 FORMULA SHEET Chapter - 7

Alternating Current

1 Alternating Emf

$$\boxed{\mathcal{E} = \mathcal{E}_0 \sin \omega t \quad \text{or} \quad \mathcal{E} = \mathcal{E}_0 \cos \omega t}$$

$\mathcal{E}_0 \rightarrow$ maximum value of emf

$\omega \rightarrow$ Angular frequency

$\omega = 2\pi f$, where f is frequency

2. Average value of AC current and AC voltage for one complete cycle

$$(I_{av})_{1\,cycle} = 0$$

and $(\mathcal{E}_{av})_{1\,cycle} = 0$

3. Average value of AC current and Voltage for half cycle

$$\boxed{(I_{av})_{half} = \frac{2}{\pi} I_0 = 0.637\, I_0}$$

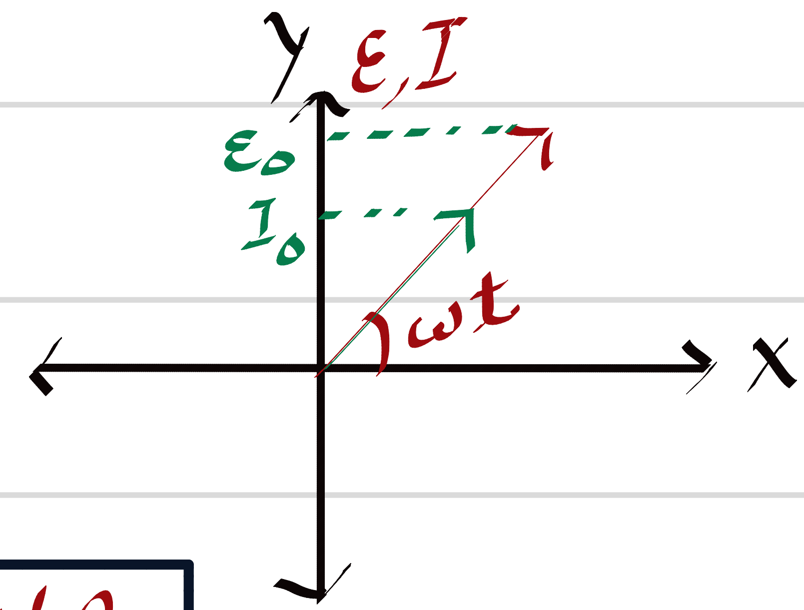
and $\boxed{(\mathcal{E}_{av})_{half} = \frac{2}{\pi} \mathcal{E}_0 = 0.637\, \mathcal{E}_0}$

4. Root mean square current (I_{rms} or I_{eff})

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}}$$

also,
$$\boxed{I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + \cdots I_n^2}{n}} = \sqrt{\frac{1}{T}\int_0^T I^2\,dt}}$$

