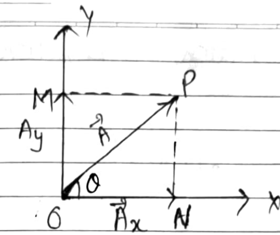

By $\|^{gm}$ law

$$\vec{OP} = \vec{OM} + \vec{ON}$$

or $$\vec{A} = \vec{A}_x + \vec{A}_y$$

Let $\hat{i}$ and $\hat{j}$ are the unit vector along x-axis and Y-axis, then

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j}} \quad \text{—— (1)}$$

If vector $\vec{A}$ makes angle θ with x-axis, then

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

so we have

$$A = \sqrt{A_x^2 + A_y^2}$$

and $$\tan\theta = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

From eq^n (1) we can find magnitude of $\vec{A}$ and dir^n of $\vec{A}$ with x-axis.

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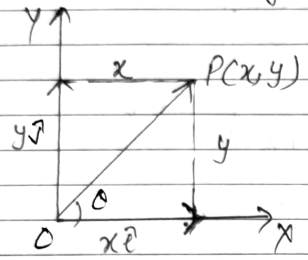
Rectangular resolution of position vector



$$\boxed{\vec{r} = x\hat{i} + y\hat{j}}$$

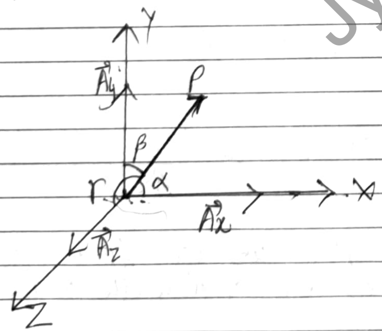
If $\vec{r}$ makes angle θ with x-axis, then

$x = r\cos\theta$, $y = r\sin\theta$

and $\boxed{r = \sqrt{x^2 + y^2}}$

also $\tan\theta = \dfrac{y}{x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\dfrac{y}{x}\right)}$

* In three dimensions



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$\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A_x = A\cos\alpha$, $A_y = A\cos\beta$ and $A_z = A\cos\gamma$

then, $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

* A Position vector $\vec{r}$ in 3 dim^n is given by

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

<u>Scalar Product of two vectors</u>: The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ defined as product of magnitude of $\vec{A}$ and $\vec{B}$ and cosine of angle θ between them.

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$

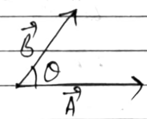
A → Magnitude of $\vec{A}$
B → " " $\vec{B}$

OR

The dot product of two vectors $\vec{A}$ and $\vec{B}$ denoted by $\vec{A} \cdot \vec{B}$ and

$\vec{A} \cdot \vec{B} = AB\cos\theta$

where θ is the angle between $\vec{A}$ and $\vec{B}$.



The dot product of $\vec{A}$ and $\vec{B}$ is scalar quantity.

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also,
$\vec{A}\cdot\vec{B} = B(A\cos\theta)$
$= B \times$ mag. of component $\vec{A}$ in the direction of $\vec{B}$.

e.g. (i) $W = \vec{F}\cdot\vec{S}$ W → Work, $\vec{F}$ → force
$\vec{S}$ → displacement
(ii) $P = \vec{F}\cdot\vec{V}$ P → Power, V → velocity

Properties:
(i) The scalar product is commutative. i.e.
$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
(ii) It is distributive over addition. i.e.
$A\cdot(B+C) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
(iii) If two vectors are ⊥, then
$\vec{A}\cdot\vec{B} = AB\cos 90$
or $\vec{A}\cdot\vec{B} = 0$
(iv) If two vectors are ∥
$\vec{A}\cdot\vec{B} = AB\cos 0°$
or $\vec{A}\cdot\vec{B} = AB$
(v) If two vectors are antiparallel
$\vec{A}\cdot\vec{B} = AB\cos 180°$
or $\vec{A}\cdot\vec{B} = -AB$
(vi) $\vec{A}\cdot\vec{A} = A^2$ [$\because \theta = 0°$]
(vii) $\hat{i}\cdot\hat{i} = 1$ [$\because \theta = 0°$]
and $\hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

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(viii) $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

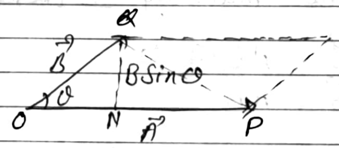
Vector Product or cross product of two vectors:

Mathematically, cross product of two vectors $\vec{A}$ and $\vec{B}$ is given by

$$\boxed{\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}}$$

where θ is the angle between $\vec{A}$ and $\vec{B}$ vectors.
and $\hat{n}$ is unit vector perpendicular to the plane of $\vec{A}\times\vec{B}$ and its dir^n is given by right hand rule.
Hence dir^n of $\vec{A}\times\vec{B}$ is same of $\hat{n}$.

