LAWS OF MOTION Chapter - 4

Formula Sheet

Momentum (p)

$p = mv$ vector quantity

or $\vec{p} = m\vec{v}$ unit $kg\,m/s$ $[MLT^{-1}]$

Newton's 1st law of motion:

If $F_{ext} = 0$, $v = u$

Newton's 2nd law of motion

$$F = \frac{\Delta p}{\Delta t} = ma$$

$m \rightarrow$ mass

$a \rightarrow$ acceleration

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

or $F = m\frac{dv}{dt}$

* If v is constant and m changes

$$F = v\frac{dm}{dt}$$

Newton's 3rd law of motion

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Action and reaction do not cancel each other because they act on different bodies.

$F_{AB} \rightarrow$ Force on object A by object B

$F_{BA} \rightarrow$ Force on object B by object A

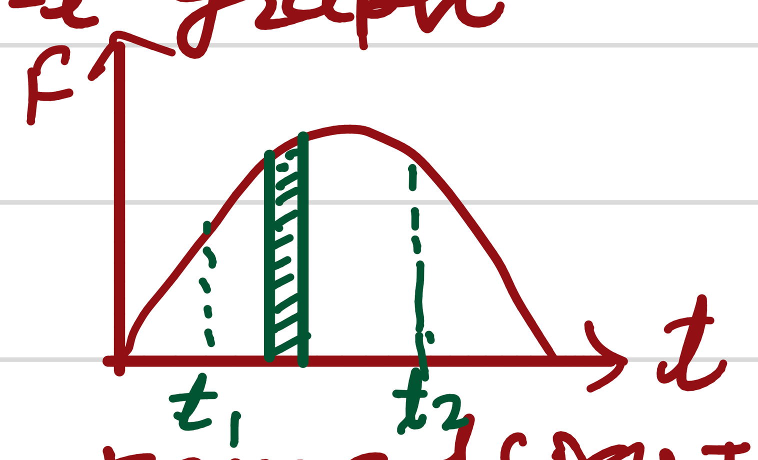
Impulse (I)

$I = F\Delta t = \Delta p$ $[\Delta p = p_2 - p_1]$

Impulse = Force x time duration

When force is changing continuously

$I = \int_{t_1}^{t_2} F(t)\,dt$ = Area under the F-t graph



When F is not constant, $I = F_{av} \times t = \Delta p$

* Impulse is vector quantity *SI unit $kg\,m\,s^{-1}$ $[MLT^{-1}]$

Apparent weight of a body in a lift

(I) Lift moves upward with uniform acceleration

$$R - mg = ma$$

or $\quad R = m(g+a) \quad$ [App. weight > Actual weight]

