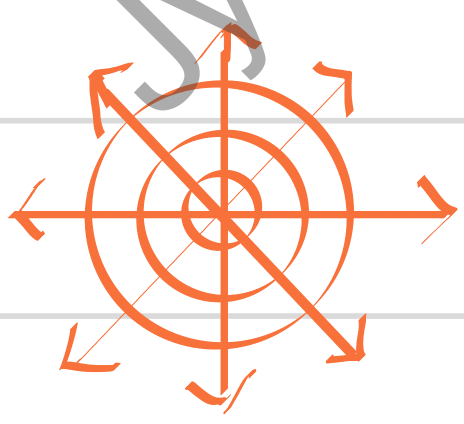
Wave Optics

Wavefront - A wavefront is defined as the surface of constant phase.
The speed with which wavefront moves outward from the source is called speed of wave.

Wavefronts

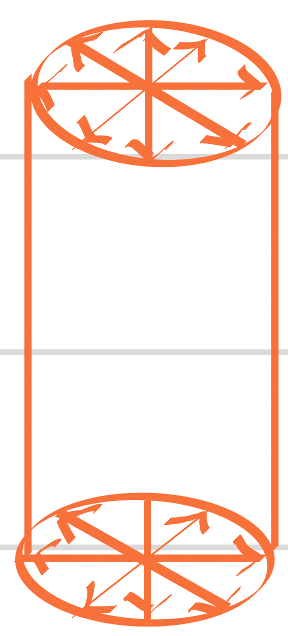
- SWF → Spherical Wavefront (Formed by a point source)
- PWF → Plane Wavefront (Small part of SWF and CWF at large distace)
- CWF → cylindrical Wavefront (Formed by a linear source)