Also



$\vec{A}\times\vec{B} = AB\sin\theta$
$=$ Area of ∥^{gm} OPRQ
or $= \frac{1}{2}\times$ Area of POQ

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e.g. (i) Torque $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) $\vec{V} = \vec{\omega} \times \vec{r}$

V → linear velocity
ω → angular "
r → radius

Properties:

(i) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

i.e anti commutative

(ii) distributive over addition, i.e

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(iii) vector product of ∥ and anti ∥ vectors is null vector. i.e.

$\vec{A} \times \vec{B} = AB \sin 0°$

or $\vec{A} \times \vec{B} = 0$

and $\vec{A} \times \vec{B} = AB \sin 180$

or $\vec{A} \times \vec{B} = 0$

(iv) For two mutually ⊥ vectors

$\vec{A} \times \vec{B} = AB \sin 90°$

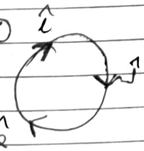
or $\vec{A} \times \vec{B} = AB$

(v) $\vec{A} \cdot \vec{A} = 0$ $\boxed{\theta = 0°}$

(vi) $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{j} = -\hat{i}$, $\hat{i} \times \hat{k} = -\hat{j}$

(vii) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$



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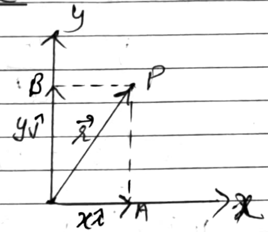
Motion in a plane

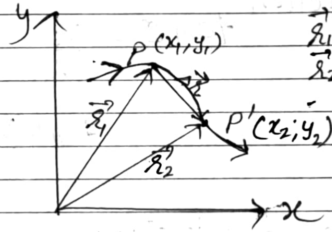
Position vector



$\vec{r} = x\hat{i} + y\hat{j}$

This equation represents position vector $\vec{r}$ in terms of its rectangular components x and y.

Displacement Vector



$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$

Here displacement vector

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$= x_2\hat{i} + y_2\hat{j} - (x_1\hat{i} + y_1\hat{j})$

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\boxed{\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}}$

Average velocity

$V_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$

or $\boxed{V_{av} = \bar{V}_x\hat{i} + \bar{V}_y\hat{j}}$

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Instantaneous Velocity

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

or $$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j}}$$

magnitude of velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

Direction

$$\tan\theta = \frac{v_y}{v_x}$$

or $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Average Acceleration

$$a_{av} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

or $$\boxed{\bar{a} = \bar{a}_x\hat{i} + \bar{a}_y\hat{j}}$$

Instantaneous Acceleration

$$a = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

or $$a = a_x\hat{i} + a_y\hat{j}$$

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Here $a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

and $a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d^2y}{dt^2}$

Motion in a plane with constant Acceleration (Uniformly accelerated motion)

→ Velocity - time equation

$$v = u + at$$

In rectangular components,

$$v_x = u_x + a_x t$$

and $$v_y = u_y + a_y t$$

→ Displacement - time Equation

$$S = ut + \frac{1}{2}at^2$$

In rectangular components

$$x = x_0 + u_x t + \frac{1}{2}a_x t^2$$

x_0 → displacement at $t=0$

and $$y = y_0 + u_y + \frac{1}{2}a_y t^2$$

→ Displacement velocity Equation

$$v^2 = u^2 + 2aS$$

$$v_x^2 = u_x^2 + 2a_x x, \quad v_y^2 = u_y^2 + 2a_y y$$

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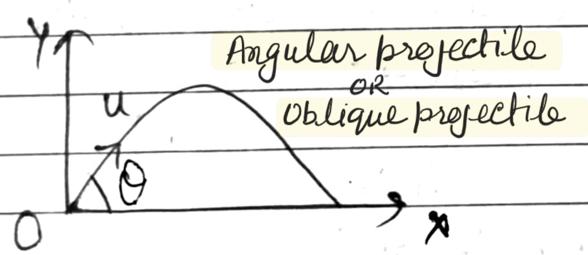
Projectile Motion - When a body moves in a plane under the effect of gravity only, called projectile and the motion of the body is called projectile motion.