(II) Lift moves downwards with uniform acceleration

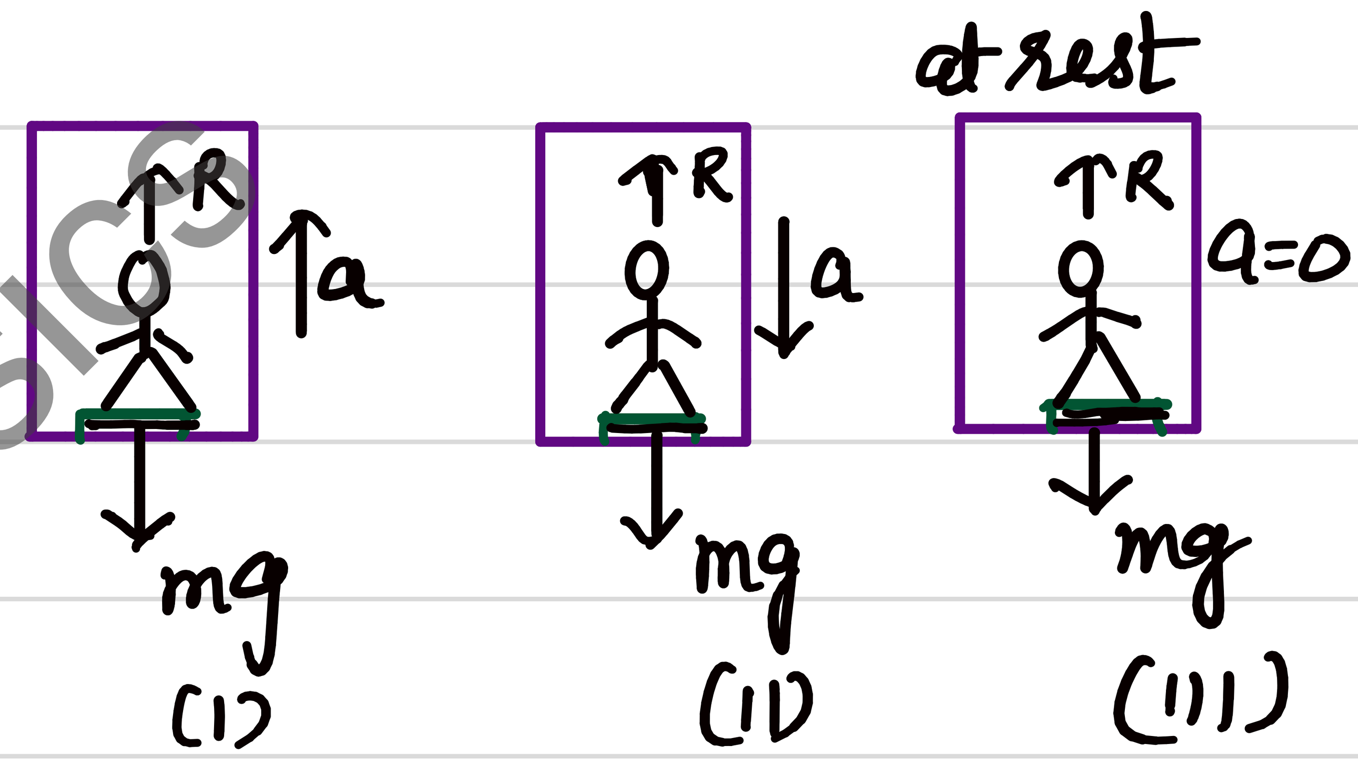
$$mg - R = ma$$

or $\quad R = m(g-a) \quad$ [App. weight < Actual weight]

(III) Lift is at rest or in uniform motion, then

$$a = 0$$

i.e $\quad R = mg$



(IV) When lift is in free fall

$$a = g$$

i.e $\quad R = m(g-a)$

$\quad R =$ (Weightlessness)

Law of conservation of linear momentum

(I) Sum of linear momentum of a system of a particles remain constant

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots \vec{p}_n = \text{Constant}$$

or $\quad m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots m_n\vec{v}_n = \text{Constant}$

(II) When two bodies

$$m_1u_1 + m_2u_2 = m_1u_1 + m_2v_2$$

(III) When a bullet of mass m is fired with velocity v from a gun of mass M, the gun recoils with velocity V