5. AC circuit containing Resistor Only

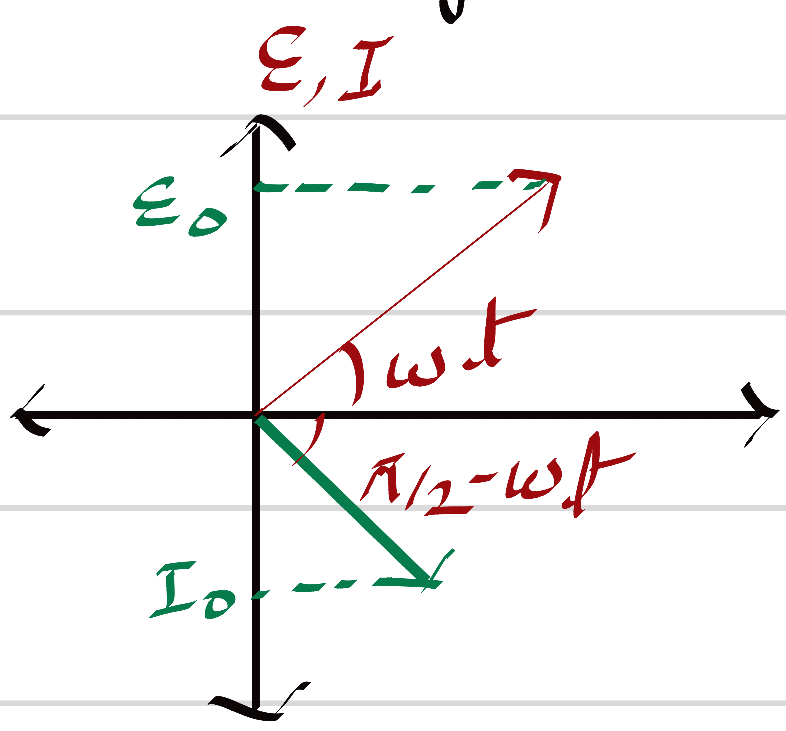


$$\boxed{\begin{aligned} &\text{Emf } \varepsilon = \varepsilon_0 \sin\omega t \\ &\text{current } I = I_0 \sin\omega t \end{aligned}}$$

Resistance $R = \frac{\varepsilon}{I} = \frac{\varepsilon_0}{I_0}$

$\boxed{* \ \varepsilon \text{ and } I \text{ are in same phase}}$

6. AC circuit containing Inductor Only



$$\boxed{\begin{aligned} &\text{Emf } \varepsilon = \varepsilon_0 \sin\omega t \\ &\text{current } I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \end{aligned}}$$

$\boxed{* \ I \text{ lags behind } \varepsilon \text{ by } \frac{\pi}{2}}$

Inductive Reactance (X_L)

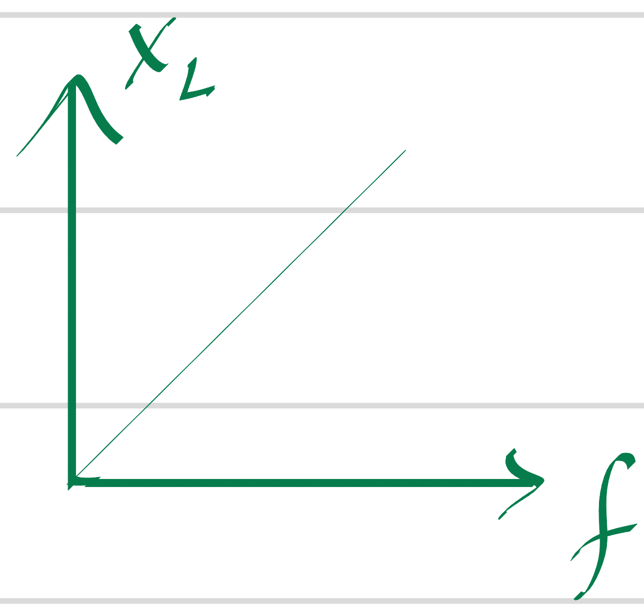
$$\boxed{X_L = \omega L = 2\pi f L}$$

SI unit of X_L is Ohm

here $\omega \rightarrow$ angular frequency in rad s^{-1}

$f \rightarrow$ frequency in rev.s^{-1} or Hz

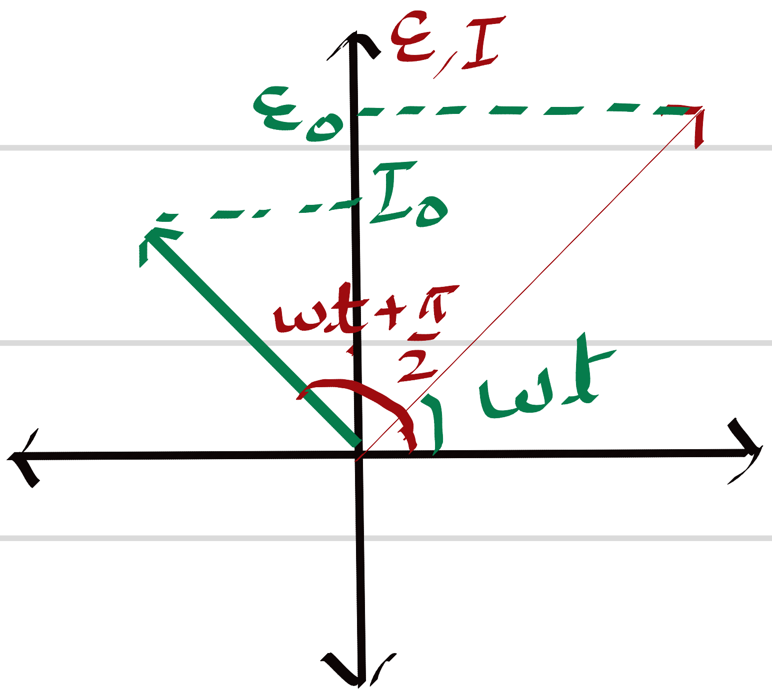
$L \rightarrow$ Inductance in henry



7. AC circuit containing capacitor only

$$\boxed{\begin{aligned} \mathcal{E} &= \mathcal{E}_0 \sin\omega t \\ I &= I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \end{aligned}}$$