Spherical Wavefront



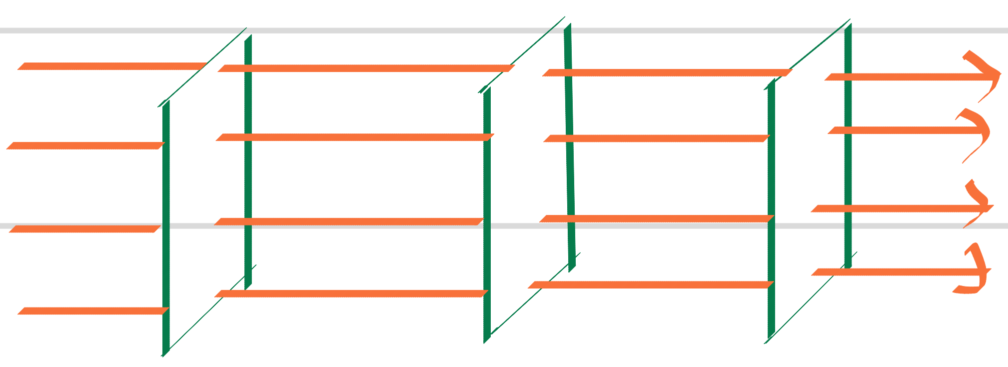
Point source

Cylindrical Wavefront

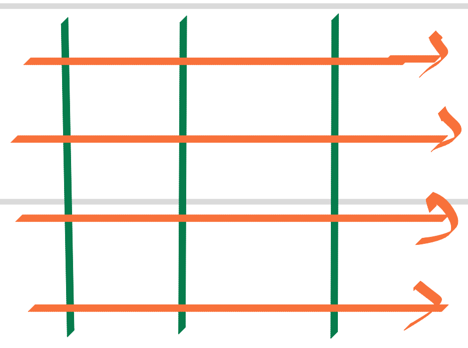


Line source

Plane Wavefronts



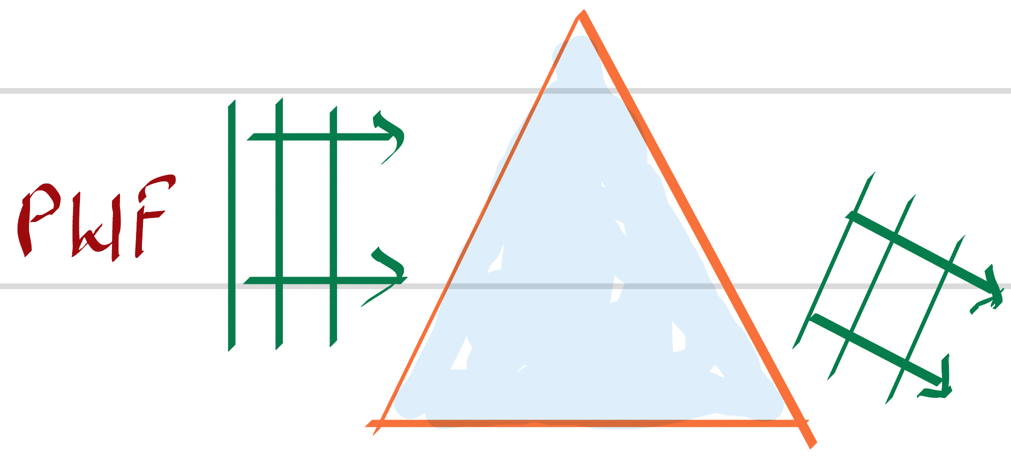
OR



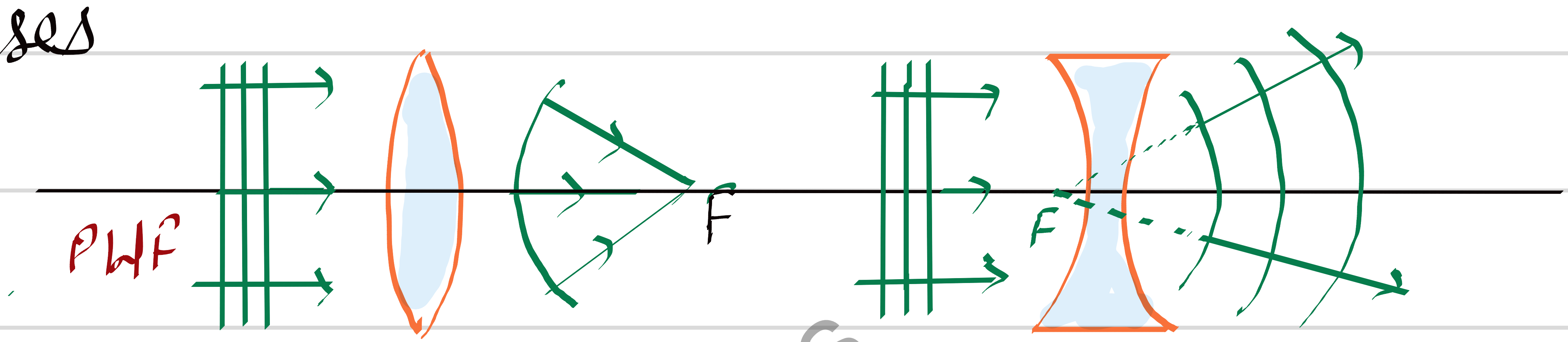
source at infinity

Behaviour of Plane Wavefronts when passes through

(a) A thin Prism



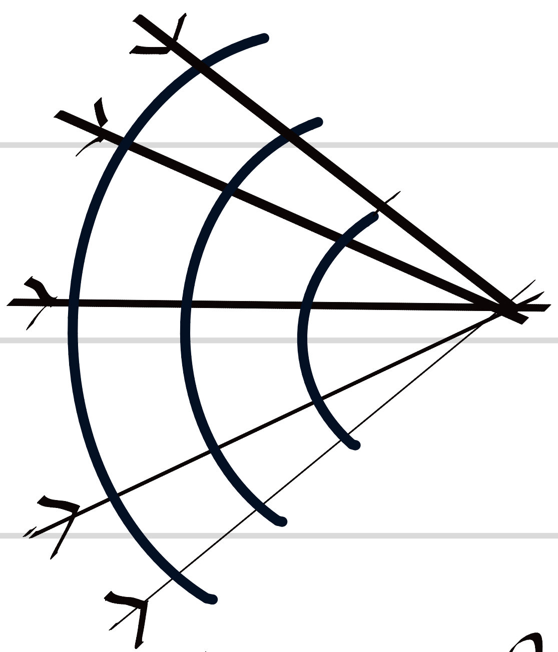
(b) Lenses



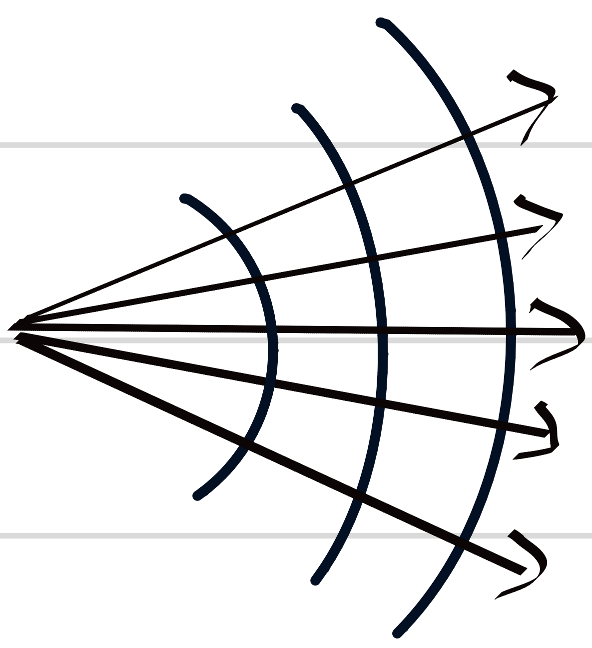
convex lens　　　　concave lens

(c) A concave Mirror

Converging and Diverging Spherical Wavefronts



Converging SWF



Diverging SWF

Huygens Principle

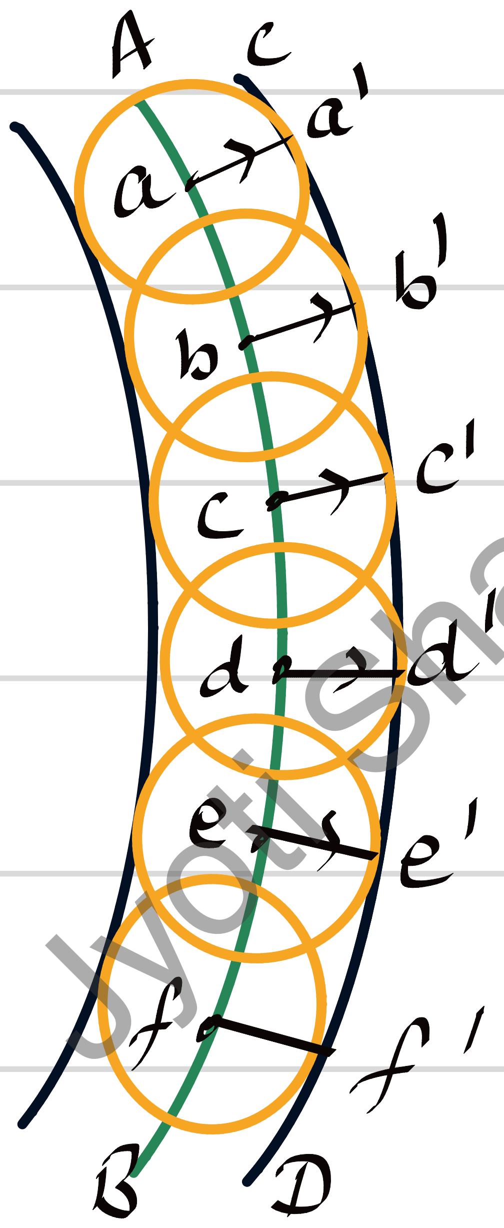
According to Huygens principle each point on a wavefront is a source of secondary waves.

Assumptions -

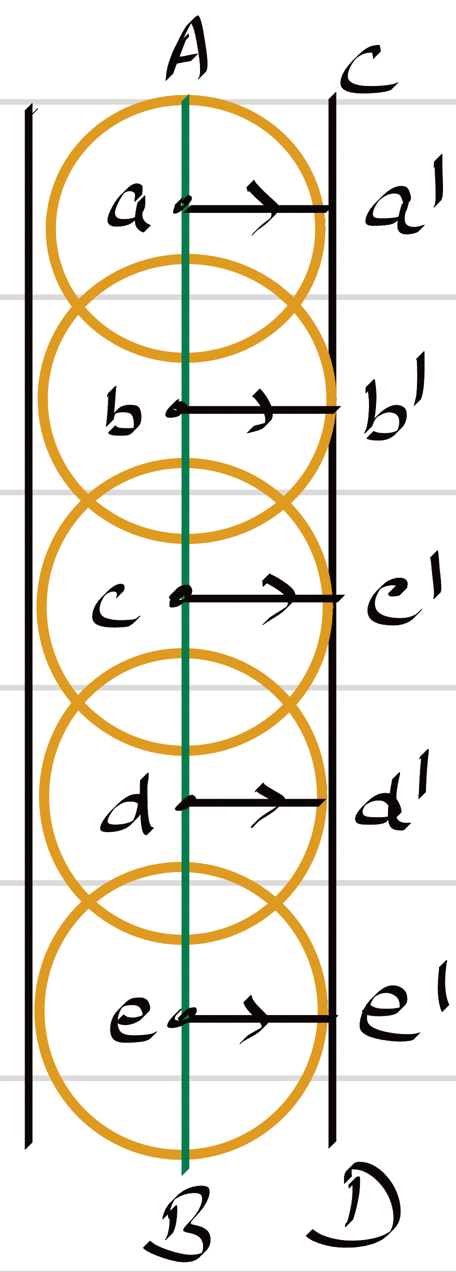
(i) Each point on a wavefront act as a fresh source of a secondary waves or wavelets.

(ii) The secondary wavelets spreads out in all directions with the speed of light.

(iii) The new wavefront at any later time is given by the forward envelop.



Propagation of SWF

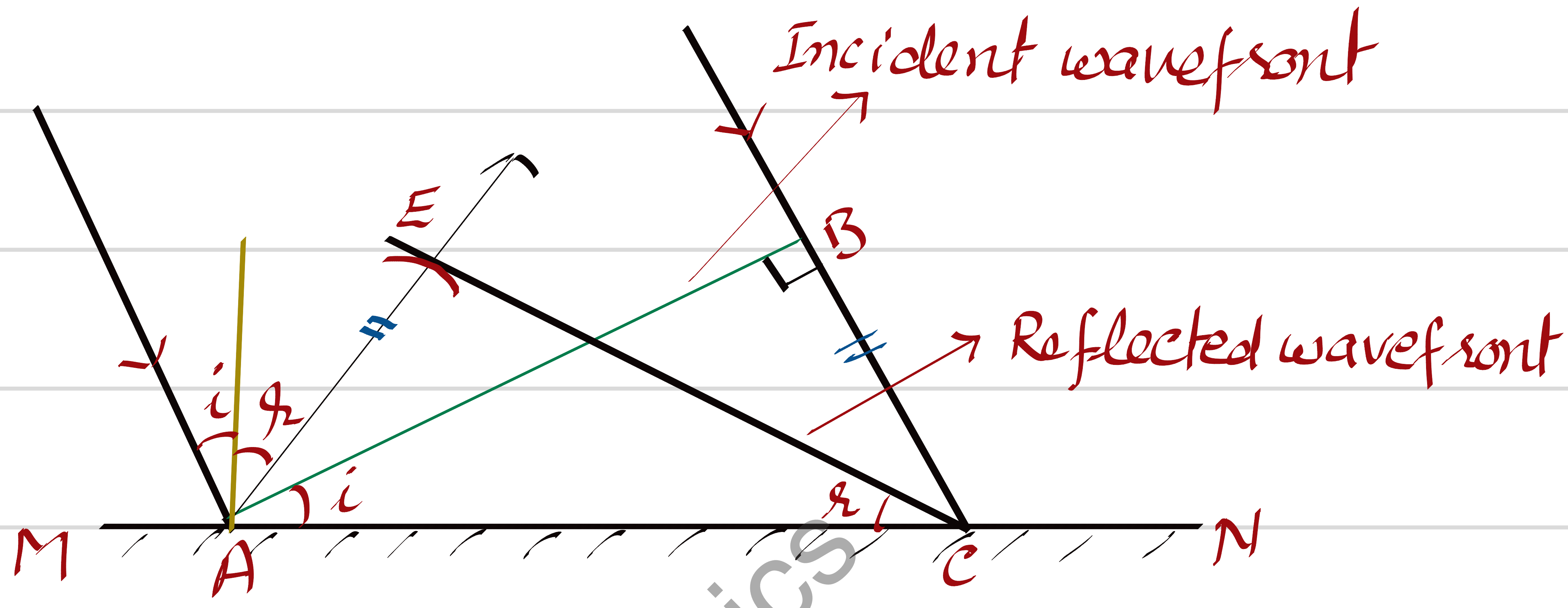


Propagation of PWF

* No backward wavefront is possible as amplitude of secondary wavelet is proportional to $(1+\cos\theta)$. For backward $\theta = \pi$ so $1+\cos\theta = 0$.