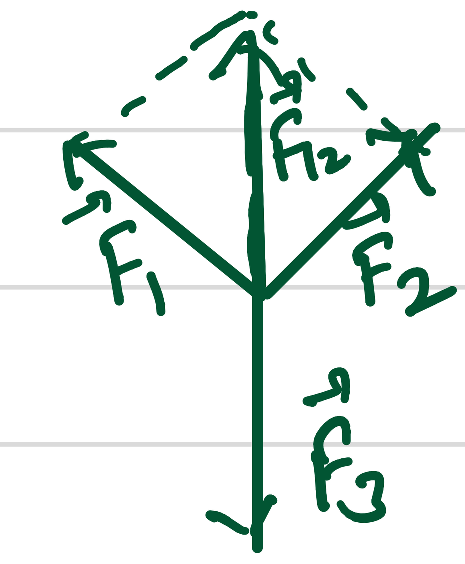
Momentum of gun = −Momentum of bullet

$$MV = -mv$$

Recoil velocity of gun

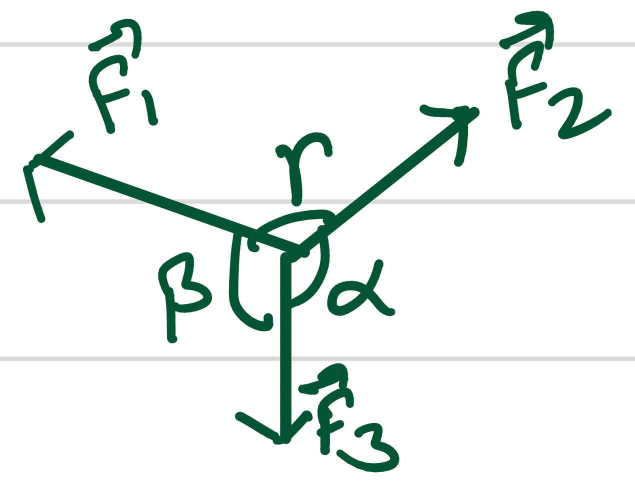
$$V = -\frac{mv}{M}$$

Equilibrium of concurrent forces



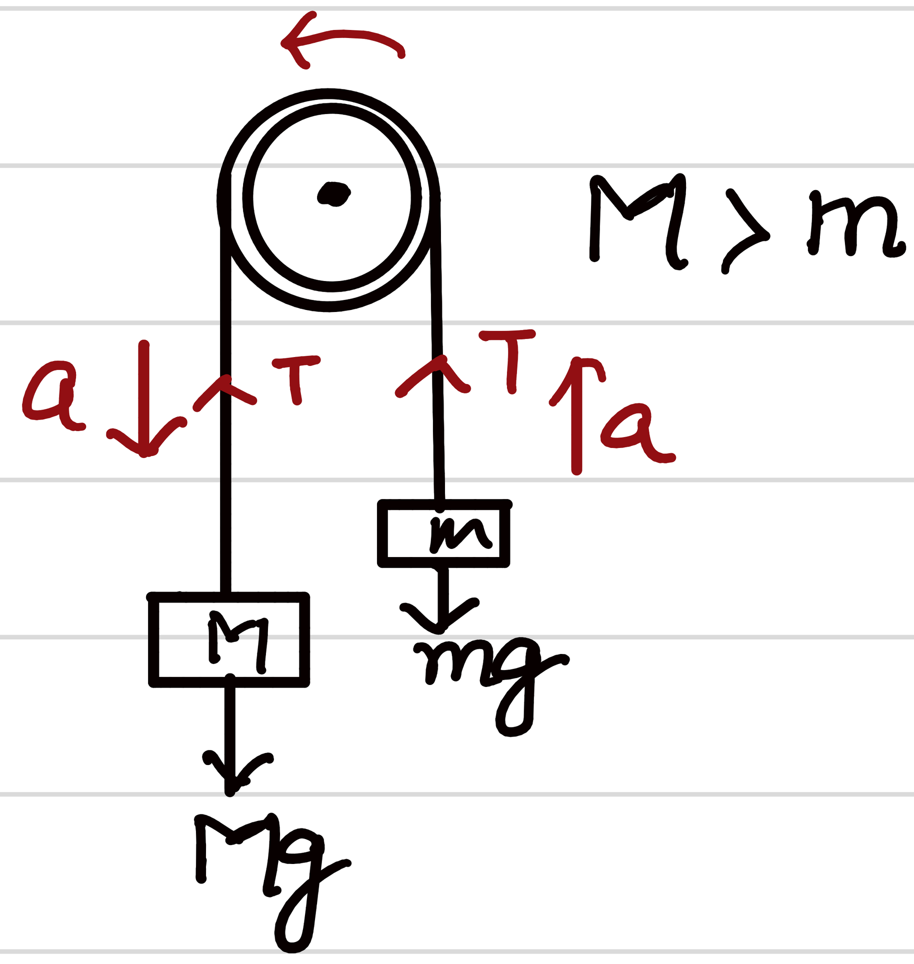
$$\vec{F}_1+\vec{F}_2+\vec{F}_3+ - - - \vec{F}_n = 0$$