* Current I is leading emf $\mathcal{E}$ by $\frac{\pi}{2}$



Capacitive Reactance (X_C)

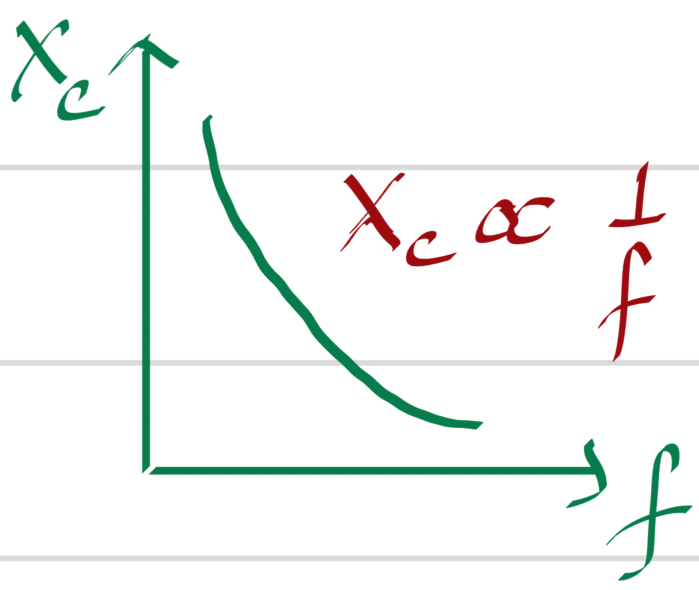
$$\boxed{\begin{aligned} X_C &= \frac{1}{\omega C} \\ &= \frac{1}{2\pi f C} \end{aligned}}$$

$C \rightarrow$ capacitance in Farad

$[\omega = 2\pi f]$

SI unit of X_C is ohm

* For AC $X_C \propto \frac{1}{f}$



* For DC $X_C = \infty$ as $f = 0$

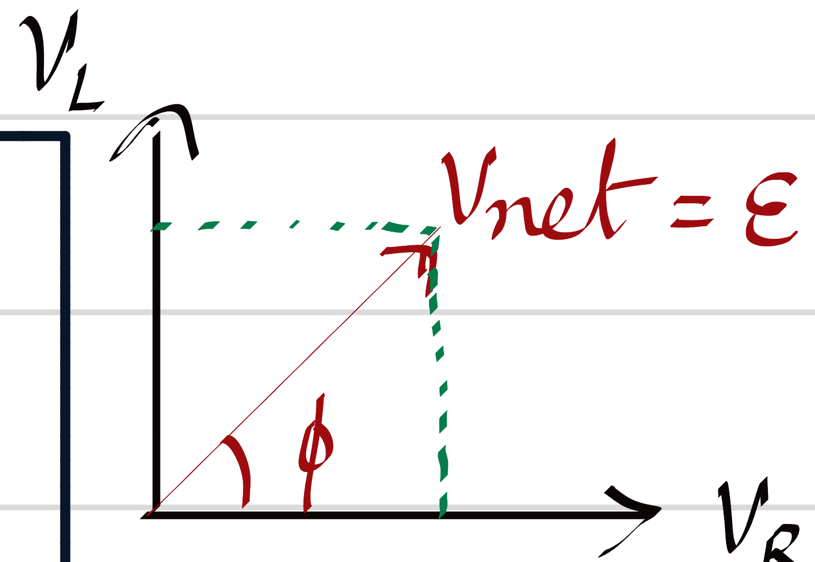
8. AC circuit containing Resistor and Inductor in series (Series RL circuit)

$$\boxed{\begin{aligned} \text{Emf} \quad \mathcal{E} &= \sqrt{V_R^2 + V_L^2} \\ &= \sqrt{(IR)^2 + (IX_L)^2} \\ &= I\sqrt{R^2 + X_L^2} \\ \mathcal{E} &= I\sqrt{R^2 + (\omega L)^2} \end{aligned}}$$

