Example ~~7.1~~ 6.1

Find the centre - - - - - - - - - - - 0.5 m long.

Solution:

Given,

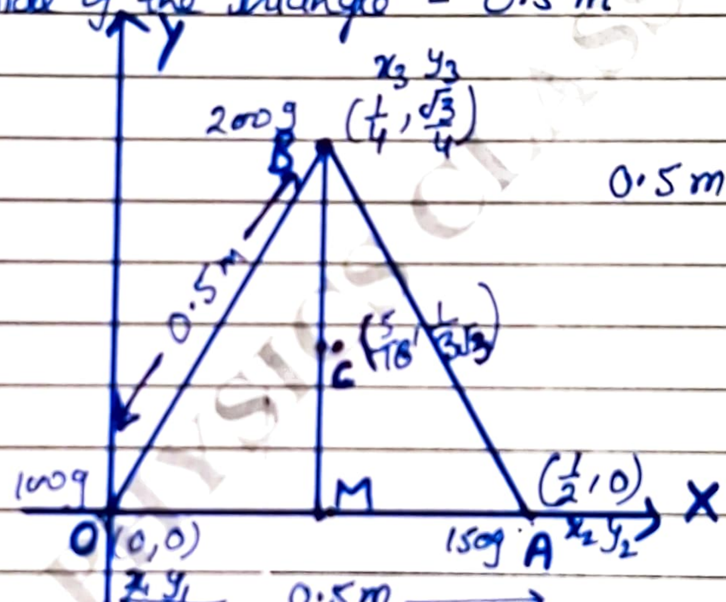
$m_1 = 100\,g$, $m_2 = 150\,g$ $m_3 = 200\,g$

So, $M = m_1 + m_2 + m_3$

$= 100 + 150 + 200$

$= 450\,g$

Each side of the triangle = 0.5 m



$0.5\,m = \frac{1}{2}\,m$

We know coordinates of centre of mass (COM) is given by

$$x = \frac{\sum m_i x_i}{M} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

and

$$y = \frac{\sum m_i y_i}{M} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{M}$$

$$BM^2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2$$

$$= \frac{1}{4} - \frac{1}{16}$$

$$= \frac{4-1}{16} = \frac{3}{16}$$

or $BM = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}\ m$

Now,

$$x = \frac{100\times 0 + 150\times\frac{1}{2} + 200\times\frac{1}{4}}{450}$$

$$= \frac{0 + 75 + 50}{450} = \frac{125}{450}$$

$$x = \frac{5}{18}$$

Now

$$y = \frac{100\times 0 + 150\times 0 + 200\times\frac{\sqrt{3}}{4}}{450}$$

$$= \frac{50\sqrt{3}}{450}$$

$$y = \frac{\sqrt{3}}{9}$$

So. $(x, y)_{com} = \left(\frac{5}{18}, \frac{\sqrt{3}}{9}\right)$

$$= \left(\frac{5}{18}, \frac{1}{3\sqrt{3}}\right)$$

NCERT PHYSICS SOLUTIONS

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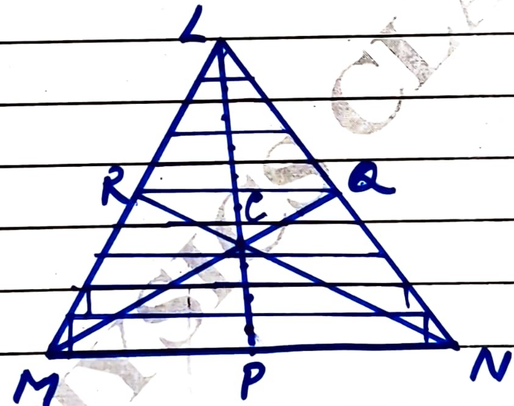
Example ~~7.2~~ 6.2

Find the centre of mass of a triangular lamina.

Solution:

The centre of the mass of triangular lamina is the point of intersection of medians. i.e. at the centroid.