Lami's theorem



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Connected Motion



Acceleration of the masses

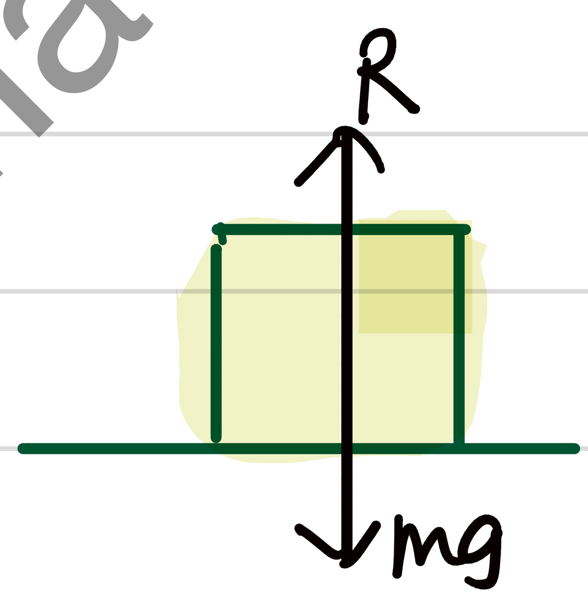
$$a = \frac{M-m}{M+m}g$$

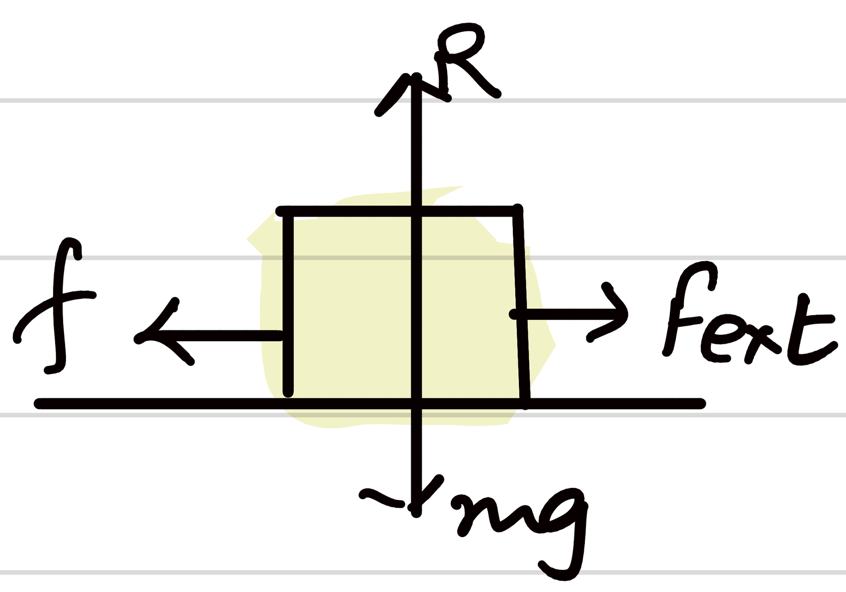
Tension in the string

$$T = \frac{2Mm}{M+m}g$$

clearly $a<g$

Friction





$$f \propto R$$

$$f = \mu R$$

$\mu \rightarrow$ coefficient of friction

Coefficient of limiting friction

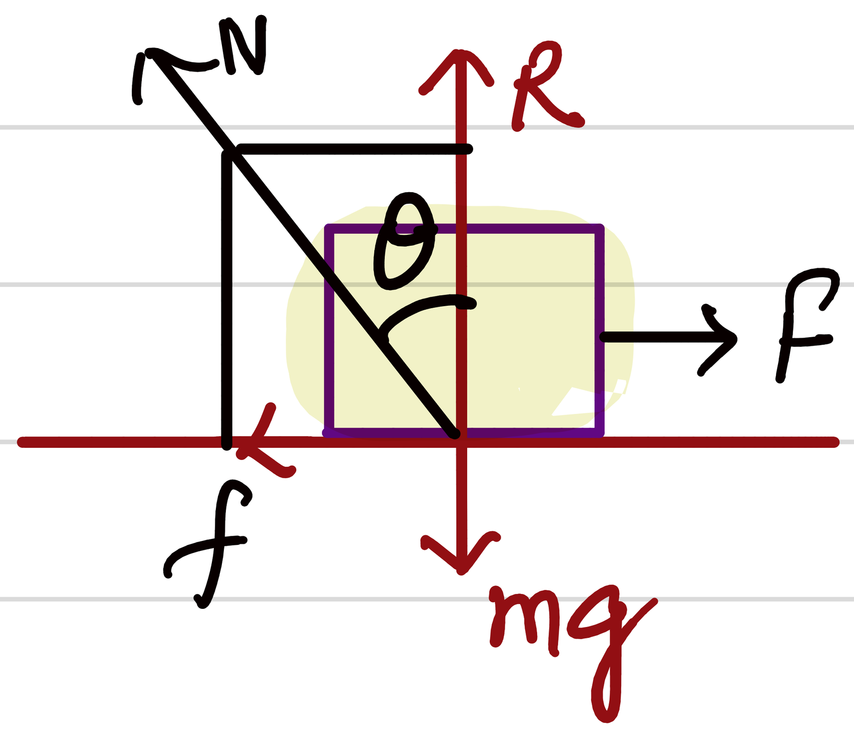
$$\mu_s = \frac{f_s^{max}}{R} = \frac{\text{Limiting fricition}}{\text{Normal reaction}}$$

Coefficient of kinetic friction

$$\mu_k = \frac{f_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

$f_k < f_s^{max}$ or $\mu_k R < \mu_s R$ $\therefore \mu_k < \mu_s$

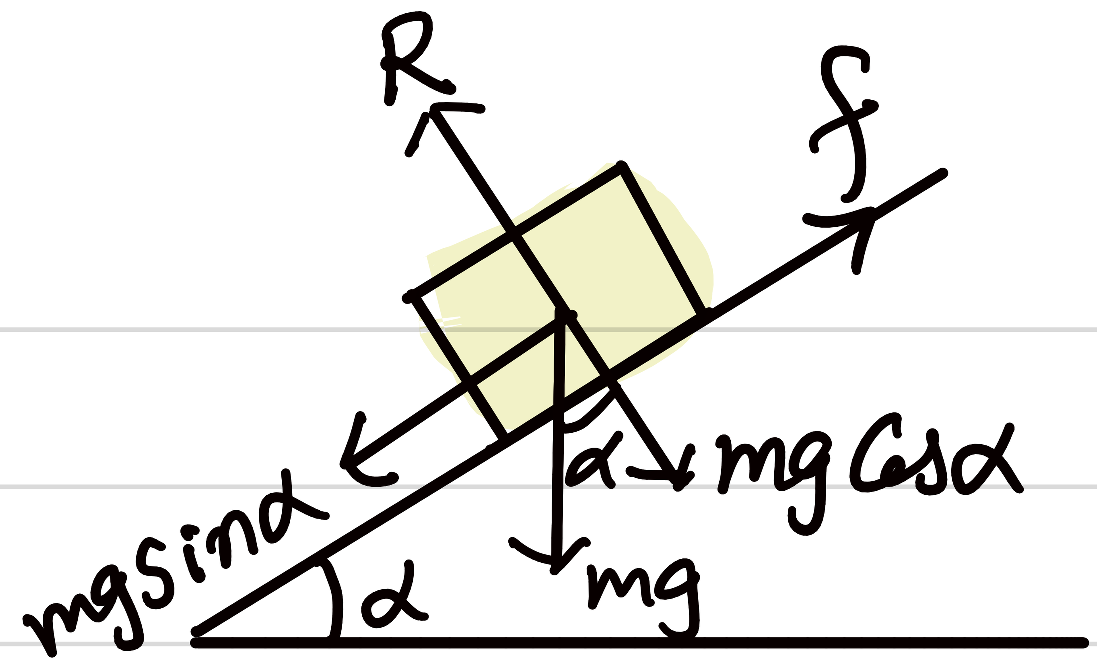
Angle of friction



$$\tan\theta = \mu_s$$

F → Applied force , f → friction force
R → Normal reaction

Angle of repose



$$\tan\phi = \mu_s$$

Motion along a rough horizontal surface

If a body moves over a rough surface through distance s, then

Friction $f = \mu R = \mu mg$

Retardation $a = \frac{f}{m}$

$= \frac{\mu R}{m}$

$a = \frac{\mu g}{m}$ $[R = mg]$

Work done against friction,

$W = f \times s$

$W = \mu mgs$

Power $P = fv$

$= \mu mgv$

Motion along a rough inclined plane

(1) When a body moves down an inclined plane with uniform velocity $(a=0)$, net downward force