Laws of Reflection by Huygens' Wave Theory

Consider a PWF on a reflecting surface MN.



In fig. , AB = Incident wavefront

and CE = Reflected wavefront

If v is the speed of wavefront and t is the time. then,

$$BC = vt \quad -(1)$$

The construct wavefront is such that

$$BC = AE$$

from (1), $BC = AE = vt$

Now in Δ's ABC and AEC

We find that

$$\Delta ABC \cong AEC$$

therefore $\angle i = \angle r$

This is the law of reflection.

OR

In ΔABC, $\sin i = \frac{BC}{AC}$

In ΔAEC, $\sin r = \frac{AE}{AC}$

but $BC = AE$, therefore

$\sin i = \sin r$

or $\angle i = \angle r$

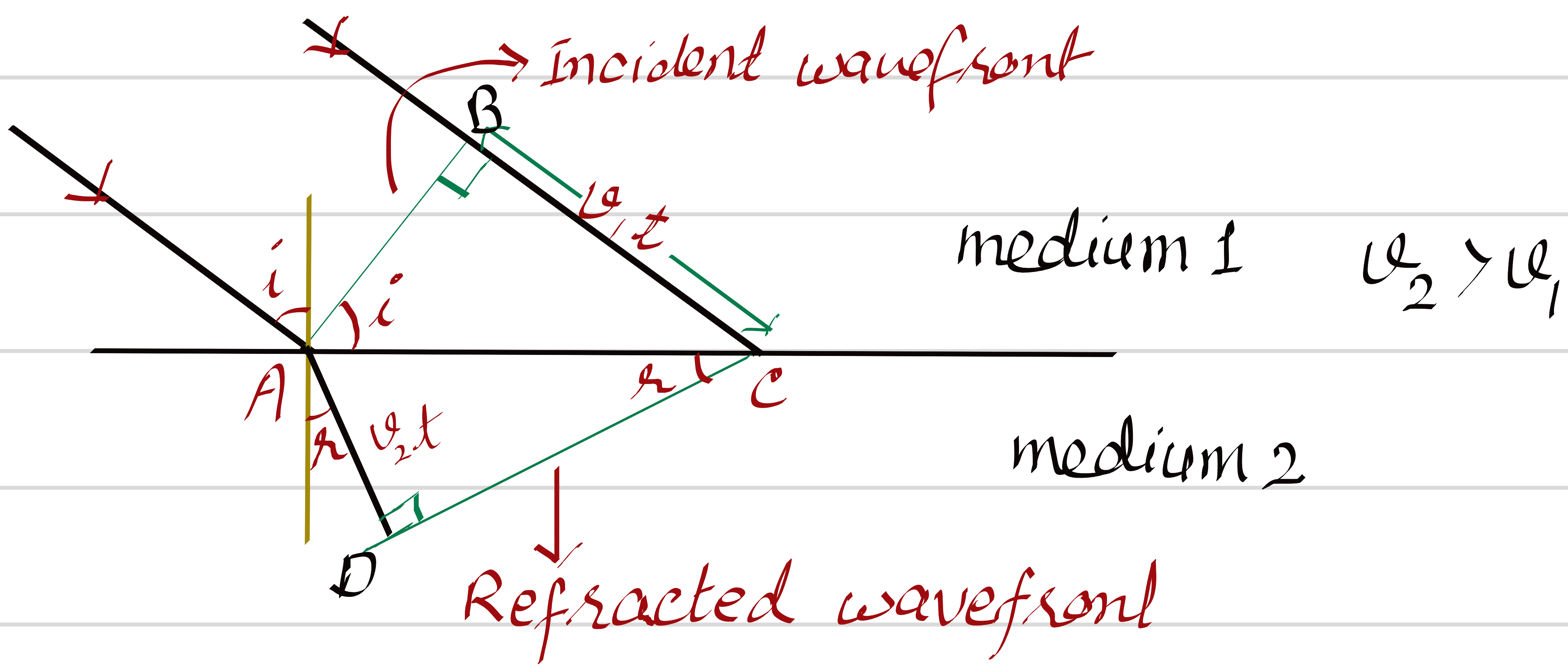
This is the law of reflection.

Laws of Refraction by Huygens' Principle:

(1) Consider a PWF AB incident on refracting surface XY and propagates from medium 1 to medium 2.

Let $v_2 > v_1$ and t is the time from B to C in medium 1 and A to E in medium 2.

* for rarer to denser



From fig.

In ΔABC, $\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$

and in ΔADC, $\sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}$

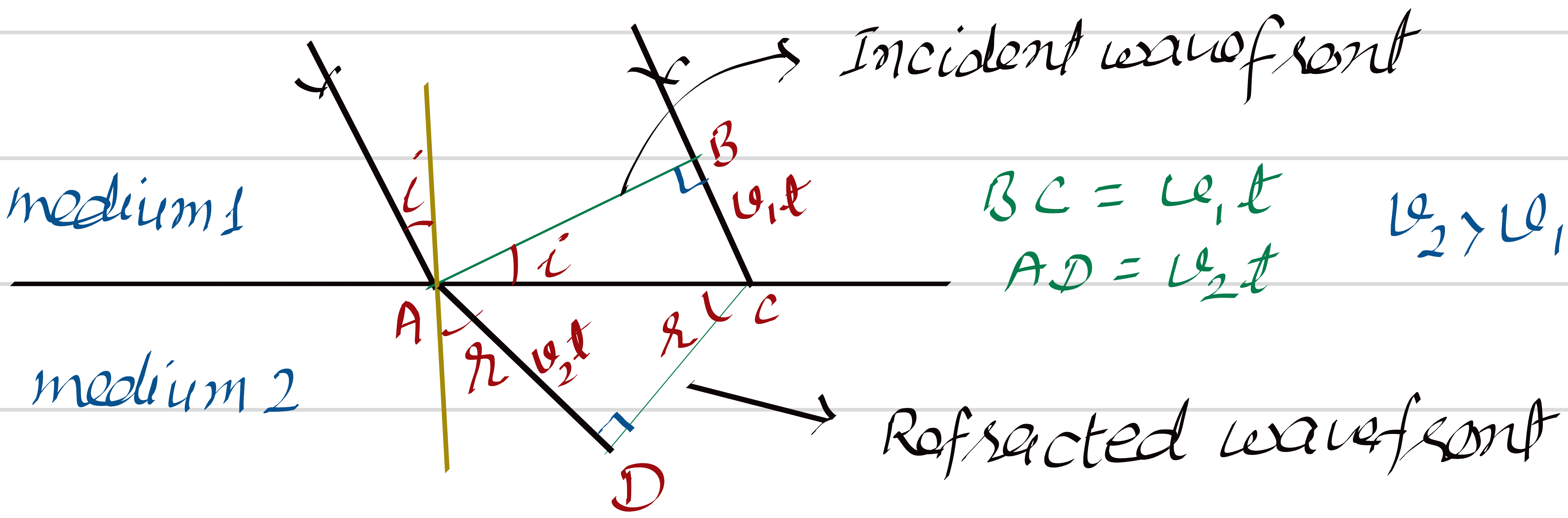
then, $\frac{\sin i}{\sin r} = \frac{v_1 t / AC}{v_2 t / AC}$

or $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} =$ constant (Refractive Index)

or $\frac{\sin i}{\sin r} = {}_1\mu_2$

This is the snell's law of refraction.