$[V_R = IR, \; V_L = IX_L]$

$[\omega = 2\pi f]$

$$\boxed{\begin{aligned} &\frac{\mathcal{E}}{I} = \underbrace{Z}_{\text{Impedance}} = \sqrt{R^2+(\omega L)^2} \\ &\text{or} \quad I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2+X_L^2}} \\ &\text{and} \quad \tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R} \end{aligned}}$$



[I lags $\mathcal{E}$ by ϕ]

9. Series RC circuit

$$\boxed{\begin{aligned} \mathcal{E} &= \sqrt{V_R^2 + V_C^2} \\ &= \sqrt{(IR)^2 + (IX_C)^2} \\ &= I\sqrt{R^2+X_C^2} \\ &= I\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2} \\ \text{or} \quad \frac{\mathcal{E}}{I} &= Z = \sqrt{R^2+X_C^2} \end{aligned}}$$

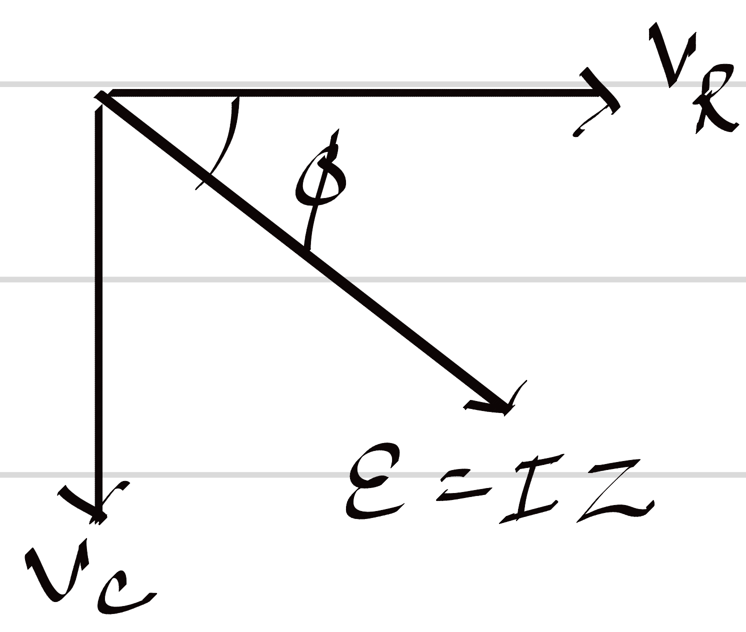
$[V_R = IR, \; V_C = IX_C]$

$\left[X_C = \frac{1}{\omega C}, \; \omega = 2\pi f\right]$

$$I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2+X_C^2}}$$

SI unit of Z is Ohm

$$\boxed{\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}}$$



[I leads $\mathcal{E}$ by ϕ]

10. Series LCR circuit

(i) When $V_L > V_C$

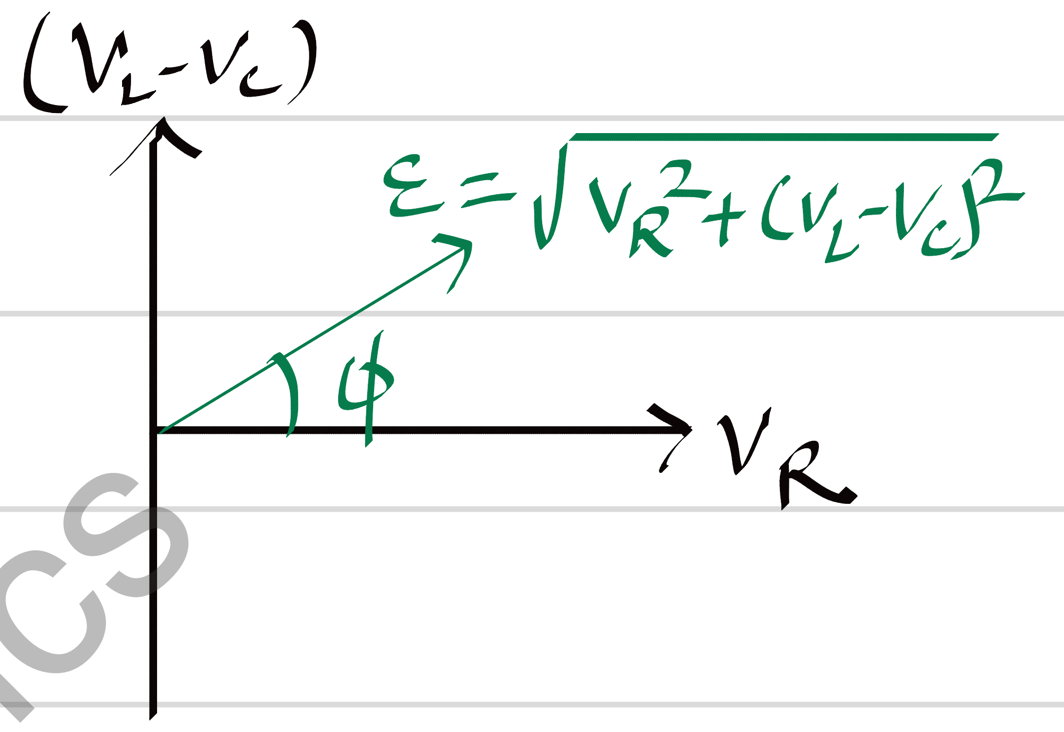
$$\boxed{\begin{aligned} \mathcal{E} &= \sqrt{V_R^2 + (V_L - V_C)^2} \\ \mathcal{E} &= I\sqrt{R^2 + (X_L - X_C)^2} \end{aligned}}$$