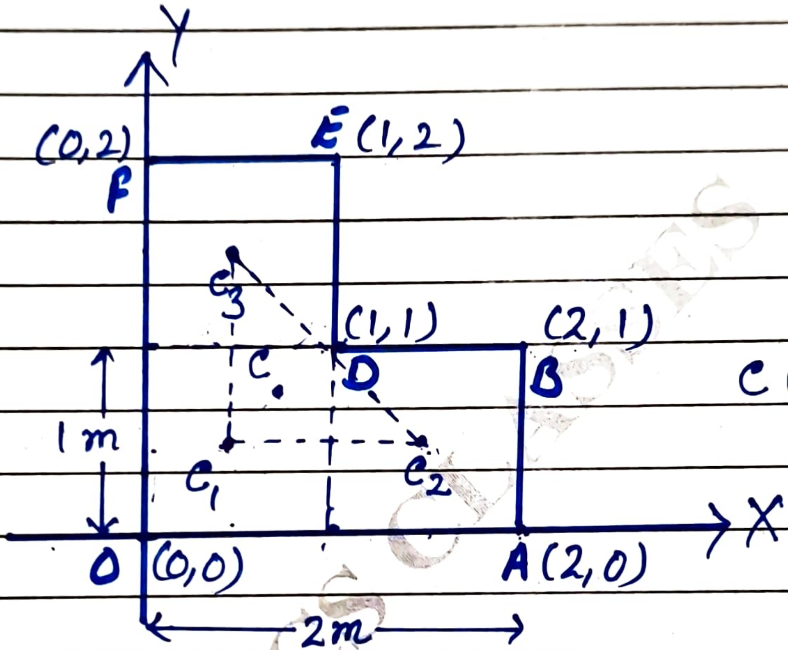
A median divides the lamina into two equal triangles. Thus equal mass is divided for uniform lamina.



Example 7·3

Find the centre - - - - - - - - - lamina is 3 kg.

Solution:



$C\left(\frac{5}{6}, \frac{5}{6}\right)$

coordinates of c_1, c_2 and c_3 are -

$C_1\left(\underset{x_1}{\frac{1}{2}}, \underset{y_1}{\frac{1}{2}}\right)$, $C_2\left(\underset{x_2}{\frac{3}{2}}, \underset{y_2}{\frac{1}{2}}\right)$ and $C_3\left(\underset{x_3}{\frac{1}{2}}, \underset{y_3}{\frac{3}{2}}\right)$

Here $M = 3\text{ kg}$

$m_1 = m_2 = m_3 = 1\text{ kg}$

Now

$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{M}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{3}{2} + 1\times\frac{1}{2}}{3}$$

$$= \frac{\frac{1}{2} + \frac{3}{2} + \frac{1}{2}}{3} = \frac{5/2}{3} = \frac{5}{6}$$

$$x = \frac{5}{6}\, m$$

and

$$y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M}$$

$$= \frac{1\times\frac{1}{2} + 1\times\frac{1}{2} + 1\times\frac{3}{2}}{3}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}+\frac{3}{2}}{3} = \frac{5/2}{3}$$

$$y = \frac{5}{6}\, m$$

$$(x, y)_{COM} = \left(\frac{5}{6}, \frac{5}{6}\right)$$

Example 7.4

Find the scalar and vector products of two vectors $\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 3\hat{k}$

Solution:

Given

$$\vec{a} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{b} = -2\hat{i} + \hat{j} - 3\hat{k}$$

Scalar product -

$$\vec{a} \cdot \vec{b} = (3\hat{i} - 4\hat{j} + 5\hat{k}) \cdot (-2\hat{i} + \hat{j} - 3\hat{k})$$
$$= -6 - 4 - 15$$
$$= -25$$

$\left[\because \hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0; \quad \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \right.$

so, $\vec{a} \cdot \vec{b} = -25$ Ans

Vector product

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -4 & 5 \\ +1 & -3 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 5 \\ -2 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & -4 \\ -2 & 1 \end{vmatrix}$$

$$= \hat{i}(12-5) - \hat{j}(-9+10) + \hat{k}(3-8)$$

$$\vec{a} \times \vec{b} = 7\hat{i} - \hat{j} - 5\hat{k}$$

also,

$$\vec{b} \times \vec{a} = -7\hat{i} + \hat{j} + 5\hat{k} \qquad [\because \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}]$$

Ans

Example 7.5

Find the torque - - - - - - is $\hat{i}-\hat{j}+\hat{k}$.