(II) For denser to rarer medium



In ΔABC, $\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$

In ΔADC

$$\sin r = \frac{AD}{AC} = \frac{v_2 t}{AC}$$

Now $$\frac{\sin i}{\sin r} = \frac{v_1 t / AC}{v_2 t / AC}$$

or $$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}_1\mu_2$$

This is the Snell's law of refraction.

wavelength

* $$\boxed{\frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}}$$

Frequency depends on the source of light only but wavelength and speed depends on the medium

* Frequency of light remains same in reflection and refraction.

Because atoms of surface oscillate with same frequency of incident light and same frequency is imparted to the light from atoms when light is reflected and refracted.

* Energy of a light wave does not change on changing medium because it depends on the amplitude of the light wave not

on the speed.

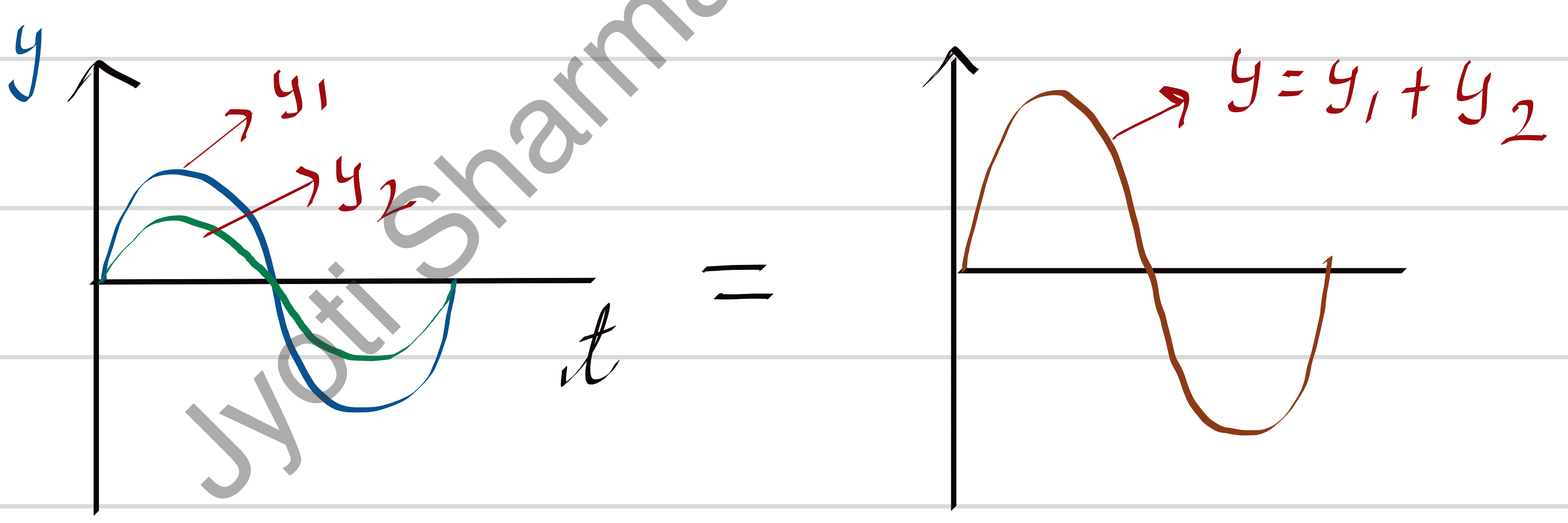
* In wave picture of light, $I \propto a^2$ $\left[\begin{array}{l} I \rightarrow \text{Intensity} \\ a \rightarrow \text{Amplitude} \end{array}\right.$

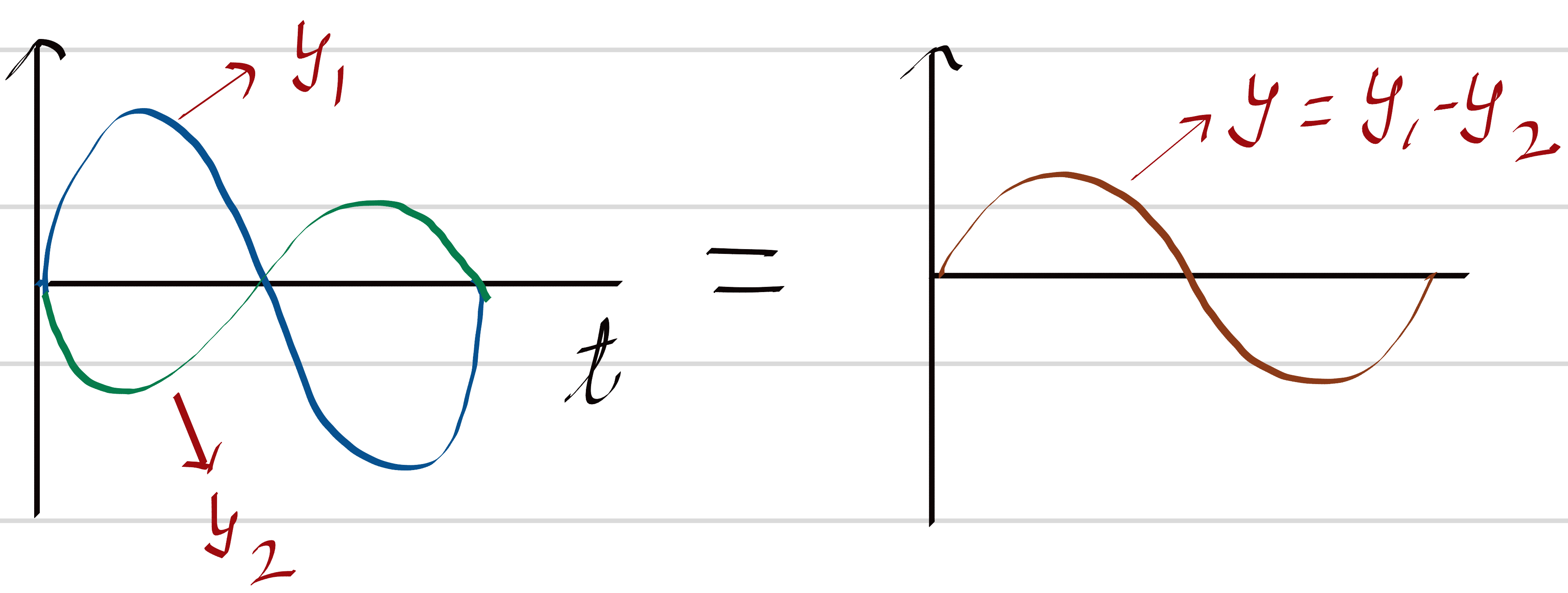
* In photon piture of light, $I \propto$ Photon density

Principle of Superposition of waves

When a number of waves superpose on each other, the resultant displacement at any point is equal to the vector sum of individual displacement.

i.e $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \cdots$





* If $y_1 = y_2 = y$, then

$$y_1 + y_2 = 2y \quad \text{and} \quad y_1 - y_2 = 0$$

Coherent and Incoherent Sources of Light

Coherent sources	Incoherent sources
Light sources having - • same frequency • same wavelength • same amplitude • same phase or constant phase difference. e.g - LASER light	Light sources having • different frequencies • different wavelengths • not in phase e.g. Electric bulb.

Need of coherent source:

To observe interference we need to have two sources of same frequency and constant phase difference. i.e we need coherent sources.

* Two independent sources can not be coherent.

Because - (I) Light is emitted by individual atom

(II) An atom emits an unbroken wave of about 10^{-8} sec.

Method to make coherent sources

Prism, lenses and mirrors with perticular parameters are used to creat a coherent source.

Conditions for obtaining two coherent sources of light:

(i) The two sources of light must be obtained from a single source.

(ii) The two sources must be monochromatic.

(iii) The path difference between the waves must not be long.

Interference of Light

The phenomenon of redistribution of intesity of light due to superposition of two coherent sources is called interference of light.

A pair of light waves pass over each other and create interference.

