Example 6.1

Given,

$\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{d} = 5\hat{i} + 4\hat{j} + 3\hat{k}$

$$\vec{F}\cdot\vec{d} = Fd\cos\theta$$

$$\cos\theta = \frac{\vec{F}\cdot\vec{d}}{Fd}$$

$$\vec{F}\cdot\vec{d} = (3\hat{i}+4\hat{j}-5\hat{k})\cdot(5\hat{i}+4\hat{j}+3\hat{k})$$
$$= 3\times5 + 4\times4 - 5\times3$$
$$= 15 + 16 - 15$$
$$\vec{F}\cdot\vec{d} = 16 \text{ unit}$$

$$|\vec{F}| = F = \sqrt{3^2+4^2+(-5)^2}$$
$$= \sqrt{9+16+25} = \sqrt{50} \text{ unit}$$

and $|\vec{d}| = d = \sqrt{5^2+4^2+3^2}$
$$= \sqrt{25+16+9} = \sqrt{50} \text{ unit}$$

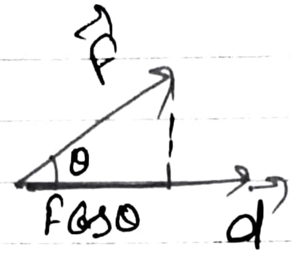
Now

$$\cos\theta = \frac{16}{\sqrt{50}\sqrt{50}} = \frac{16}{50} = 0.32$$

$$\therefore \theta = \cos^{-1}(0.32)$$ Ans

Projection of F on d

$F \cos\theta = \sqrt{50} \times 0.32$



$= 5\sqrt{2} \times 0.32$

$= 5 \times 1.414 \times 0.32$

$F\cos\theta = 2.26$

Ans

Example 6.2

$m = 1\,gm = 1 \times 10^{-3}\,kg$

$h = 1\,Km = 10^{3}\,m$

$v = 50\,m/s, \; g = 10\,m/s^2$

$W = mgh$

$= 10^{-3} \times 10 \times 10^{3}$

$W = 10\,J$

i.e work done by the gravitational force is 10 joule.

Ans

Now work done by the resistive force

By work energy theorem

$\Delta K = W_g + W_r$ [$W_r \rightarrow$ work done by resistive force]

here $\Delta K = W$

or $W = \Delta K$

ΔK = change in K.E

$= \frac{1}{2} m v^2 - \frac{1}{2} m u^2$

$= \frac{1}{2} \times 10^{-3} \times (50)^2 - 0 \qquad [\because u = 0]$

$= \frac{1}{2} \times 10^{-3} \times 2500$

$= 1250 \times 10^{-3}$

$\Delta K = 1.25\ J$

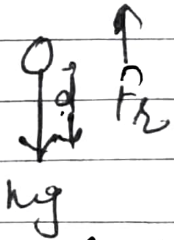
Now

$\Delta K = W_g + W_r$

or $W_r = \Delta K - W_g$

$= 1.25 - 10$

$W_r = -8.75\ J$



$\theta = 180'$

-ve sign shows that F_r and displacement of drop are in opposite dir^n.

Example 6.3

$S = 10\ m$

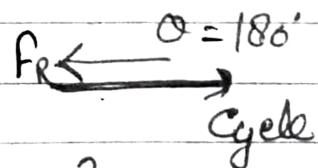
$V = 0$

Force applied by road on cycle = 200N

(a) $W = F d \cos\theta$

$= 200 \times 10 \times \cos 180'$

$= -2000\ J \qquad [\cos 180' = -1]$



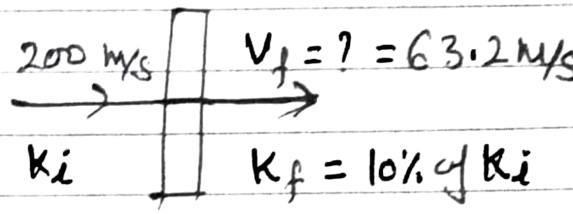
Example 6.4

Given,

$m = 50\,g = 50 \times 10^{-3}\,kg$

$= 5 \times 10^{2}\,kg$

$V_i = 200\,m/s$



$K_i = \frac{1}{2} m V_i^2$

$= \frac{1}{\cancel{2}} \times 5 \times 10^{-2} \times \overset{100}{\cancel{200}} \times 200$

$K_i = 1000\,J$

$K_f = 10\%$ of 1000

$= 1000 \times \frac{10}{100} = 100\,J$

$\frac{1}{2} m V_f^2 = K_f$

$\frac{1}{2} m V_f^2 = 100$

$V_f^2 = \frac{200}{m} = \frac{\overset{40}{\cancel{200}}}{\cancel{5} \times 10^{-2}}$

$V_f^2 = 40 \times 10^2 = 400 \times 10$

$V_f = \sqrt{400 \times 10} = 20\sqrt{10}$

$V_f = 20 \times 3.16 = 63.2\,m/s$

$V_f = 63.2\,m/s$ Ans.