Solution:

Given,

$$\vec{F} = 7\hat{i}+3\hat{j}-5\hat{k}$$

and $\vec{r} = \hat{i}-\hat{j}+\hat{k}$

We know

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & +1 \\ 7 & 3 & -5 \end{vmatrix}$$

or $\vec{\tau} = \hat{i}(5-3)-\hat{j}(-5-7)+\hat{k}(3+7)$

$$= 2\hat{i}+12\hat{j}+10\hat{k}$$

i.e the torque $\vec{\tau} = 2\hat{i}+12\hat{j}+10\hat{k}$

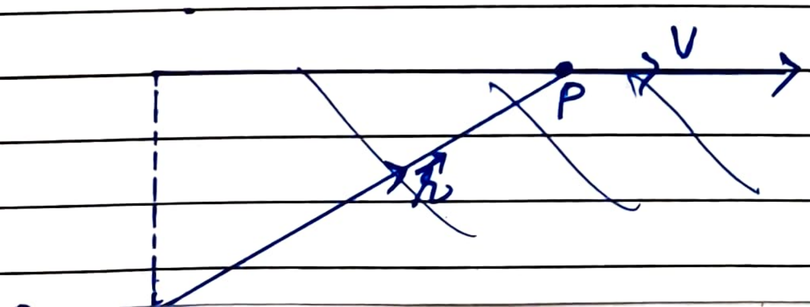
Ans

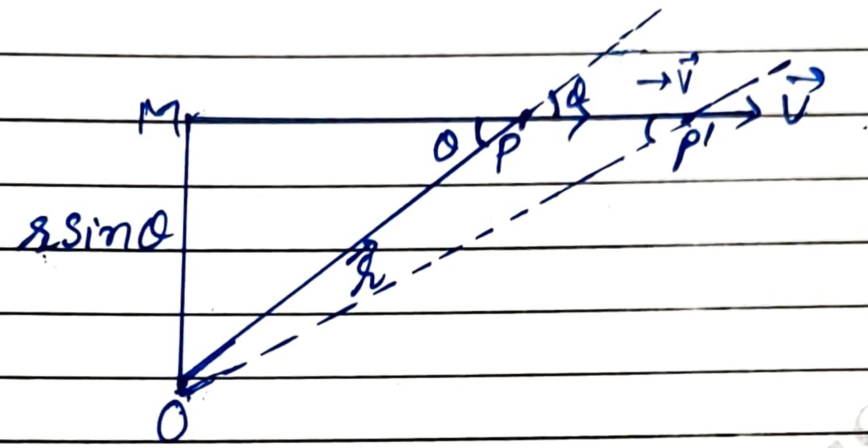
Example 7.6

Show that the - - - - - - - - - - the motion.

Solution:

Let the particle P, with velocity v is moving in a straight line as shown in fig. Here we calculate the angular momentum of the particle about point O.





We have,

$$\vec{L} = \vec{r} \times \vec{p}$$

or $L = rp\sin\theta$

or $L = mvr\sin\theta$ $[\because p = mv]$

here m is constant

v is constant (given)

and for any position of P, $r\sin\theta$ also remains constant. Therefore

$$L = mvr\sin\theta = \text{Constant}$$

i.e

L is Constant.

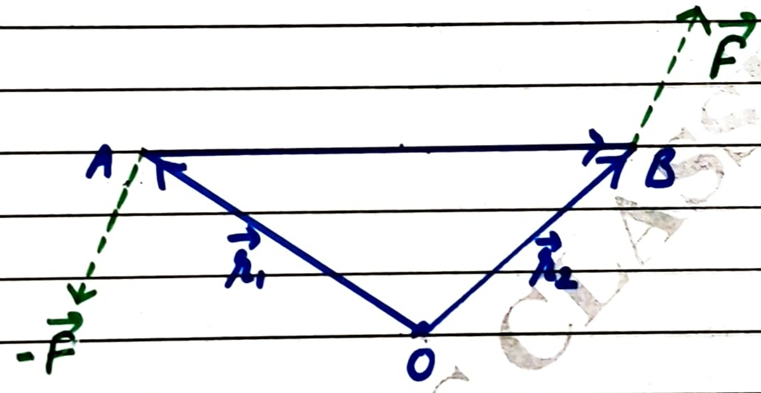
Thus L remains the same in magnitude and direction and therefore conserved.