$$F = mg\sin\theta - f = mg(\sin\theta - \mu\cos\theta)$$

Work done

$$W = Fs = mg(\sin\theta - \mu\cos\theta)s$$

When a body moves up with $a=0$

$$F = mg\sin\theta + f = mg(\sin\theta + \mu\cos\theta)$$

$$W = Fs = mg(\sin\theta + \mu\cos\theta)s$$

(ii) When body moves up with acceleration a, net upward force

$$F = ma + mg\sin\theta + f$$
$$= m(a + g\sin\theta + \mu g\cos\theta) \quad [f = \mu mg\cos\theta]$$
$$W = m(a + g\sin\theta + \mu g\cos\theta)s$$

Acceleration of a body sliding down a rough inclined plane

$$a = g(\sin\theta - \mu_k\cos\theta)$$

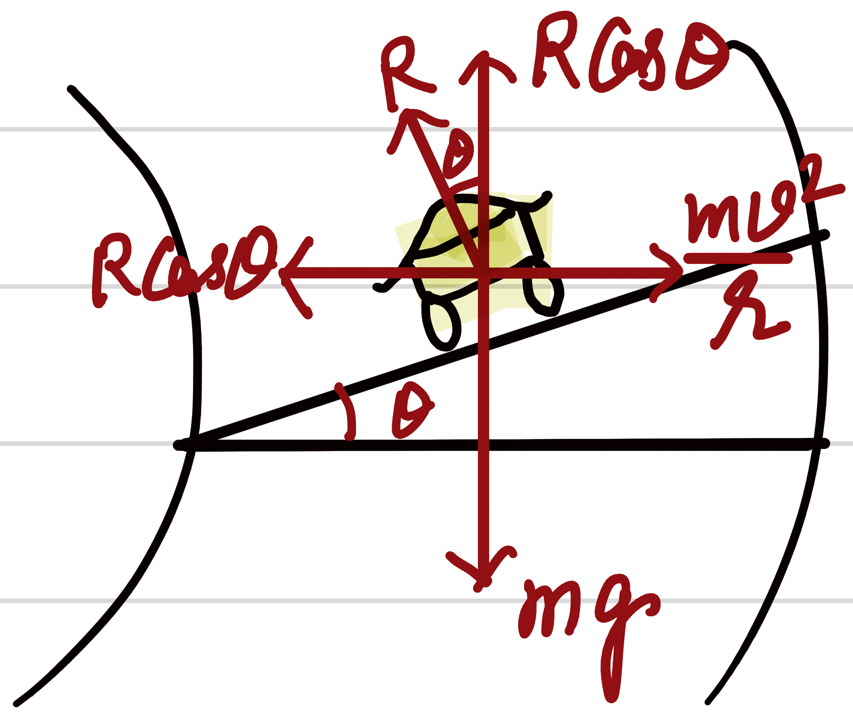
Centripetal force

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi\nu)^2 = mr\left(\frac{2\pi}{T}\right)^2$$