Mathematical Interpretation of Interference of Two Waves (Constructive and Destructive Interference)

Consider two simple harmonic progressive waves of same frequency. Let a_1 and a_2 are amplitudes and ϕ is the phase difference at any point. Displacements at that points are given by -

$y_1 = a_1 \sin\omega t \quad -(I)$

$y_2 = a_2 \sin(\omega t + \phi) \quad -(II)$

$y = y_1 + y_2$

$y = a_1 \sin\omega t + a_2 \sin(\omega t + \phi)$

$= a_1 \sin\omega t + a_2 \sin\omega t \cos\phi + a_2 \cos\omega t \sin\phi$

$y = \sin\omega t (a_1 + a_2 \cos\phi) + a_2 \sin\phi \cos\omega t$

Let $a_1 + a_2 \cos\phi = R\cos\theta \quad -(III)$

and $a_2 \sin\phi = R\sin\theta \quad -(IV)$

Where R and θ are new constants. then

$y = R\sin\omega t \cos\theta + R\cos\omega t \sin\theta$

or $y = R\sin(\omega t + \phi)$

This equation is also a wave equation of amplitude R.

To determine R, we square and add eqn (III) and eqn (IV), we get

$$R^2\cos^2\theta + R^2\sin^2\theta = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

or $R^2 = a_1^2 + \underline{a_2^2\cos^2\phi} + 2a_1a_2\cos\phi + \underline{a_2^2\sin^2\phi}$

or $R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi \quad [\because \cos^2\phi + \sin^2\phi = 1]$

but $I \propto a^2 \quad$ [$I \rightarrow$ Intensity, $a \rightarrow$ Amplitude] then, $\boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

and phase angle

$$\boxed{\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}}$$

* If phase difference ϕ is equivalent to path difference Δx and Δt is time difference then,

$$\frac{\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}, \text{ we get}$$

(i) $\boxed{\phi = \frac{2\pi}{\lambda}\Delta x} \longrightarrow$ Path diff.

(ii) $\boxed{\Delta x = \frac{\lambda}{2\pi} \times \phi} \longrightarrow$ Phase diff.

(iii) $\boxed{\phi = \frac{2\pi}{T}\Delta t}$

maximum intensity

Condition for Constructive Interference

from equation

$$\boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

For constructive interference

$\cos\phi = +1$, $\phi = 0, 2\pi, 4\pi$ - - -

i.e Phase diff $\boxed{\phi = 2n\pi}$ $[n = 0, 1, 2, 3 \ldots]$

Path difference for constructive interference

By path diff. $(\Delta x) = \frac{\lambda}{2\pi} \times$ phase diff (ϕ)

$$= \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

For constructive interference

$$\boxed{\Delta x = n\lambda} \quad [n = 0, 1, 2, 3 \ldots]$$

Condition for destructive Interference

minimum intensity

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos\phi$$

For destructive interference

$\cos\phi = -1$ $\phi = \pi, 3\pi, 5\pi, \ldots$

i.e $\boxed{\phi = (2n+1)\pi}$, where $n = 0, 1, 2, 3 \ldots$

phase diff.

and path diff $\boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$ $[n = 0, 1, 2, \ldots]$

* If $\cos\phi = +1$, then

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$

or $I_{max} = (a_1 + a_2)^2$

* If $\cos\phi = -1$, then

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

or $I_{min} = (a_1 - a_2)^2$

* $\boxed{I = a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

or $\boxed{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi}$

* $I_{av} = \dfrac{I_{max} + I_{min}}{2} = a_1^2 + a_2^2$

* $\dfrac{I_{max}}{I_{min}} = \dfrac{(a_1+a_2)^2}{(a_1-a_2)^2} = \dfrac{a^2_{max}}{a^2_{min}}$

* For two light source of equal amplitude

$$y = a\sin\omega t + a\sin(\omega t + \phi)$$

$$= a\left[2\sin\frac{2\omega t + \phi}{2} \cdot \cos\left(-\frac{\phi}{2}\right)\right] \quad \left[\because \sin A + \sin B = \sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}\right]$$

$$y = \underbrace{2a\cos\frac{\phi}{2}}\sin\left(\omega t + \frac{\phi}{2}\right) \quad [\because \cos(-\theta) = \cos\theta]$$

here amplitude $A = 2a\cos\frac{\phi}{2}$ [compare with $y = A\sin\omega t$]

or $A^2 = 4a^2\cos^2\frac{\phi}{2}$

or $\boxed{I = 4I_0\cos^2\frac{\phi}{2}}$ $[I \propto a^2]$

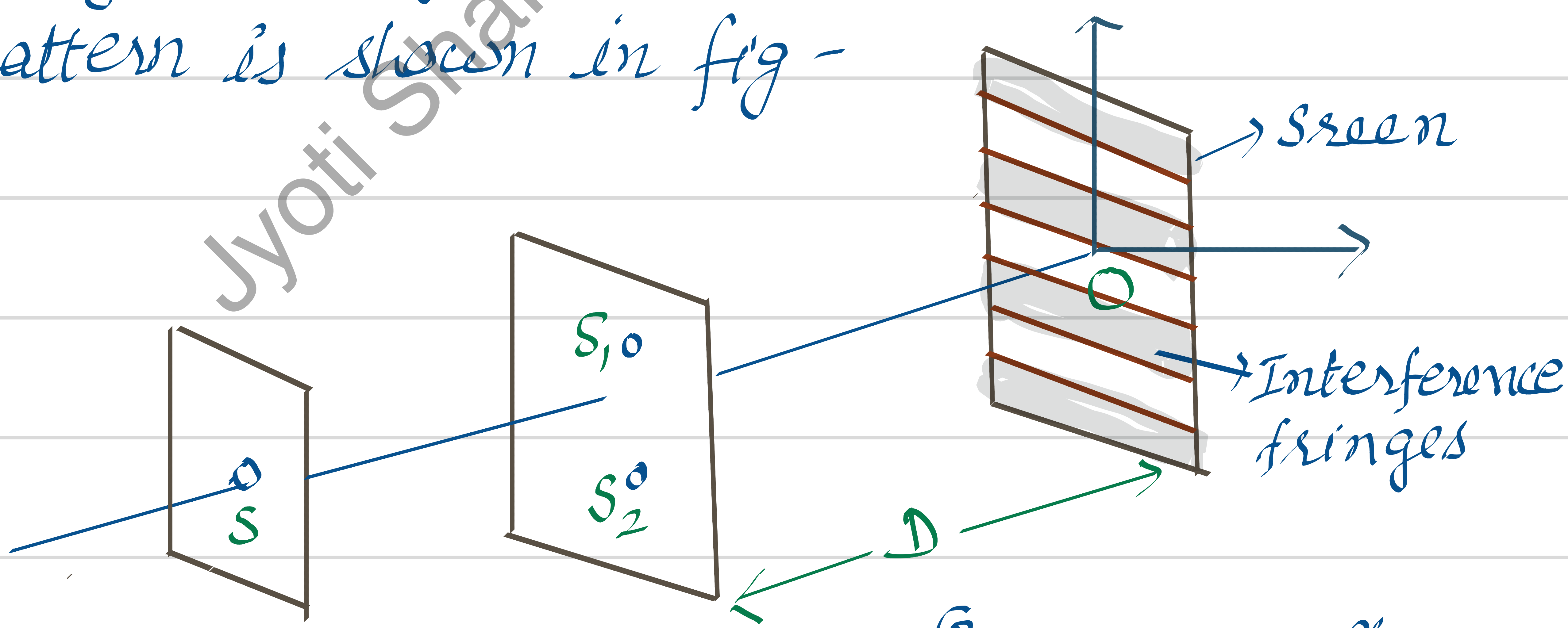
* <u>Sustained Interference</u> - The positions of the maxima and minima of light intensity stay constant