Example 7.7

Show that moment - - - - - - - - - the moments.

Solution:

Consider a couple acting on a rigid body at points A and B with position vectors $\vec{r}_1$ and $\vec{r}_2$ with respect to origin O respectively.



The moment of couple = moment of force at A + moment of force at B

or $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

$= \vec{r}_1 \times (-\vec{F}) + \vec{r}_2 \times \vec{F}$ $\quad [\because \vec{\tau} = \vec{r} \times \vec{F}]$

or $= (\vec{r}_2 - \vec{r}_1) \times \vec{F}$

but here

$\vec{r}_1 + \vec{AB} = \vec{r}_2$ $\quad$ [$\because$ By triangle law]

or $\vec{r}_2 - \vec{r}_1 = \vec{AB}$

so

$$\vec{\tau} = \vec{AB} \times \vec{F}$$

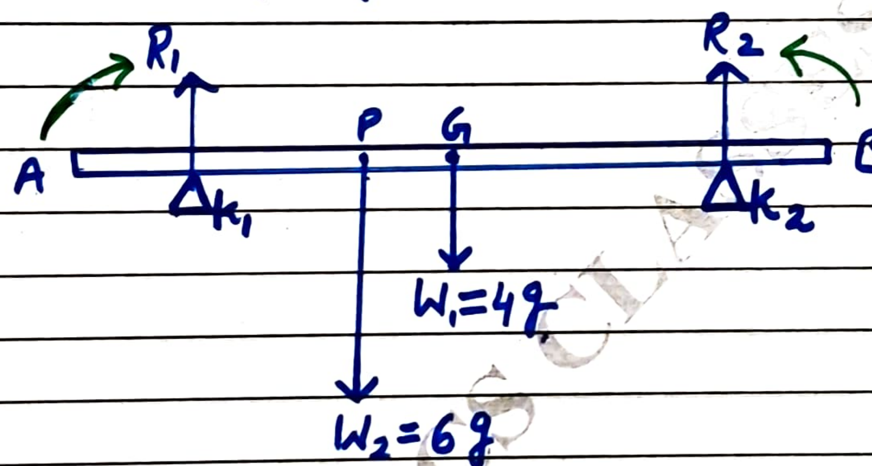
clearly moment of couple τ is independent of origin O.

Example 7.8

A meter bar 70 cm - - - - - - knife edges.

Solution:

Figure shows the rod AB with knife edges at K_1 and K_2. G is the centre of the gravity of the rod.



In fig

$AB = 70\,cm$, $AG = BG = 35\,cm$

$AK_1 = BK_2 = 10\,cm$, $PG = 5\,cm$

$K_1G = K_2G = 25\,cm$

$g = 9.8\,m/s^2$

$W_1 = 4g = 4 \times 9.8\,N$, $W_2 = 6g = 6 \times 9.8\,N$

For translational equilibrium of rod

Net force $= 0$

i.e

$R_1 + R_2 - W_1 - W_2 = 0$ [∵ upward forces → +ve and downward forces → -ve]

or $R_1 + R_2 = W_1 + W_2$

or $R_1 + R_2 = 4g + 6g$

or $R_1 + R_2 = 10g$ — (1)

For rotational equilibrium of the rod

Net torque $= 0$

i.e.

$-R_1 \times K_1G + R_2 \times K_2G + W_2 \times PG = 0$

or $-R_1 \times \frac{25}{100} + R_2 \times \frac{25}{100} + 6g \times \frac{5}{100} = 0$

or [∵ clockwise → −ve, anticlockwise → +ve]

$(R_2 - R_1)25 + 30g = 0$

or $(R_2 - R_1)25 = -30g$

or

$R_1 - R_2 = \frac{30}{25} g$

or $R_1 - R_2 = \frac{6}{5} g$ — (2)

from eq^n (1) and (2)

$R_1 + \cancel{R_2} = 10g$ [on adding]

$R_1 - \cancel{R_2} = \frac{6}{5} g$

$2R_1 = \left(10 + \frac{6}{5}\right) g$

or $2R_1 = \frac{56}{5} \times g$

or $R_1 = \frac{56 \times 9.8}{5 \times 2} = \frac{56 \times 9.8}{10}$

$= 54.88\ N$

Now from eqn (1)