$$\boxed{\frac{\mathcal{E}}{I} = Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

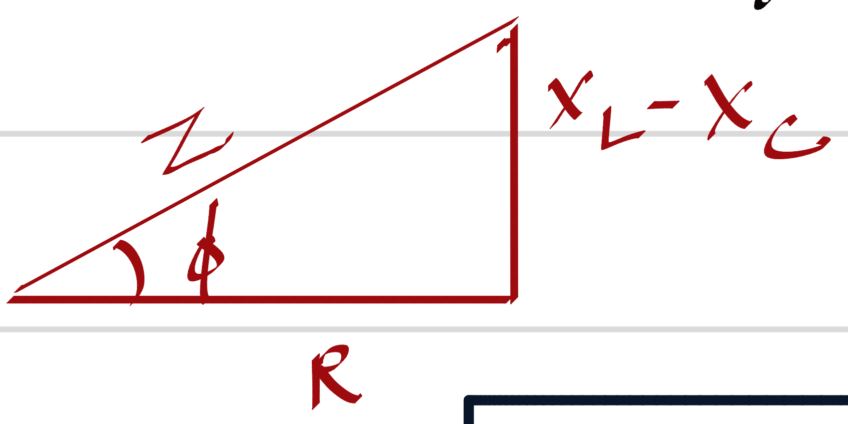
or

$$\boxed{I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}}$$

Z is impedance



Impedance Triangle



$$\boxed{\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}}$$

(II) When $V_C > V_L$

$$\boxed{I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{\mathcal{E}}{\sqrt{R^2 + (\frac{1}{\omega C} - \omega L)^2}}}$$

$$\boxed{\tan\phi = \frac{X_C - X_L}{R}}$$

(III) Resonance condition $(X_L = X_C)$

When $X_L = X_C \Rightarrow \boxed{\omega L = \frac{1}{\omega C}}$

$$\omega^2 = \frac{1}{LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \qquad [\omega = 2\pi f]$$