* 68.4% of V_i (reduced speed)

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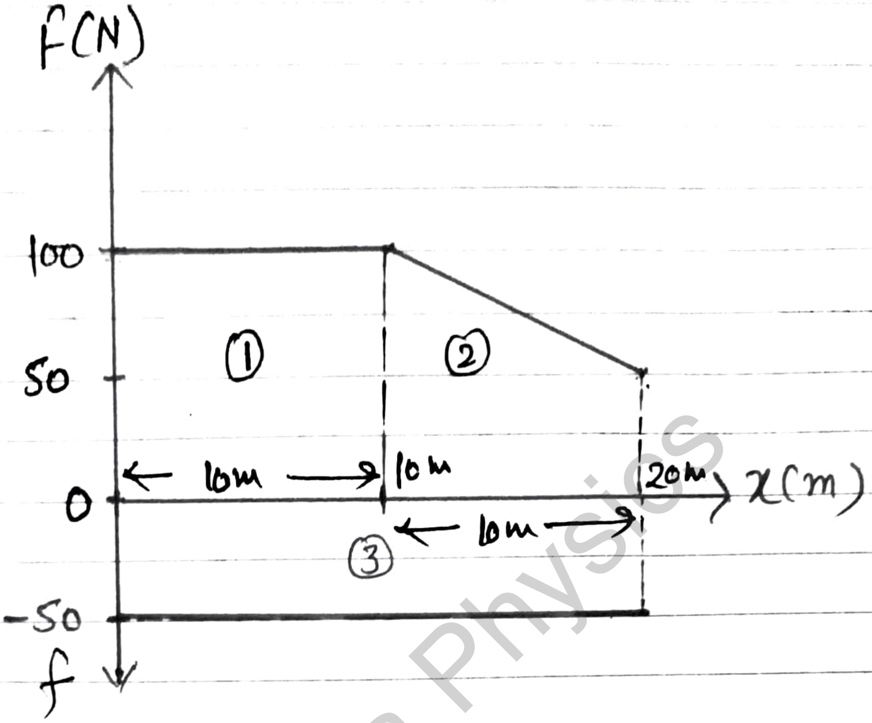
Example 6.5



Work done by the woman

$$W_F = \text{Area of rectangle ①} + \text{Area of trapezium ②}$$

$$= 100 \times 10 + \frac{(100+50)}{2} \times 10$$

$$= 1000 + 750$$

$$W_F = 1750\,J$$

Work done by friction force

$$W_f = \text{Area of ③}$$

$$= (-50) \times 20$$

$$= -1000\,J$$

-ve sign show that friction force and displacement are in opposite direction.

Teacher's Sign ....

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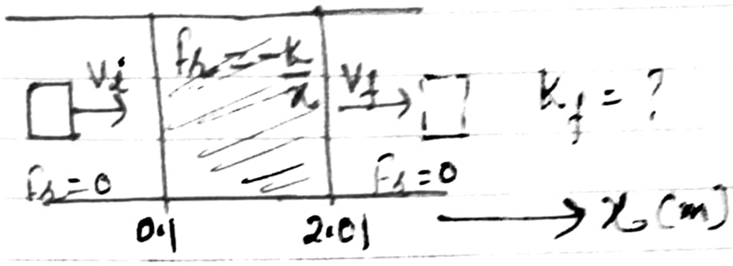
Example 6.6