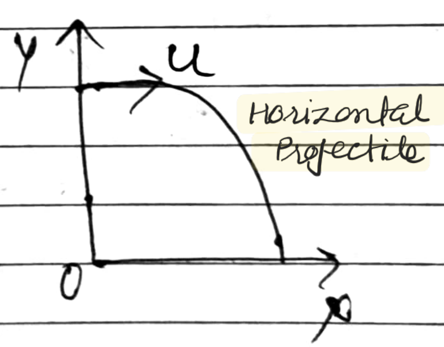
e.g. (I) An arrow released from bow.
(II) A bullet fired from a gun.
(III) A javelin throw
(IV) A jet of water coming out from a hole of a vessle.

Projectile

↙ Projectile fired at an angle from horizontal

↘ Horizontal projectile

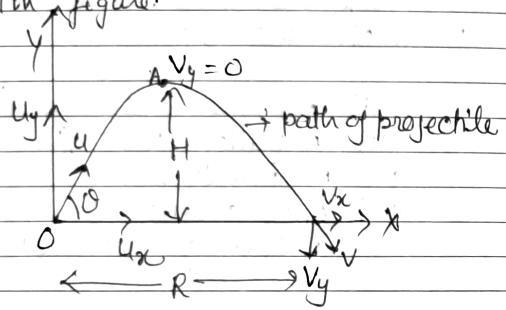




* In projectile motion we assume that there is no air resistance.
* Acceleration due to gravity (g) is taken constant at all the points of path of projectile.

Projectile fired at an angle θ with the horizontal.

Let a projectile is fired at an angle θ as shown in figure.



Here $u_x = u\cos\theta$ $\quad a_x = 0 \quad V_y = 0$ [at A]

$u_y = u\sin\theta \quad a_y = -g$

Motion along x axis.

By $x = x_0 + u_x t + \frac{1}{2} a_x t^2$

$x_0 = 0$, $u_x = u\cos\theta$, $a_x = 0$, then

$x = u\cos\theta \cdot t + 0$

or $t = \frac{x}{u\cos\theta}$ — (1)

Motion along Y axis

$y = y_0 + u_y t + \frac{1}{2} a_y t^2$

$y_0 = 0$, $u_y = u\sin\theta$, $a_y = -g$, $t = \frac{x}{u\cos\theta}$ [from (1)]

then

$y = 0 + u\sin\theta \cdot \frac{x}{u\cos\theta} + \frac{1}{2}(g)\left(\frac{x}{u\cos\theta}\right)^2$

or $y = \tan\theta \cdot x - \frac{g}{2u^2\cos^2\theta} x^2$

Which is an equation of parabola. Therefore path (trajectory) of the projectile is parabolic.

Time of flight: (T) →

By $V_y = u_y + a_y t$

$V_y = 0$, $u_y = u\sin\theta$ $\quad a_y = -g$

then $0 = u\sin\theta - gt$

or $t = \frac{u\sin\theta}{g}$

this is the time from O to A. So the total time

$T = 2t$

or $\boxed{T = \frac{2u\sin\theta}{g}}$

Maximum Height (H)

By $V_y^2 = u_y^2 + 2a_y S$

$V_y = 0$, $u_y = u\sin\theta$, $a_y = -g$, $S = H$

$0 = (u\sin\theta)^2 - 2gH$

or $\boxed{H = \frac{u^2\sin^2\theta}{2g}}$

Horizontal Range (R)

horizontal range = horizontal velocity × Time of flight

$$R = u_x \times T$$

or $$R = u\cos\theta \times \frac{2u\sin\theta}{g}$$

or $$R = \frac{u^2(2\sin\theta\cos\theta)}{g}$$

or $$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$ $[\because 2\sin\theta\cos\theta = \sin 2\theta]$

condition for maximum height (H)