$R_1 + R_2 = 10g$

or $R_2 = 10g - R_1$

$= 10g - 5.6g$

$= 4.4g$

$= 4.4 \times 9.8$

$R_2 = 43.12\ N$

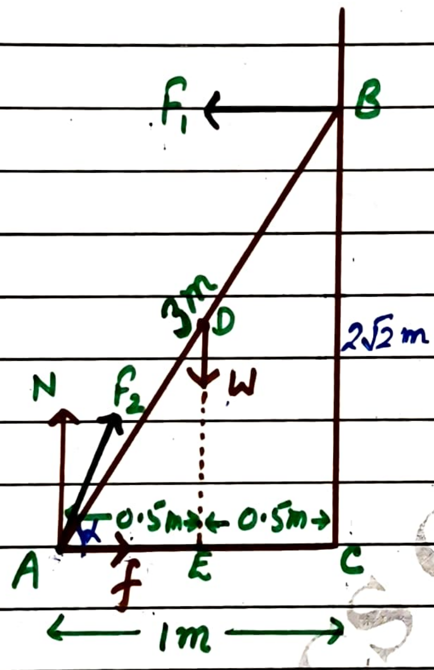
i.e $R_1 = 54.88\ N$ and $R_2 = 43.12\ N$ Ans

Thus the reactions of the support are about 55 N and 43 N at K_1 and K_2 respectively.

Example 7.9

A 3 m long ladder - - - - - - - - - and the floor.

Solution:



$$BC^2 = AB^2 - AC^2$$
$$= 3^2 - 1^2$$
$$= 9 - 1$$
$$BC^2 = 8$$
$$BC = 2\sqrt{2}\ m$$

Given,

Length of ladder $AB = 3\ m$

Weight of ladder $W = 20 \times 9.8$ $[\because W = mg]$

$= 196\ N$

Distance of ladder from wall $AC = 1\ m$

For translational equilibrium

$$F_{net} = 0$$

$$N - W = 0$$

or $N = W$

$= mg$

$= 20 \times 9.8 = 196\ N$

and $f - F_1 = 0$

or $f = F_1$

Now

For rotational Equilibrium

$\tau_{net} = 0$ (about point 'A')

$$F_1 \times BC - W \times AE = 0$$

$$F_1 \times 2\sqrt{2} - mg \times 0.5 = 0$$

or

$$F_1 = \frac{mg \times 0.5}{2\sqrt{2}}$$

$$= \frac{196 \times 0.5}{2\sqrt{2}}$$

$$F_1 = \frac{49}{\sqrt{2}} = \frac{49}{1.414} = 34.75\,N$$

$$f = F_1 = \frac{49}{\sqrt{2}} = 34.75\,N$$

$$F_1 = 34.75\,N$$ Ans

Now resultant of N and f (say F_2)