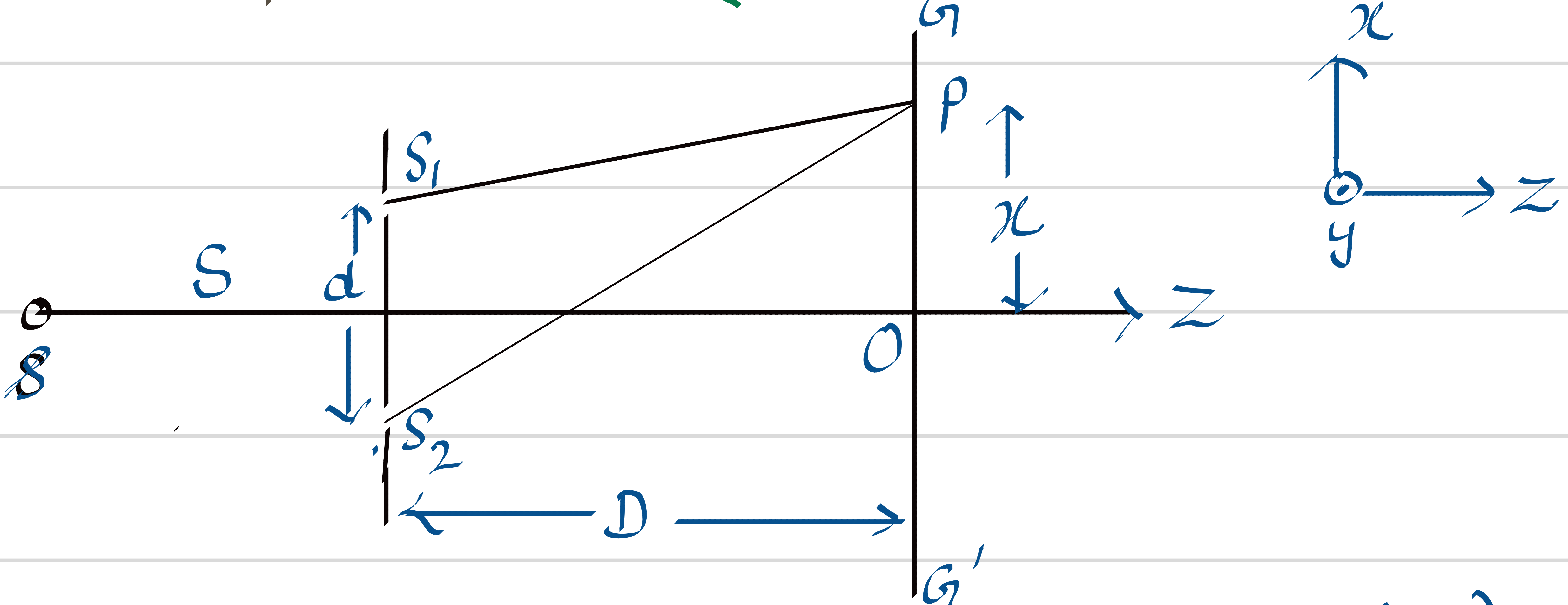
<u>Conditions for sustained interference pattern</u> - (i) The two sources of light must be coherent.

(II) The two sources have same amplitude.
(III) The two sources must be very close.
(IV) Sources should emit light waves continuously.

Young's Double Slit Experiment

YDSE uses two coherent sources of light placed at a small distance apart. YDSE helped in understanding the wave theory of light.
Young's arrangement to produce interference pattern is shown in fig -





*(For interference fringes to be seen - $\frac{s}{S} < \frac{\lambda}{d}$)

* S_1 and S_2 are two coherent sources of light.
* The spherical waves emanating from S_1 and S_2 will produce interference fringes on screen.
* The positions of maximum and minimum intensities can be calculated by using constructive and destructive interference conditions.

→ We have constructive interference when,

$$\frac{xd}{D} = n\lambda$$

i.e $\boxed{x = x_n = \frac{n\lambda D}{d}}$, $n = 0, \pm 1, \pm 2 \ldots$

→ We have destructive interference when,

$$\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$$

i.e $\boxed{x = x_n = (2n+1)\frac{\lambda D}{2d}}$

where $n = 0, \pm 1, \pm 2 \ldots$

The dark and bright bands appear on the screen. Such bands are called fringes.

* The dark and bright fringes are equally spaced.

Fringe width (β)

($\beta_{dark} = \beta_{bright}$)