A vehical taking circular turn on a level road

$$v = \sqrt{\mu r g}$$

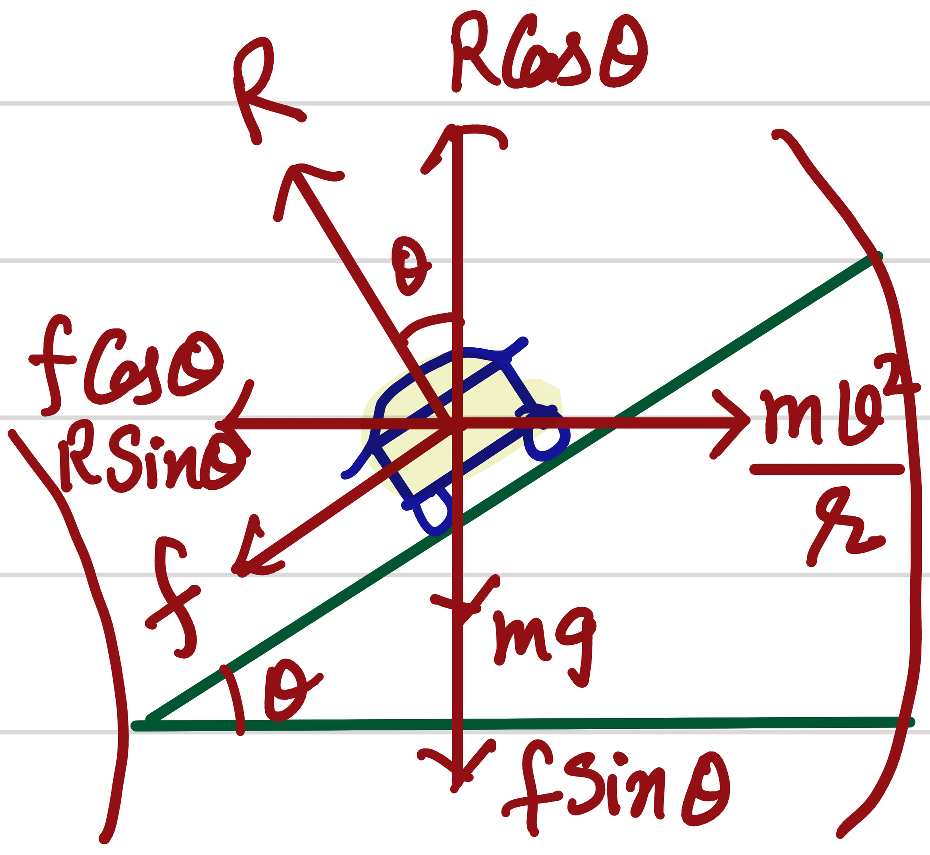
Banking of roads (tracks)



In absence of friction

$$v = \sqrt{rg\tan\theta}$$

When friction is considered

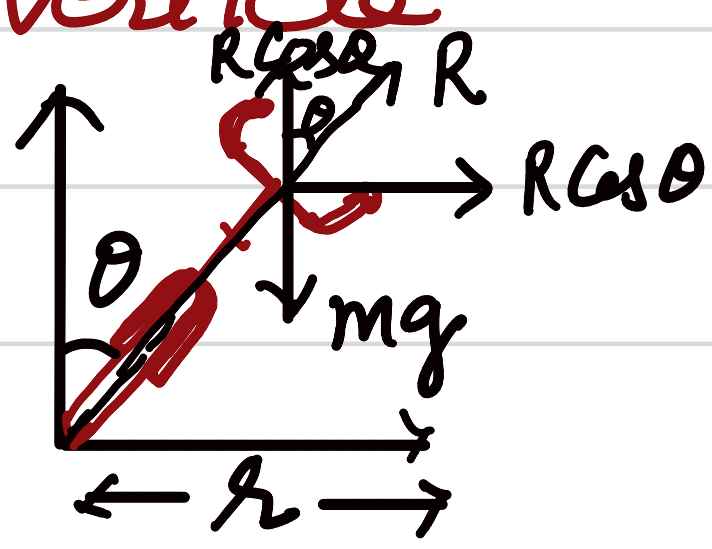


$$v = \sqrt{rg\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)}$$

Bending of a cyclist

In order to take a circular turn, the cyclist bend himself through an angle θ from verticle