Resonance frequency

11. Power factor ($\cos\phi$)

$$\boxed{\cos\phi = \frac{P}{\mathcal{E}_{rms} I_{rms}}}$$

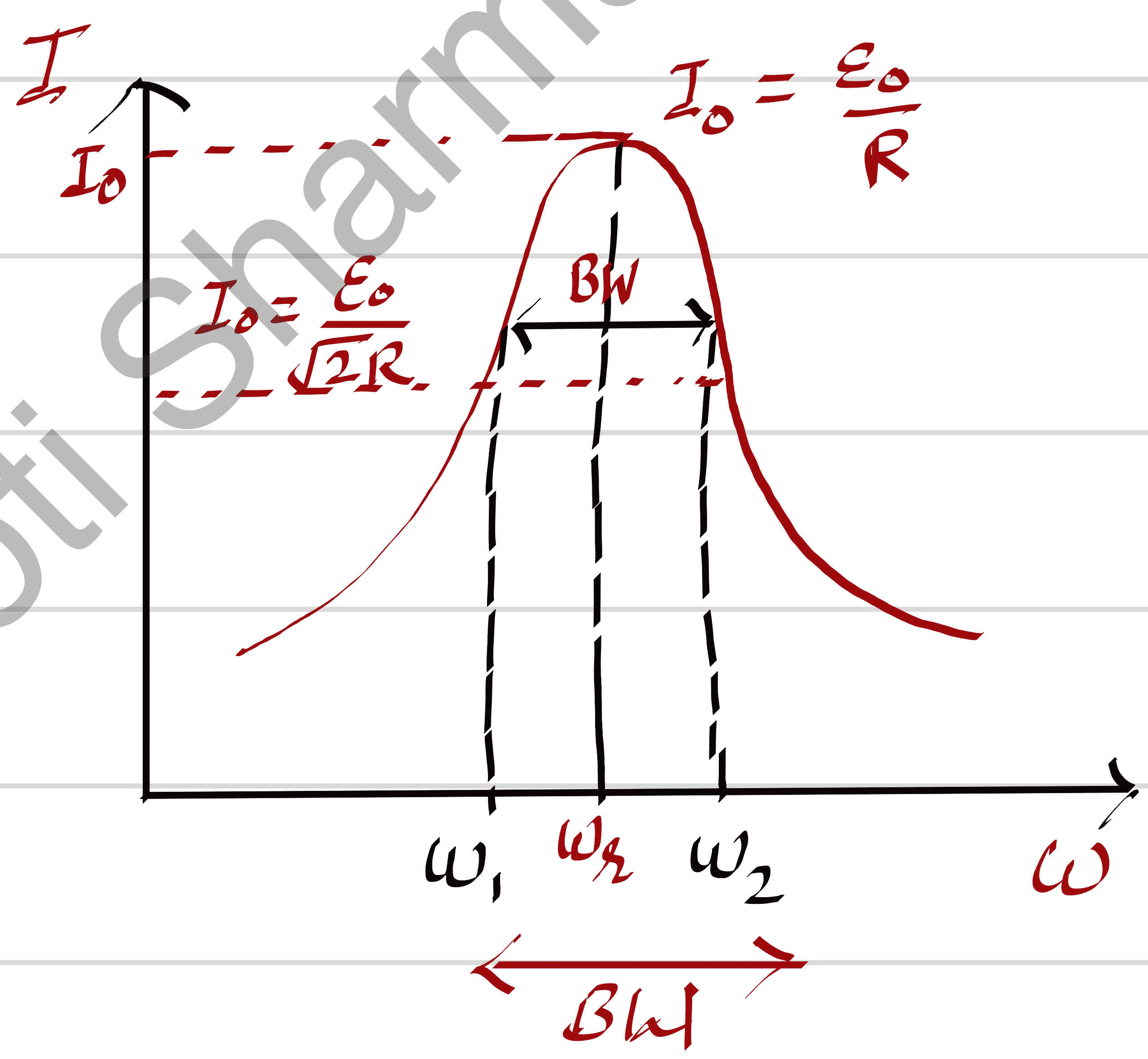
$P \rightarrow$ True Power

* Its value is b/w 0 to 1

$$\boxed{\cos\phi = \frac{R}{Z}}$$

12. Quality factor (Q-factor)

$$\boxed{Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{2\Delta\omega} = \frac{\text{Resonance frequency}}{\text{Bandwidth}}}$$



(more I, less R)

13. Power in AC circuit

$$\boxed{P = \frac{1}{2}\mathcal{E}_0 I_0 \cos\phi}$$

$$\boxed{P = \mathcal{E}_{rms} I_{rms} \cos\phi}$$

Cases -

(I) For Pure Resistive circuit

$$\boxed{P = VI = \frac{V^2}{R} = I^2R}$$ (In watts)

(II) For Pure capacitive or Pure inductive circuit

$$\boxed{P = 0}$$ [Wattless current]

(III) For LCR circuit

$$\boxed{P = \frac{\varepsilon_0 I_0}{2} \cos\phi = \frac{\varepsilon_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\phi}$$

$$\boxed{P = \varepsilon_{rms} I_{rms} \cos\phi}$$ $\left[\cos\phi = \frac{R}{Z} \right]$

14 Transformer

$$\boxed{\varepsilon_1 = -N_1 \frac{d\phi}{dt}, \quad \varepsilon_2 = -N_2 \frac{d\phi}{dt}}$$