$$F_2 = \sqrt{N^2 + f^2}$$

$$= \sqrt{(196)^2 + \left(\frac{49}{\sqrt{2}}\right)^2}$$

$$F_2 = \sqrt{(49\times4)^2 + \left(\frac{49}{\sqrt{2}}\right)^2}$$

$$= 49\sqrt{16+\frac{1}{2}}$$

$$= 49\sqrt{16+0.5}$$

$$= 49\sqrt{16.5}$$

$$= 49\times4.06$$

$$F_2 = 198.94\ N$$

Now

$$\tan\alpha = \frac{N}{f} = \frac{196}{\frac{49}{\sqrt{2}}}$$

$$= 4\sqrt{2}$$

$$= 4\times1.414$$

$$\tan\alpha = 5.656$$

or $\alpha = \tan^{-1}(5.656)$

or $\alpha \simeq 80^\circ$

So $F_1 = 34.75\ N$

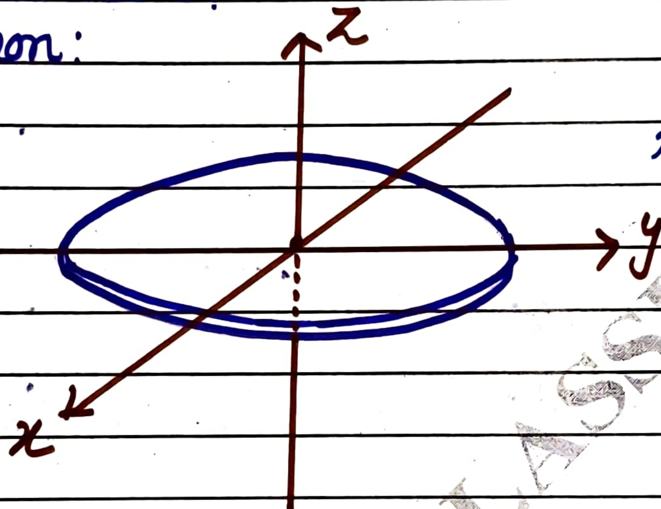
$F_2 = 198.94\ N$

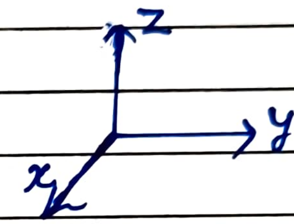
$\tan\alpha = 5.656 \Rightarrow \alpha = 80^\circ$ Ans

Example 7.10

What is the moment of inertia of a disc about one of its diameter?

Solution:





We know moment of inertia (M.I.) of a disc about its centre is given by

$$I = \frac{MR^2}{2} \quad , \quad R \to \text{Radius of the disc}$$

here $I_z = \frac{MR^2}{2}$

By the theorem of perpendicular axes -

$$I_z = I_x + I_y$$

Now x and y axes are along two diameters of the disc.

hence $I_x = I_y$

so, $I_z = 2I_x \Rightarrow I_x = \frac{I_z}{2}$

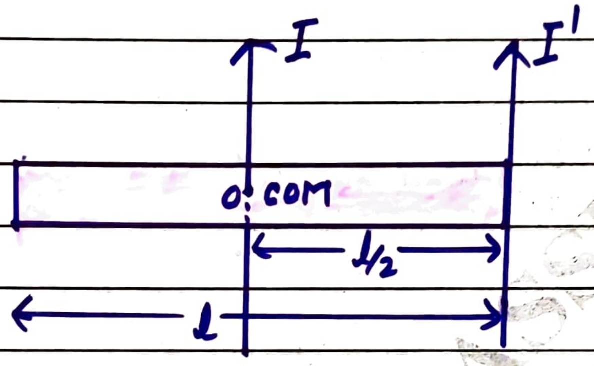
put $I_z = \frac{MR^2}{2}$, then

$$I_x = \frac{MR^2}{4} \quad \text{also, } I_y = \frac{MR^2}{4}$$

Example 7.11

What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it through one end?

Solution:



For the rod of mass M and length l, M.I is given by

$$I = \frac{Ml^2}{12}$$

By parallel axes theorem

$$I' = I + Ma^2$$

here $a = \frac{l}{2}$, then

$$I' = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2$$

$$= \frac{Ml^2}{12} + \frac{Ml^2}{4}$$

$$= \frac{Ml^2}{4}\left(\frac{1}{3} + 1\right)$$

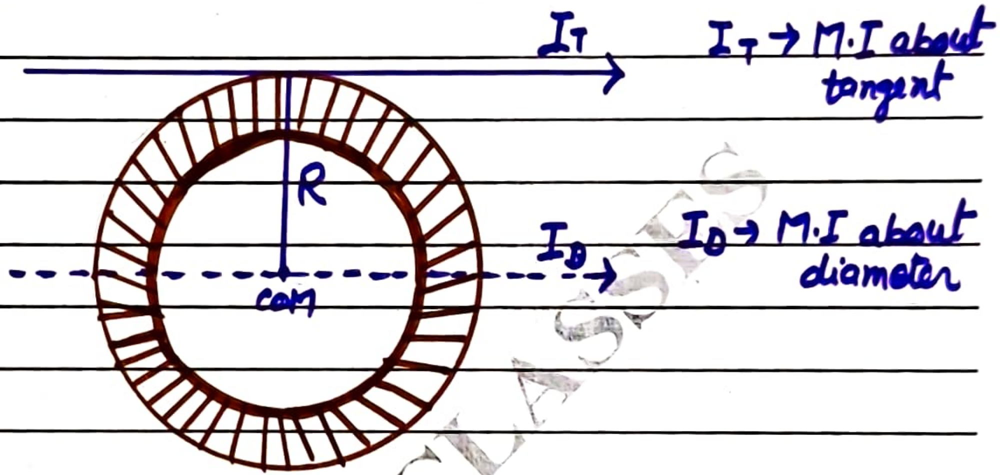
$$= \frac{4Ml^2}{4 \times 3}$$

or $$I' = \frac{Ml^2}{3}$$

Example 7.12

What is the moment of inertia of a ring about a tangent to the circle of the ring?