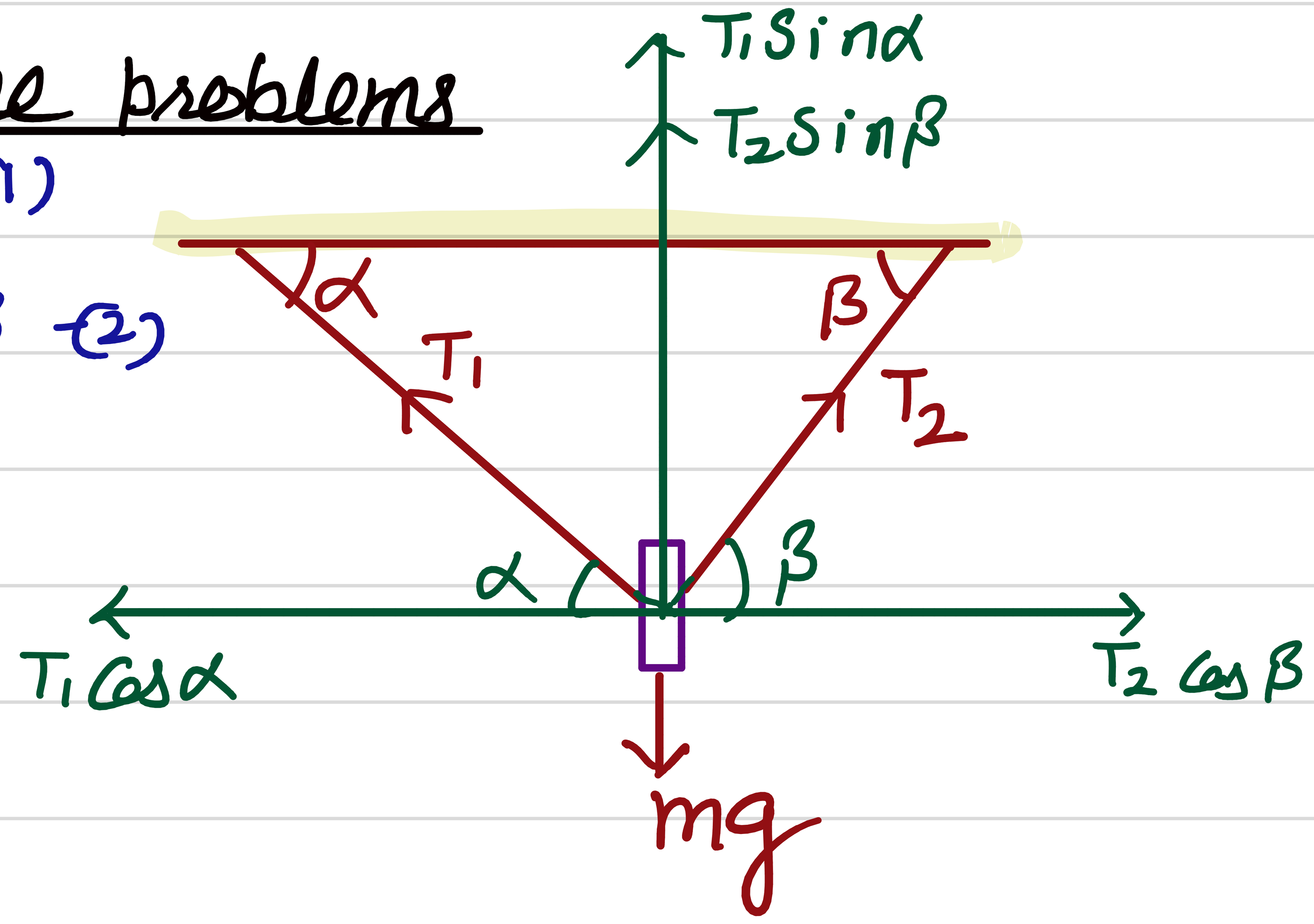
$$\tan\theta = \frac{v^2}{rg}$$



✻ Hints to solve some problems

1. $T_1 \cos\alpha = T_2 \cos\beta \quad -(1)$

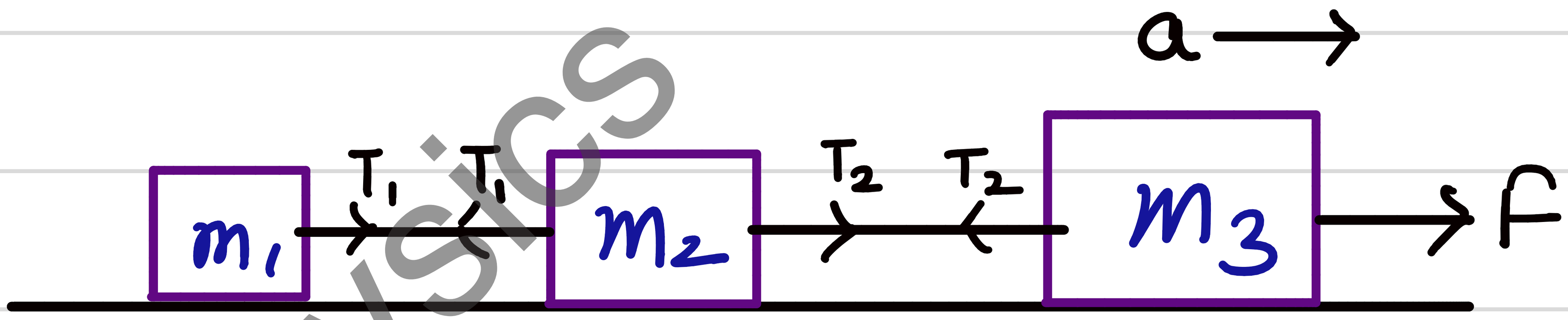
$mg = T_1 \sin\alpha + T_2 \sin\beta \quad -(2)$



2. $a = \dfrac{F}{m_1 + m_2 + m_3}$

$T_1 = m_1 a$

$T_2 = (m_1 + m_2) a$



3. For stationary system

Acc. to Newton's 2nd law (for $m_1 + m_2 + m_3$)