$K_f = ?$

Given,

$$F_r = -\frac{k}{x} \quad \text{where } k = 0.5\ J$$

By WE theorem

$$K_f - K_i = W_{net}$$

here $K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} \times 1 \times 2^2 = 2\,J$

$$\boxed{K_i = 2J}$$

Now $K_f - K_i = \int_{0.1}^{2.01} F_r \, dx$

$$= \int_{0.1}^{2.01} \left(-\frac{k}{x}\right) dx$$

$$= -k \int_{0.1}^{2.01} \frac{1}{x} dx$$

$$= -k \left[\log x\right]_{0.1}^{2.01}$$

$$= -k \left[\log 2.01 - \log 0.1\right]$$

$$= -k \log \frac{2.01}{0.1}$$

$$k_f - k_i = -k \log_e 20.1 \qquad [\because \log_e x = \ln x]$$

$$= -k \times 3.00$$

$$= -0.5 \times 3.00$$

$$k_f - k_i = -1.5$$

$$k_f = k_i - 1.5$$

$$= 2 - 1.5 = 0.5 \text{ J}$$

$$k_f = 0.5 \text{ J}$$

Ans

$$k_f = \frac{1}{2} m v_f^2 = 0.5$$

$$v_f^2 = \frac{0.5 \times 2}{m}$$

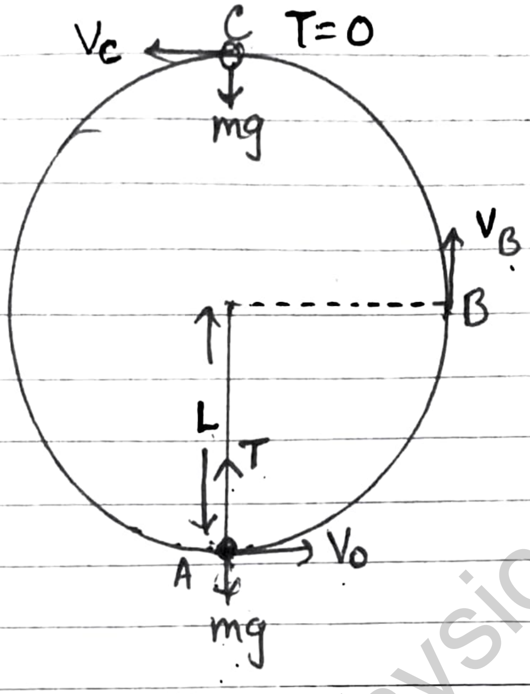
$$= \frac{0.5 \times 2}{1} = 1$$

$$v_f^2 = 1$$

or $v_f = 1$ m/s

Ans

Example 6.7



At point 'A'

$$K.E = \frac{1}{2} m v_0^2$$

$$P.E = 0 \quad [\because \text{lowest point}]$$

Total energy E_A

$$E_A = K.E + P.E$$

$$E_A = \frac{1}{2} m v_0^2 \quad -(1) \quad [\because P.E = 0]$$

and $\dfrac{m v_0^2}{L} = T - mg \quad -(2)$ [from II law of newton]

At point 'C' [highest point]

$$K.E = \frac{1}{2} m v_c^2$$

$$P.E = mg(2L) = 2mgL$$

so, $E_C = \frac{1}{2} m v_c^2 + 2mgL \quad -(3)$

and

$$\frac{mv_c^2}{L} = mg$$

$$mv_c^2 = mgL \quad -(4)$$

By the conservation of energy

$$E_A = E_C$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_c^2 + 2mgL$$

or $$v_0^2 = v_c^2 + 4gL$$

from eq^n (4) $v_c^2 = gL$, then

$$v_0^2 = gL + 4gL$$

$$v_0^2 = 5gL$$

$$\boxed{v_0 = \sqrt{5gL}}$$

(ii) From eq^n (4)

$$\cancel{m}v_c^2 = \cancel{m}gL$$

$$v_c = \sqrt{gL}$$

At point 'B'

$$E_B = K.E + P.E$$

$$= \frac{1}{2}mv_B^2 + mgL$$

By the conservation of energy

$$E_B = E_A$$

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$$E_B = E_A$$

$$\frac{1}{2} m v_B^2 + mgL = \frac{1}{2} m v_0^2 \quad [\text{from eq}^n (1)]$$

$$v_B^2 + 2gL = v_0^2$$

$$v_B^2 + 2gL = 5gL$$

$$v_B^2 = 3gL$$

$$\boxed{v_B = \sqrt{3gL}}$$

Velocities at B and C points are

$$v_B = \sqrt{3gL}$$

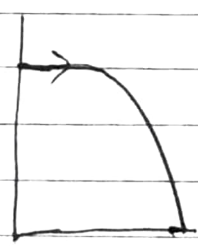
$$v_C = \sqrt{gL}$$

(iii)

$$\frac{K_B}{K_C} = \frac{v_B^2}{v_C^2} \quad \left[K = \frac{1}{2} m v^2\right]$$

$$\frac{K_B}{K_C} = \frac{3\cancel{gL}}{\cancel{gL}} = \frac{3}{1}$$

$$K_B : K_C = 3 : 1$$

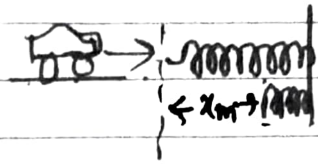


Teacher's Sign ....