Solution:



The tangent of the ring is $\parallel$ to the one of its diameter.

By parallel axes theorem, we have,

$$I_T = I_D + MR^2$$, $R \to$ Radius of the ring

here, we know that

$$I_D = \frac{MR^2}{2}$$

then $$I_T = \frac{MR^2}{2} + MR^2$$

$$I_T = \frac{3MR^2}{2}$$

Example 7·13

Obtain equation $\omega = \omega_0 + \alpha t$ from first principles.

Solution:

Here angular acceleration α is uniform.

Hence

$$\alpha = \frac{d\omega}{dt}$$

or $$d\omega = \alpha dt$$

On integrating,

$$\int_{\omega_0}^{\omega} d\omega = \alpha \int_0^t dt$$

or $$[\omega]_{\omega_0}^{\omega} = \alpha [t]_0^t$$

or $$\omega - \omega_0 = \alpha t$$

Thus $$\boxed{\omega = \omega_0 + \alpha t}$$

Example 7·14

The angular speed - - - - - - - during this time?

Solution:

Given,

Initial angular speed $\omega_0 = 1200$ rpm

$$= \frac{1200 \times 2\pi}{60} \quad [1m = 60s]$$

$$= 40\pi \text{ Rad/s}$$

Final angular speed $\omega = 3120$ rpm

$$= \frac{3120 \times 2\pi}{60}$$

$$= 104\pi \text{ rad/s}$$

and $t = 16$ s

(i) Angular acceleration

$$\alpha = \frac{\omega - \omega_0}{t}$$

$$= \frac{104\pi - 40\pi}{16}$$

$$= \frac{64\pi}{16}$$

$$= 4\pi \text{ rad/s}$$ Ans

(ii) To find number of revolutions we find angular displacement.

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = 40\pi \times 16 + \frac{1}{2} \times 4\pi \times (16)^2$$

$$= 640\pi + 2\pi \times 256$$

$$= 640\pi + 512\pi$$

$$\theta = 1152\pi$$

Number of revolutions $n = \frac{1152\pi}{2\pi} = 576$

Thus the engine makes 576 revolutions Ans

Example 7.15

A cord of negligible- - - - - - -parts (b) and (d)

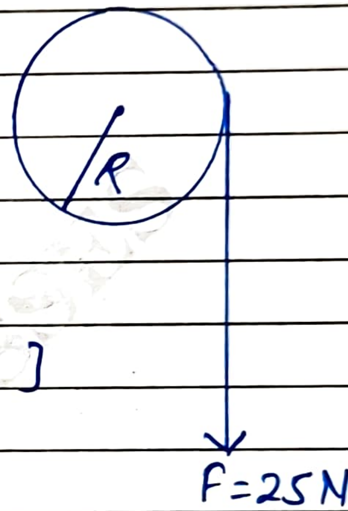
Solution :

Given.

$m = 20 kg$

$R = 20 cm = 0.2 m$

$F = 25 N$



(a) Angular acceleration

$\alpha = \frac{\tau}{I}$ [$\because \tau = I\alpha$]

here $\tau = F \times R$

$= 25 \times 0.2$

$= 5 \ Nm$

and $I = mR^2/2$ [$\because I_{flywheel} = \frac{MR^2}{2}$]

$= 20 \times (0.2)^2/2$

$= 10 \times 0.04$

$= 0.40 \ kg\text{-}m^2$

so $\alpha = \frac{5}{0.4} = 12.5 \ rad/s^2$

$\alpha = 12.5 \ rad/s^2$

(b) Work done by the pull

$W = F \cdot d$

$= 25 \times 2$ [$\because d = 2m$

$= 50 J$

(c) Kinetic energy of the wheel

$$K.E = \frac{1}{2} I \omega^2$$

to find ω we use.