$T_1 - (m_1 + m_2 + m_3) g = (m_1 + m_2 + m_3) a \quad -(1)$

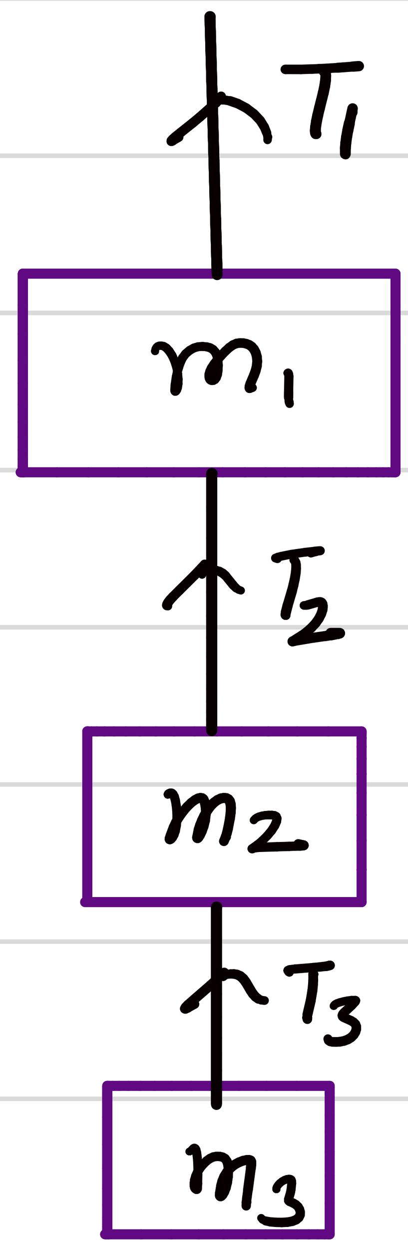
or $T_1 = (m_1 + m_2 + m_3)(g + a)$

Similarly for $m_2 + m_3$

$T_2 = (m_2 + m_3)(g + a)$

and for m_3

$T_3 = m_3 (g + a)$



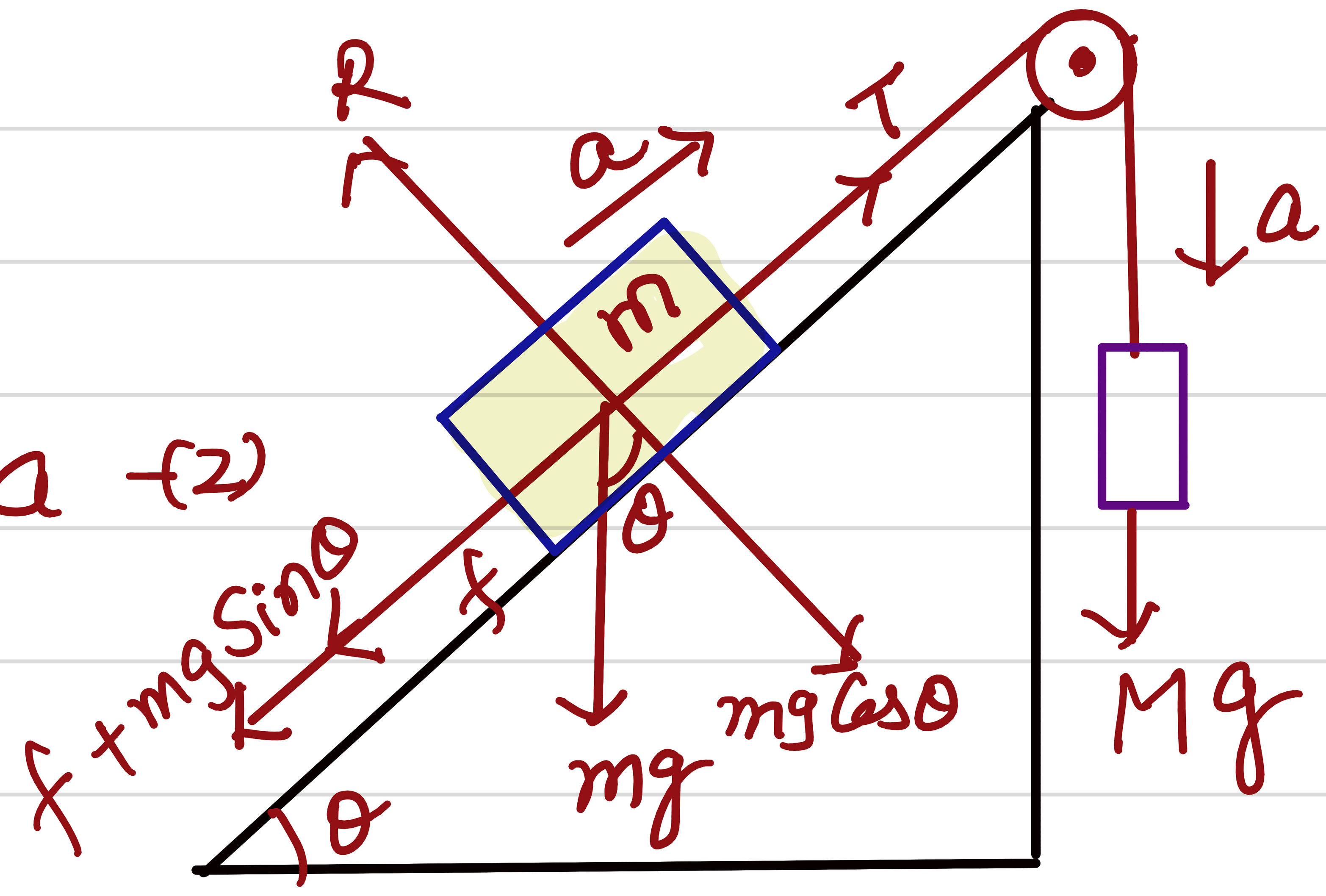
4. $Mg - T = Ma \quad -(1)$

and

$T - (f + mg \sin\theta) = ma \quad -(2)$

here

$f = \mu R = \mu mg \cos\theta$



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