Example 6.8

Given,

$m = 1000\ Kg$



$v = 18\ Km/h$

$v = 18 \times \frac{5}{18} = 5\ m/s$

spring constant $k = 6.25 \times 10^3\ N m^{-1}$

At the time of compression (max^m) by the conservation of energy

$K.E = P.E$

$\frac{1}{2} m v^2 = \frac{1}{2} k x_m^2$

$x_m^2 = \frac{m v^2}{k}$

$= \frac{1000 \times 5^2}{6.25 \times 10^3}$

$= \frac{2500}{6.25} = 4$

$x_m^2 = 4$

$x_m = 2\ m$ Ans

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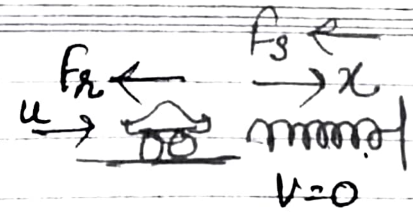
Example 6.9



By WE theorem

$$K_f - K_i = W_{net}$$

$$0 - \frac{1}{2}mu^2 = -\mu mg x_m - \frac{1}{2}kx_m^2$$

or $$\frac{1}{2}kx_m^2 + \mu mg x_m - \frac{1}{2}mu^2 = 0$$

or $$kx_m^2 + 2\mu mg x_m - mu^2 = 0$$

$$6.25\times\underline{10^3}\, x_m^2 + 2\times 0.5\times\underline{10^3}\times 10\, x_m - \underline{10^3}\times 5^2 = 0$$

or $$6.25x_m^2 + 10x_m - 25 = 0$$

or $$\underset{a}{1.25}x_m^2 + \underset{b}{2}x_m - \underset{c}{5} = 0$$

$$x_m = \frac{-2 \pm \sqrt{4 - 4\times 1.25\times(-5)}}{2\times 1.25}$$

$$= \frac{-2 + \sqrt{4+25}}{2.5}$$ [drop -ve sign as x is +ve]

$$= \frac{-2+\sqrt{29}}{2.5}$$

$$= \frac{-2+5.38}{2.5}$$

$$= \frac{3.38}{2.5} = 1.35\text{ m}$$

$$x_m = 1.35\text{ m}$$

Teacher's Sign........

Example 6.10

Energy required to break one bond in DNA

(a) $E = 10^{-20}$ J

$1\,eV = 1.6 \times 10^{-19}$ J

so $1\,J = \dfrac{1}{1.6 \times 10^{-19}}$ eV

then,

$$10^{-20}\,J = \frac{10^{-20}}{1.6 \times 10^{-19}}\,eV$$

$$= \frac{10^{-1}}{1.6} = \frac{1}{16} = 0.0625\,eV$$

i.e. $10^{-20}\,J = 0.0625\,eV$

$= 6.25 \times 10^{-2}\,eV$ Ans

(b) K.E of an air molecule $= 10^{-21}$ J

$$= \frac{10^{-21}}{1.6 \times 10^{-19}}$$

$$= \frac{1}{1.6 \times 10^{2}} = \frac{1}{160}$$

$= 0.00625$ eV

$= 6.25 \times 10^{-3}$ eV

Ans

(C) Daily food intake of a human adult $= 10^{7}$ J

here $1\,cal = 4.2$ J [$1\,cal = 4.186$ J]

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$$10^7 J = \frac{10^7}{4.2} cal$$

$$= 2.4 \times 10^6 cal$$

$$10^7 J = 2.4 \times 10^3 Kcal$$

$$10^7 J = 2400 Kcal$$

Example 6.11

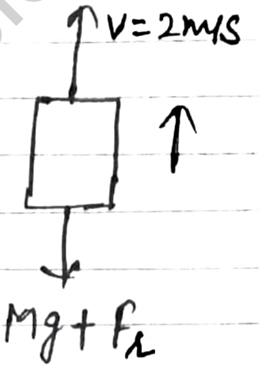
The downward force

$F = Mg + f_r$

$= 1800 \times 10 + 4000$ [$\because g = 10 m/s^2$]

$= 18000 + 4000$

$F = 22000 N$



We know

$P = F V$

$= 22000 \times 2$ [$\because V = 2 m/s$]