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

given $\omega_0 = 0$, so

$$\omega^2 = 2 \times 12.5 \times \theta$$

$$= 25\theta$$

now we find θ (angular displacement) by

$$\theta = \frac{arc}{radius} = \frac{\text{length of unwound string}}{radius}$$

$$= \frac{2}{0.2}$$

$$\theta = 10 \text{ rad}$$

then by $\omega^2 = 25\theta$

$$\omega^2 = 25 \times 10$$

$$\omega^2 = 250 \ (rad/s)^2$$

hence

$$K.E = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} \times 0.4 \times 250$$

$$K.E = 50 \text{ J}$$

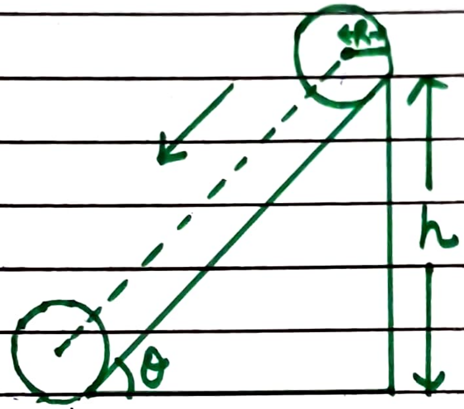
(d) On comparing (b) and (c) part we see that

K.E = Work done [No loss of energy due to friction]

Example 7.16

Three bodies - - - - - - - maximum velocity?

Solution:



$$V = \sqrt{\frac{2gh}{1 + K^2/R^2}}$$

For Ring, $K^2 = R^2$

$$V_{ring} = \sqrt{\frac{2gh}{1 + R^2/R^2}} = \sqrt{\frac{2gh}{2}}$$

$$V_{ring} = \sqrt{gh} \quad \text{— (1)}$$

For solid cylinder, $K^2 = R^2/2$

$$V_{disc} = \sqrt{\frac{2gh}{1 + \frac{R^2/2}{R^2}}} = \sqrt{\frac{2gh}{1+\frac{1}{2}}}$$

$$V_{disc} = \sqrt{\frac{4gh}{3}} \quad \text{— (2)}$$

For solid sphere , $K^2 = \frac{2}{5} R^2$

$$V_{sphere} = \sqrt{\frac{2gh}{1 + \frac{2}{5}R^2/R^2}}$$

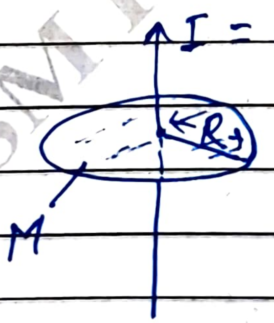
$$= \sqrt{\frac{2gh}{1 + \frac{2}{5}}}$$

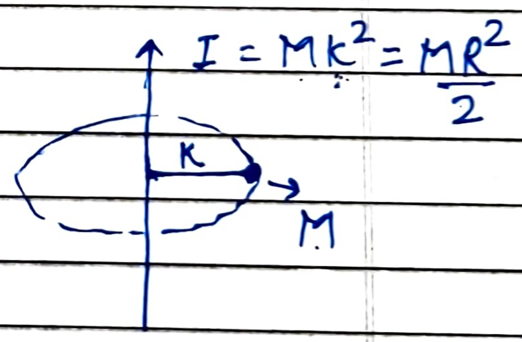
$$V_{sphere} = \sqrt{\frac{10gh}{7}} \quad \text{--- (3)}$$

$V_s > V_d > V_r$

i.e the solid sphere will reach the ground first as its velocity is greatest.

* Radius of Gyration





k → radius of gyration

For sphere, $K^2 = \frac{2}{5} R^2$

$$V_{sphere} = \sqrt{\frac{2gh}{1 + \frac{2}{5}R^2/R^2}}$$

$$V_{sphere} = \sqrt{\frac{2gh}{1 + 2/5}}$$

$$V_{sphere} = \sqrt{\frac{10gh}{7}} \quad \text{—(3)}$$

From eqn (1), (2) and (3), on comparing, we get,

$$V_{sphere} > V_{disc} > V_{ring}$$

i.e the velocity of sphere is greatest. So it will reach the ground first.