We have

$$H = \frac{u^2\sin^2\theta}{2g}$$

clearly H will be max^m when

$$\sin^2\theta = 1 = \sin 90^\circ$$

or $$\sin\theta = \sin 90^\circ$$

i.e $\boxed{\theta = 90^\circ}$ $\Rightarrow$ $H = \frac{u^2}{2g}$

Thus H will be max when projectile is projected at $\theta = 90^\circ$ (vertically upward).

condition for maximum Range (R)

We have

$$R = \frac{u^2\sin 2\theta}{g}$$

It is clear for R_{max}

$$\sin 2\theta = 1 = \sin 90^\circ$$

or $2\theta = 90^\circ$

or $\theta = 45^\circ$

Thus the horizontal range of a projectile is maximum when it is projected at an angle of 45° with the horizontal.

$$R_{max} = \frac{u^2}{g}$$

Two Angles of Projection for the same Horizontal Range:

(I) $R_\theta = R_{(90-\theta)}$

(II) $R_{45+\theta} = R_{45-\theta}$

(I) $R_\theta = R_{(90-\theta)}$

By $R = \frac{u^2 \sin 2\theta}{g}$

L.H.S

$$R_\theta = \frac{u^2 \sin 2\theta}{g} \quad -(1)$$

R.H.S

$$R_{(90-\theta)} = \frac{u^2 \sin 2(90-\theta)}{g}$$

$$= \frac{u^2 \sin(180-2\theta)}{g}$$

or $R_{(90-\theta)} = \frac{u^2 \sin 2\theta}{g}$ (2) [$\because \sin(180-\theta) = \sin\theta$]

From (1) and (2)

L.H.S = R.H.S

or $R_\theta = R_{(90-\theta)}$

(II) $R_{45+\theta} = R_{45-\theta}$

L.H.S $R_{45+\theta} = \frac{u^2 \sin 2(45+\theta)}{g}$

$$= \frac{u^2 \sin(90+2\theta)}{g}$$

$$= \frac{u^2 \cos 2\theta}{g} \quad -(1)$$

R.H.S

$$R_{45-\theta} = \frac{u^2 \sin 2(45-\theta)}{g}$$

$$= \frac{u^2 \sin(90-2\theta)}{g}$$

$$= \frac{u^2 \cos 2\theta}{g} \quad -(2)$$

From (1) and (2)

L.H.S = R.H.S

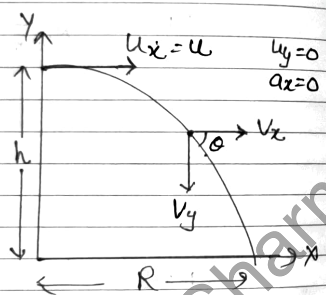
i.e. $R_{45+\theta} = R_{45-\theta}$

* $R \tan\theta = 4H$ is very important relation b/w horizontal range and maximum height.

Horizontal Projectile

When a body is projected horizontally from a certain height above the ground then it is called horizontal projectile.

Trajectory of the projectile -



Motion along x-axis

$x = x_0 + u_x t + \frac{1}{2} a_x t^2$

here $x_0 = 0$, $u_x = u$, $a_x = 0$

then

$x = ut$

or $t = \frac{x}{u}$ —— (1)

Motion along y axis

$y = y_0 + u_y t + \frac{1}{2} a_y t^2$

here $y_0 = 0$, $u_y = 0$, $a_y = g$

then

$y = \frac{1}{2} g t^2$

put the value of t from eq^n (1)

$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2$

or $y = \left(\frac{g}{2u^2}\right) x^2$

or $y = kx^2$, where $k = \frac{g}{2u^2}$ = const

Which is a equation of parabola.
Therefore path of projectile is parabolic.

Time of flight :

by $s = ut + \frac{1}{2} a t^2 \Rightarrow y = u_y t + \frac{1}{2} a_y t^2$

here $y = s = h$, $u_y = 0$ and $a_y = g$, then

$h = \frac{1}{2} g t^2$

or $t^2 = \frac{2h}{g}$ or $\boxed{t = \sqrt{\frac{2h}{g}}}$

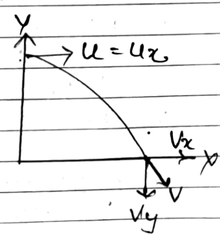
Horizontal Range :