N_1 and N_2 are no. of turns in primary, and the secondary coil

$$\boxed{\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}}$$

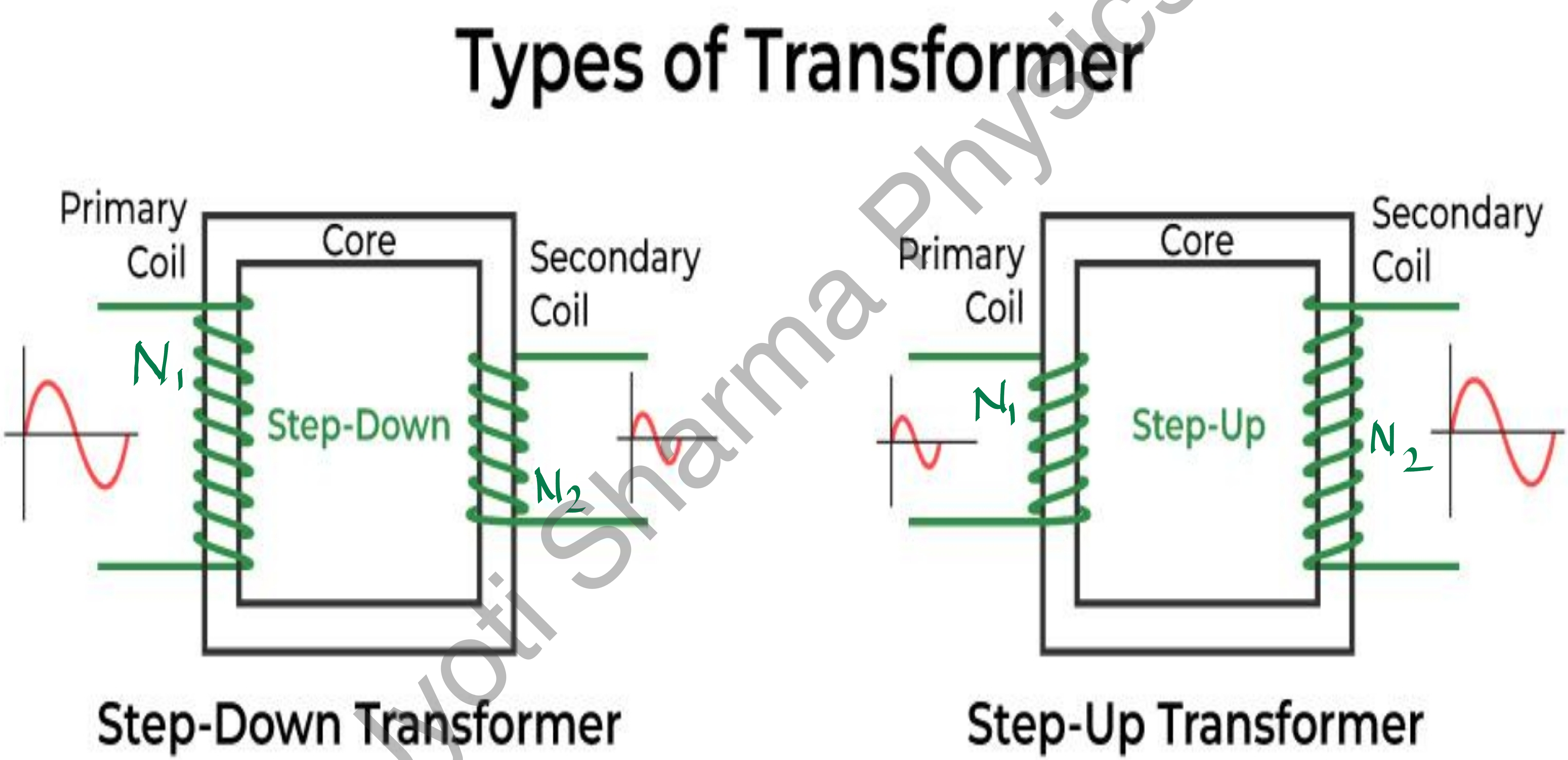
(For ideal transformer $P = \varepsilon_1 I_1 = \varepsilon_2 I_2$)

Efficiency of transformer

$$\boxed{\eta = \frac{\text{Output power}}{\text{Input power}} \times 100}$$

Flux linked with the coil

$$\boxed{\phi = NBA\cos\omega t}$$



$N_2 < N_1$

$N_2 > N_1$