$P = 44000$ Watt Ans

1 hp = 746 Watt

$$P = \frac{44000}{746} = 58.98$$

$P \simeq 59$ hp Ans

Example 6.12

$$(n) \xrightarrow{u_1} \quad (D)\ u_2 = 0$$

befor collision

$$(n) \xrightarrow{v_1} \quad (D) \rightarrow v_2$$

after collision

for elastic collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

here $u_2 = 0$, then

$$m_1u_1 = m_1v_1 + m_2v_2 \quad -(1)$$

We know that for elastic coll^n

$$e = 1 = \frac{v_2 - v_1}{u_1 - u_2}$$

or $u_1 = v_2 - v_1$

or $v_2 = u_1 + v_1$

put this value in eq^n(1)

$$m_1u_1 = m_1v_1 + m_2(u_1 + v_1)$$

$$m_1u_1 = m_1v_1 + m_2u_1 + m_2v_1$$

$$(m_1 - m_2)u_1 = (m_1 + m_2)v_1$$

or $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

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For carbon

$m_2 = 12 m_1$

then

$$\frac{K_f}{K_i} = \left(\frac{m_1 - 12m_1}{m_1 + 12m_1}\right)^2$$

$$= \left(\frac{-11}{13}\right)^2 = \frac{121}{169}$$

% change in K.E

$$= \frac{K_i - K_f}{K_i} \times 100$$

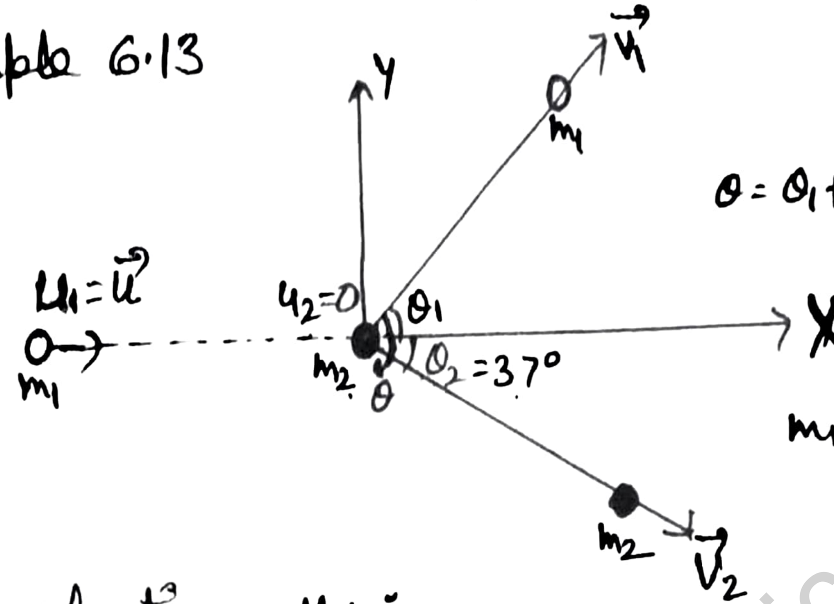
$$= \left(1 - \frac{121}{169}\right) \times 100$$

$$= \frac{48}{169} \times 100$$

$$= 28.4\%$$

Teacher's Sign ....

Example 6.13



$\theta = \theta_1 + \theta_2$

$m_1 = m_2 = m$

For elastic collision

(i) Momentum is conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$u_1 = u$, $u_2 = 0$, $m_1 = m_2 = m$

$$\vec{u} = \vec{V_1} + \vec{V_2} \qquad [\vec{A} + \vec{B} = \vec{R}]$$

$$u^2 = v_1^2 + v_2^2 + 2v_1v_2\cos\theta \quad \text{--- (1)} \qquad [\because R^2 = A^2 + B^2 + 2AB\cos\theta]$$

here $\theta = \theta_1 + \theta_2$

(ii) K.E is also conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$u^2 = v_1^2 + v_2^2 \quad \text{--- (2)}$$

from eqn (1)

$$\cancel{v_1^2} + \cancel{v_2^2} + 2v_1v_2\cos\theta = \cancel{v_1^2} + \cancel{v_2^2}$$

or $2v_1v_2\cos\theta = 0$

or $\cos\theta = 0$ $[\because 2v_1v_2 \neq 0]$

$\cos(\theta_1 + \theta_2) = 0 = \cos 90°$

or $\theta_1 + \theta_2 = 90°$

$\theta_1 = 90 - \theta_2 = 90 - 37° = 53°$

$\boxed{\theta_1 = 53°}$ Ans