$R = u_x \times t$ [$\because a_x = 0$]

or $\boxed{R = u\sqrt{\frac{2h}{g}}}$

Velocity of the projectile at any instant

Resultant velocity at any point



$V = \sqrt{V_x^2 + V_y^2}$

here $V_x = u_x = u$

and $V_y = u_y + a_y t$

so $V_y = 0 + gt = gt$

then $V = \sqrt{u^2 + (gt)^2}$

or $\boxed{V = \sqrt{u^2 + g^2t^2}}$

direction of the resultant velocity

$$\tan\theta = \frac{V_y}{V_x} = \frac{gt}{u}$$

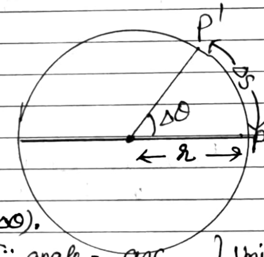
or $\boxed{\theta = \tan^{-1}\left(\frac{gt}{u}\right)}$

* In projectile motion, the horizontal and vertical motion are independent of each other.
* Horizontal range R is max. for $\theta = 45°$.
* Horizontal range R is same for (i) θ and $(90-\theta)$ (ii) $45° + \theta$ and $45° - \theta$.
* At highest of the parabolic path the velocity and acceleration are perpendicular to each other.
* Kinetic energy is max. at initial point and final point (reaching the ground) and min^m at at highest point.
* for $\theta = 45°$ $H_{max} = \frac{R_{max}}{4} = \frac{u^2}{4g}$
* A highest point there is only horizontal velocity $V_x (= u_x)$, no vertical velocity. ($V_y = 0$).

Uniform Circular Motion (UCM)

When a particle moves along circular path with constant speed, then its motion is said to be a uniform circular motion.
e.g. Motion of needle of a clock, motion of a fan, wheel etc.
UCM is an accelerated motion because the velocity changes at every point due to the change in direction.

Some Related Terms -



(i) Angular displacement ($\Delta\theta$)
The angle swept out by the radius vector in the given time interval is called angular displacement ($\Delta\theta$).

$$\Delta\theta = \frac{\Delta s}{r} \quad \left[\because \text{angle} = \frac{\text{arc}}{\text{radius}}\right]$$ Unit → Rad.

(ii) Angular Velocity (ω)
The rate of change of angular displacement is called angular velocity of the object

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity (ω)

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

or $$\omega = \frac{d\theta}{dt}$$

Unit - Rad./s

Time Period(T): The time taken to complete the one revolution is called time period of the object.

$$T = \frac{1}{n}$$

where n is frequency. Unit - sec.

Frequency(n): It is the number of revolutions completed per unit time.
It is also denoted by ν (nu)

$$n = \frac{1}{T}$$ $T \rightarrow$ Time period

Unit - rev./sec

Relation between ω, n and T

we know

$$\omega = \frac{\Delta\theta}{\Delta t}$$

for 1 revolution $\Delta\theta = 2\pi$
and $\Delta t = T$ (time period)

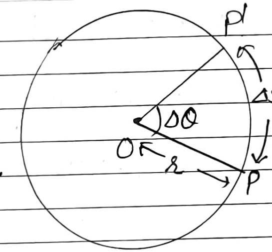
so,

$$\omega = \frac{2\pi}{T} = 2\pi n \quad \left[\because T = \frac{1}{n}\right]$$

Relation between linear Velocity and Angular velocity:

Consider a particle moving along a circular path of radius r as shown in fig.

The angular displacement



$$\Delta\theta = \frac{\Delta S}{r}$$

dividing both sides by Δt, we get

$$\frac{\Delta\theta}{\Delta t} = \left(\frac{\Delta S}{\Delta t}\right) \cdot \frac{1}{r}$$

or

$$\omega = \frac{v}{r} \quad \left[\because \frac{\Delta\theta}{\Delta t} = \omega \text{ and } \frac{\Delta S}{\Delta t} = v\right]$$

or $\boxed{v = r\omega}$

i.e Linear velocity = Angular velocity x radius

vector for $\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$

* $v \propto r$ as ω is constant.
* Direction of v is given by right hand thumb rule.