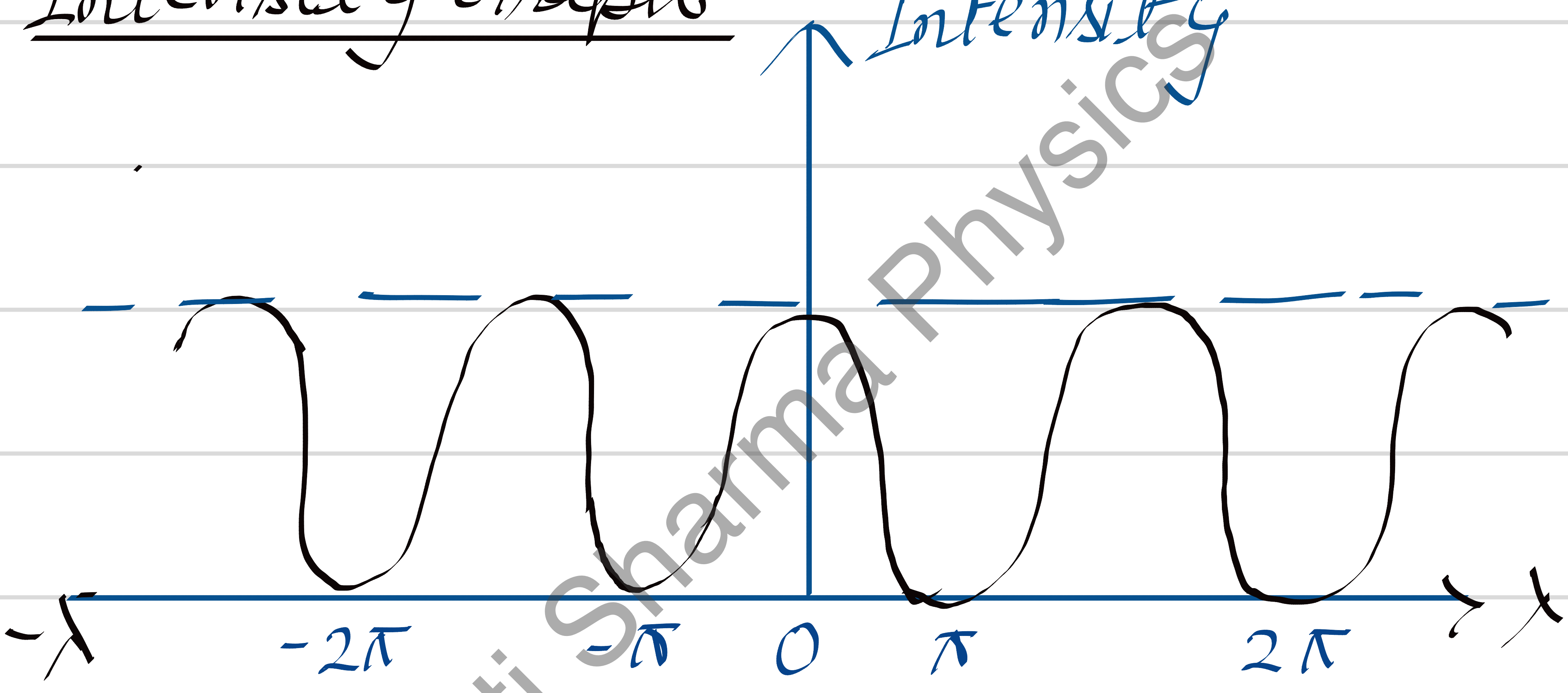
distance b/w slits and scree

$$\boxed{\beta = \frac{D\lambda}{d}}$$

wavelength

distance b/w S_1 & S_2

Intensity Graph



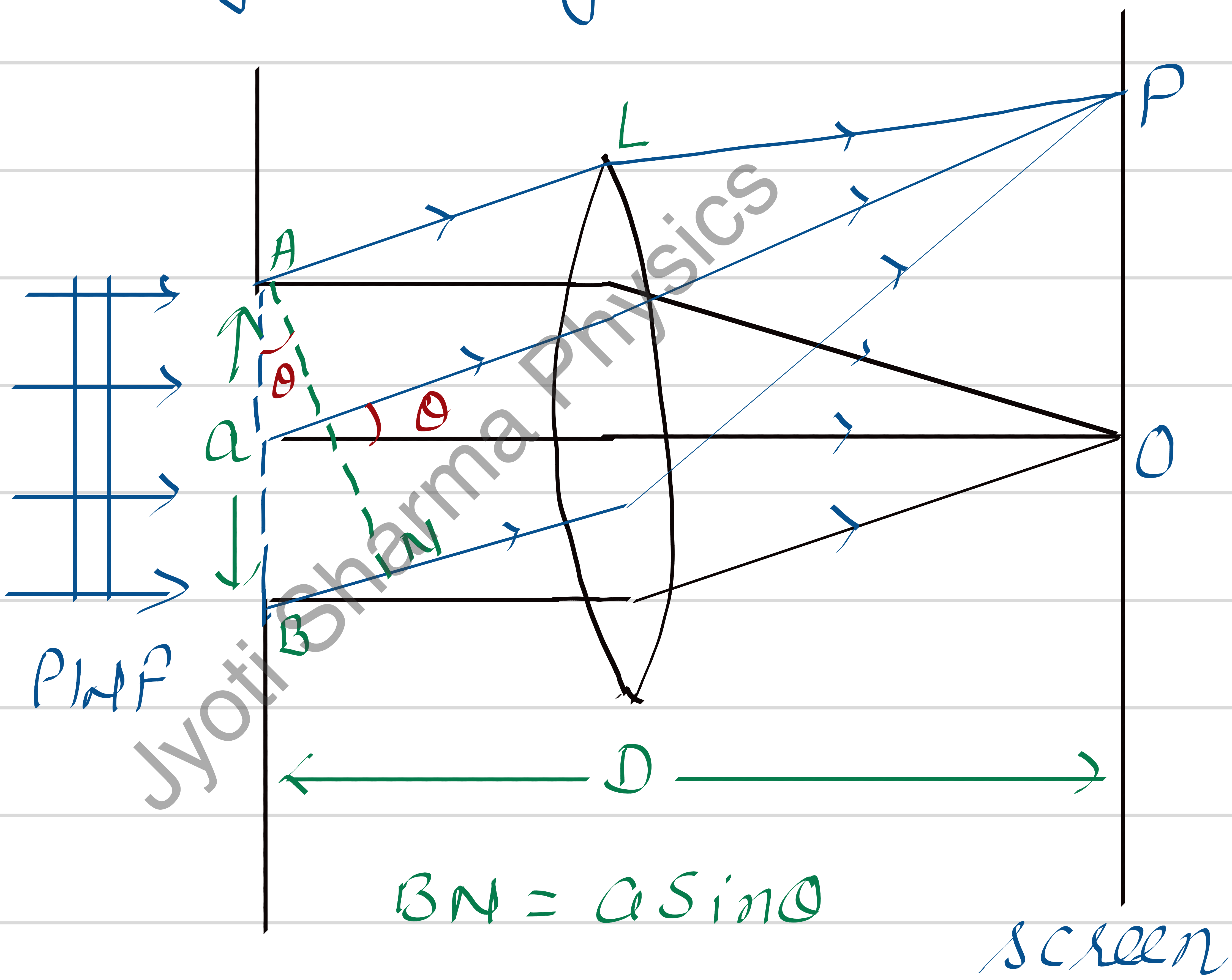
Diffraction: The pheomenon of bending of light around the corners of small obstacales or apertures is called diffraction of light.

Two types of diffraction are:-

Fresnel's diffraction	Fraunhaffer's diffraction
• Source and screen are placed closed	• Source and screen are at far distance
• No lens is required.	• converging lens is used.

Young's Single Slit Experiment (YSSE)

In YSSE a broad pattern with a central bright is seen. On both sides there are alternate dark and bright regions with decreasing intensity are seen.



Central Maximum

When wavelets are focused at point 'O'
path diff. b/w A and B = 0
so a central bright fringe is formed.

Width of central maximum

Angular width

$$2\theta = \frac{\lambda}{a}$$

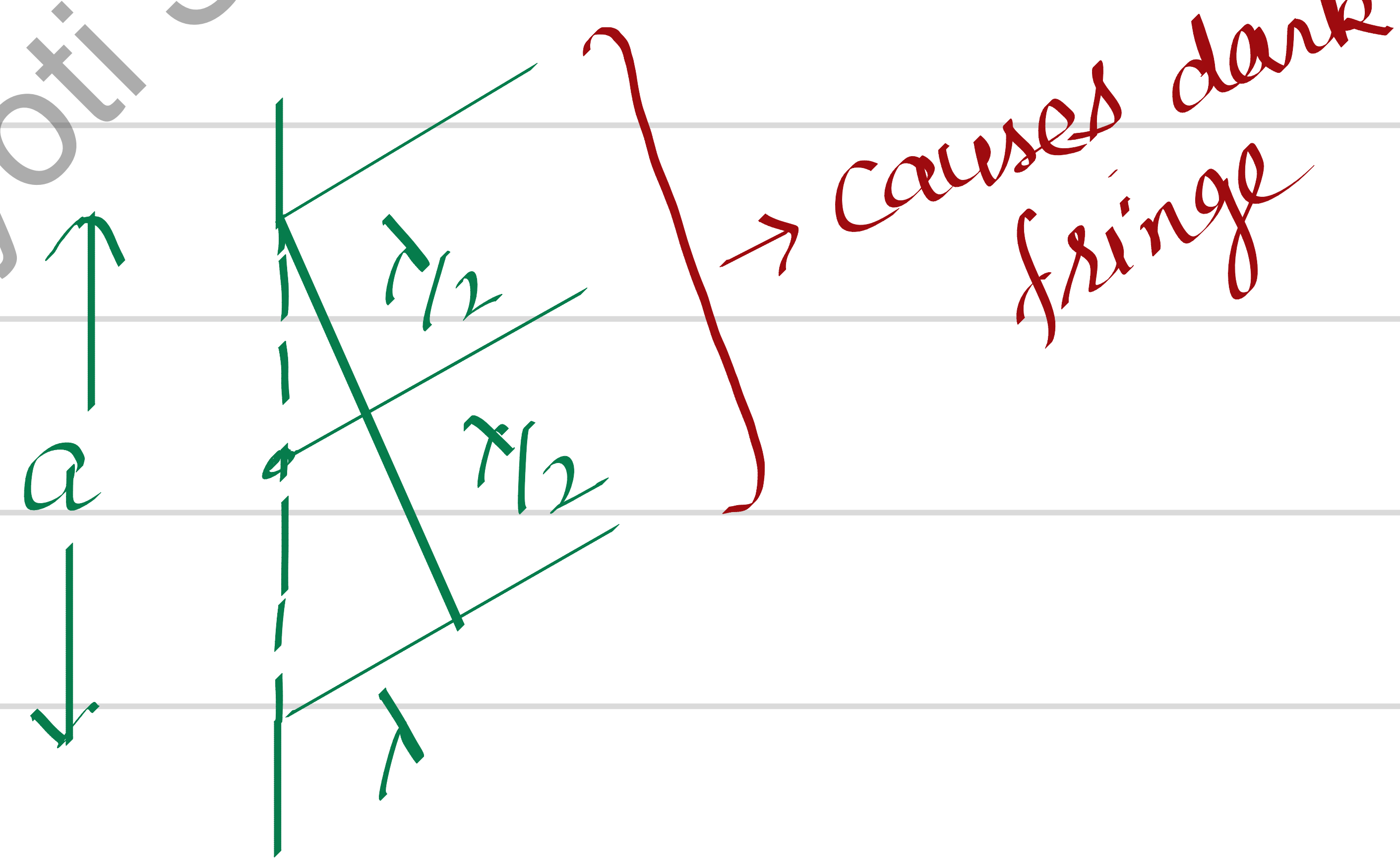
Linear width

$$\beta = D \times 2\theta = \frac{2D\lambda}{a}$$

Angular position of nth minimum

$$\boxed{\theta_n = \sin\theta_n = \frac{n\lambda}{a} \quad \text{where } n = \pm 1, \pm 2, \pm 3 \ldots}$$