Angular Acceleration (α):
The rate of change of angular velocity is called angular acceleration.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration

$$\alpha = \frac{d\omega}{dt}$$

Relation between linear acceleration and angular acceleration:

We have

$$v = r\omega$$

On differentiating both sides w.r.to time t

$$\frac{dv}{dt} = r\frac{d\omega}{dt}$$

or $\boxed{a = r\alpha}$ $\left[\because \frac{dv}{dt} = a,\ \frac{d\omega}{dt} = \alpha\right]$

Linear acceleration = angular acceleration × radius

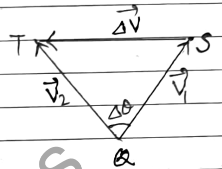
and vector form -

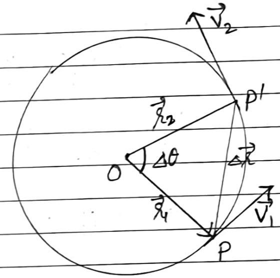
$$\boxed{\vec{a} = \vec{\alpha} \times \vec{r}}$$

Centripetal Acceleration: In uniform circular motion the acceleration directed along radius is called centripetal acceleration.

Expression for Centripetal Acceleration

Consider a object moving on circular path of radius r and uniform speed v as shown in fig. Let initially object is at point P and after Δt time it reaches at P'. From P to P' the velocity changes v_1 to v_2.
The direction of velocities are tangential at P and P'.





Here $|\vec{r_1}| = |\vec{r_2}| = r$

and $|\vec{v_1}| = |\vec{v_2}| = v$

In $\Delta OPP'$ and ΔQST, we have

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \qquad [\because \Delta OPP' \sim \Delta QST]$$

dividing both sides by Δt, we get

$$\frac{\Delta v}{\Delta t}\cdot\frac{1}{v} = \frac{\Delta r}{\Delta t}\cdot\frac{1}{r}$$

$$\frac{a}{v} = \frac{v}{r} \qquad \left[\because \frac{\Delta v}{\Delta t} = a \text{ and } \frac{\Delta r}{\Delta t} = v\right]$$

or $\boxed{a = \frac{v^2}{r}}$

put $v = r\omega$, then

$$a = \frac{(r\omega)^2}{r}$$

or $\boxed{a = r\omega^2}$

* The direction of centripetal acc^n is changing continuously but the magnitude remains constant.
Its direction is always towards centre of the ~~circular~~ circle.

Centripetal Force : A force that acts on a body moving in a circular path and is directed towards the centre of the circle.

By $F = ma$

put $a = \frac{v^2}{r}$

we get $\boxed{F = \frac{mv^2}{r}}$ unit - Newton

Direction of centripetal force is towards centre of the circle, same as centripetal acceleration

e.g. (i) Turning a car - Here centripetal force is provided by friction force.
(ii) Planets orbiting around the sun - Here the centripetal force is provided by gravity
(iii) Orbiting of an electron - Here the centripetal force is provided by electrostatic force.

Centrifugal Force : If an object moving in a circle and experiences an outward force then the force is called centrifugal force.
* It is a fictitious force.
* This force arises from the body's inertia and appears to act on a body away from the centre.

Mathematically
centrifugal force $F = -\frac{mv^2}{r} = mr\omega^2$

-ve shows that direction of centrifugal force is opposite to centripetal force.

e.g. (i) When a car takes turn, the passengers experience an apparent force in opposite direction.
(ii) Passengers feeling pushed outward on a merry-go-round.

Important points

→ In horizontal projectile

- Initial vertical velocity $u_y = 0$
- Horizontal velocity is constant throughout the motion because no horizontal force acts.

- In vertical direction motion is – free fall.
- Time of flight depends only on height.
- Vertical velocity during the fall increases due to gravity.

→ In angular projectile

- Horizontal velocity is constant because no force is in horizontal direction.
- At highest point vertical velocity is zero.
- At highest point acceleration is not zero it is g always acts downward. ($g = 9.8\ m/s^2$)
- Time of ascent = Time of descent (In ideal condition) It is symmetric about the highest point.
- At highest point total velocity is minimum (only horizontal velocity remains)

→ Uniform circular motion

- In UCM speed is constant but velocity changes due to change in direction.
- UCM is an accelerated motion ⇒ centripetal acceleration
- In UCM acceleration is toward the centre of circle always (centripetal acceleration)
- Direction of velocity in UCM is tangential.
- In UCM an object cannot be in equilibrium because there is net acceleration towards center.