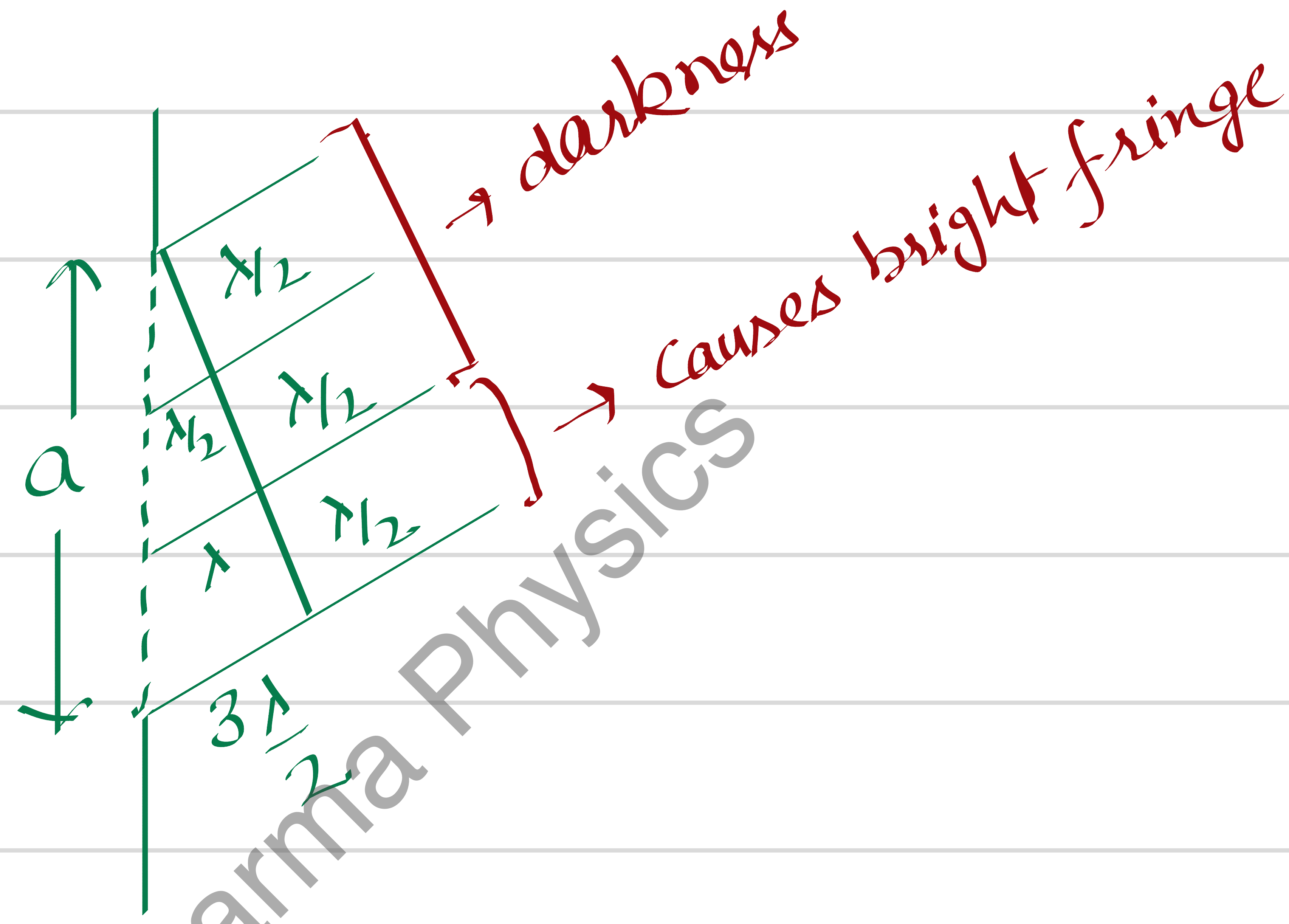
Slit AB is suppose to be divided into even numbers (2, 4, 6 ...). So the path diff $= \frac{\lambda}{2}$.
Therefore dark fringe is formed.



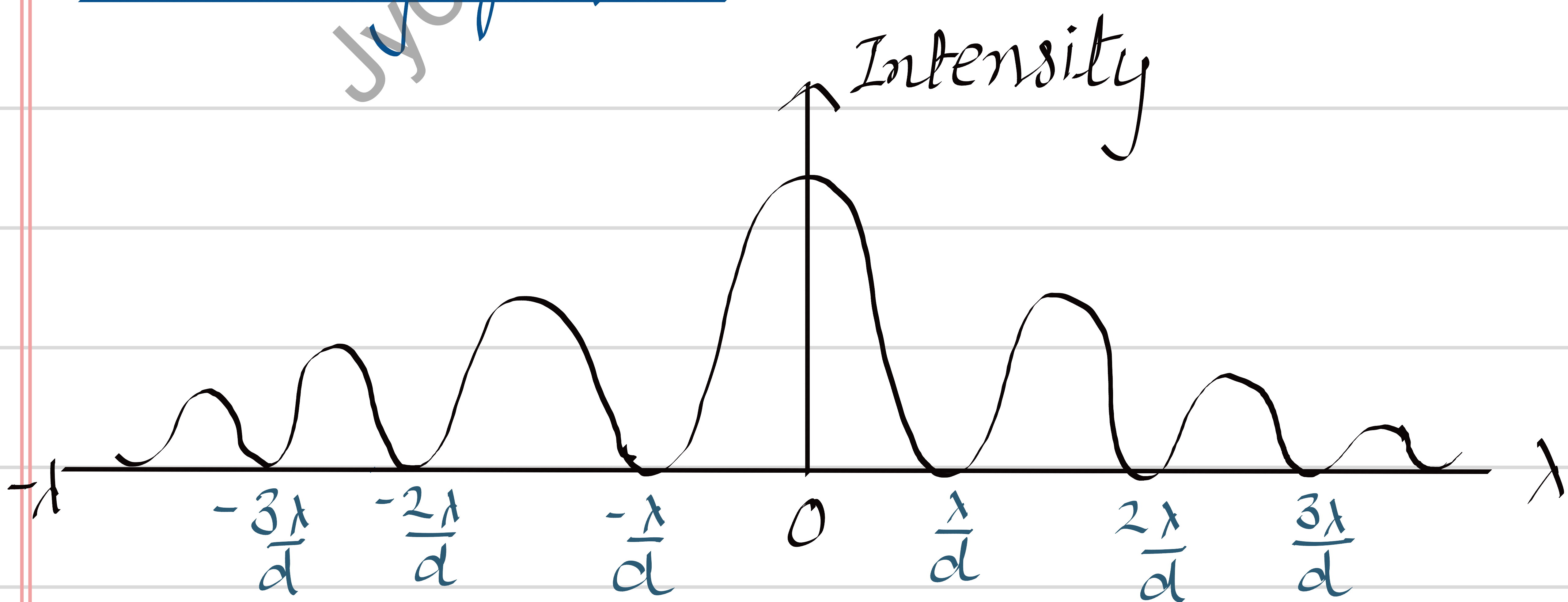
Angular position of nth maximum

$$\boxed{\theta_n = \sin\theta_n = \frac{(2n+1)\lambda}{2a} \quad \text{where } n = \pm 1, \pm 2, \pm 3 \ldots}$$

Slit AB is suppose to be divided into odd numbers (3, 5, 7 - - - - -). The 3rd, 5th - - part form bright fringe of decreasing intensity.



Intensity graph



* When monochromatic source is replaced by white light, we get a coloured diffraction pattern.

* For YDSE, fringe width

$$\beta = \frac{D\lambda}{d} \quad [\text{for all fringes}]$$

i.e If $D \uparrow$, $\beta \uparrow$

If $d \uparrow$, $\beta \downarrow$

If $\lambda \uparrow$, $\beta \downarrow$

here λ is wavelength of monochromatic light.

* $\beta' \rightarrow \frac{\beta}{\mu}$, $\mu \rightarrow$ Refractive index.

* For YSSE

The diffraction pattern has a central bright maximum which is twice as wide as other maxima.

* In YDSE and YSSE the 'd' and 'a' respectively should be quite small to be observe good interference and diffraction pattern (of the order 0.1 or 0.2 mm)