

ALTERNATING CURRENT

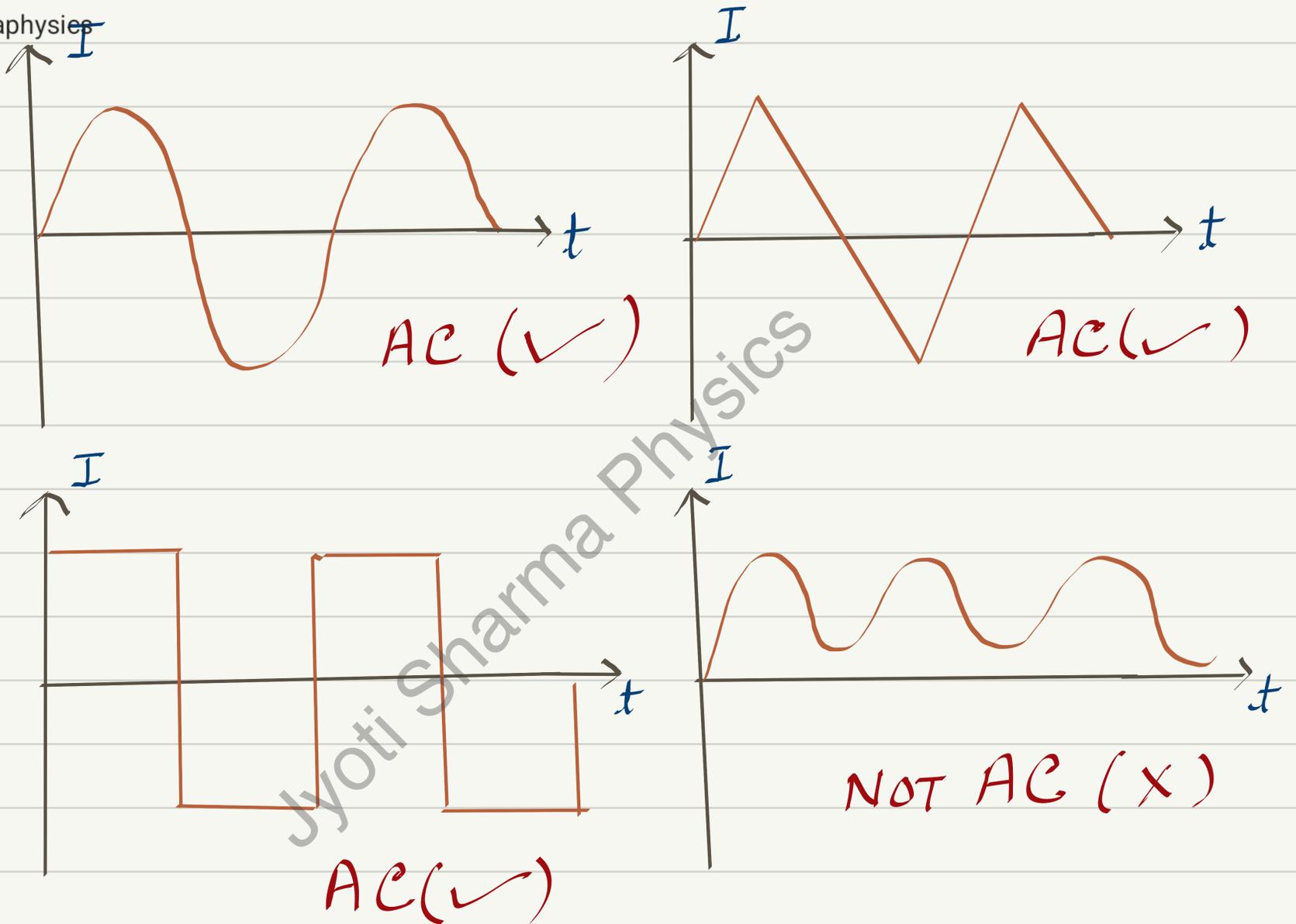
Ch. 7

Alternating current: The current whose direction changes alternately with time is known as alternating current.

OR

A current that changes its magnitude and polarity at regular intervals of time is called alternating current.

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Alternating Emf: An emf whose magnitude and direction changes periodically is known as alternating emf.

Alternating emf $\boxed{\mathcal{E} = \mathcal{E}_0 \sin \omega t}$

where $\mathcal{E}_0 = NBA\omega$

↓
peak or maximum value of emf

and $\omega = 2\pi f$

$N \rightarrow$ No. of turns

$B \rightarrow$ Mag. field

$A \rightarrow$ Area of coil

$\omega \rightarrow$ Angular velocity

↗ Frequency

Alternating current is given by

$$I = \frac{\epsilon}{R}$$

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$$I = \frac{\epsilon_0}{R} \sin \omega t$$

or $I = I_0 \sin \omega t$

where $I_0 = \frac{\epsilon_0}{R} =$ Peak value of AC current

$$\omega = 2\pi f, \quad f \rightarrow \text{frequency}$$

Value of Alternating current and voltage

1. Instantaneous current \rightarrow current at any instant
2. Peak value \rightarrow The maximum value of current in one cycle
3. Mean value \rightarrow The average value of current for one complete cycle.
4. RMS current \rightarrow Root mean square (rms) value is the effective value of current and also known as virtual value.

Advantages of AC over DC

1. Generation of AC and DC
2. Heat loss can be minimized.
3. AC is easily converted into DC
4. AC machines are small in size.

Average value of AC for one complete cycle

We know $I = \frac{dq}{dt}$

$$\text{or } dq = I dt$$

or

$$q = \int dq \\ = \int_0^T I dt$$

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$$= \int_0^T I_0 \sin \omega t dt \quad [\because I = I_0 \sin \omega t]$$

$$= I_0 \int_0^T \sin \omega t dt$$

$$= \frac{I_0}{\omega} [-\cos \omega t]_0^T \quad [\because \int \sin \omega t = -\frac{\cos \omega t}{\omega}]$$

$$= \frac{I_0}{\omega} [-\cos \omega T + \cos 0]$$

$$= \frac{I_0}{\omega} \left[-\cos \frac{2\pi}{T} \cdot T + 1 \right] \quad [\because \omega = \frac{2\pi}{T}]$$

$$= \frac{I_0}{\omega} [-\cos 2\pi + 1]$$

$$= \frac{I_0}{\omega} [-1 + 1]$$

i.e.

$$q = 0$$

so,

$$I_{av} = \frac{q}{T}$$

$$\boxed{I_{av} = 0}$$

Hence for one complete cycle, mean current is zero.

Also the average emf over one complete cycle is zero.

$$E_{av} = 0$$

* Since the average value of $\sin \omega t$ over a cycle is zero, the average value over a full cycle is always zero.

Average value of current for half cycle

$$q = \int dq = \int_0^{T/2} I dt$$

$$= I_0 \int_0^{T/2} \sin \omega t dt \quad [\because I = I_0 \sin \omega t]$$

$$= \frac{I_0}{\omega} \left[-\cos \omega t \right]_0^{T/2} \quad [\int \sin \omega t = -\frac{\cos \omega t}{\omega}]$$

$$= \frac{I_0}{\omega} \left[-\cos \omega \frac{T}{2} + \cos 0 \right]$$

$$= \frac{I_0}{\omega} \left[-\cos \frac{2\pi \cdot T}{T \cdot 2} + 1 \right]$$

$$= \frac{I_0}{\omega} \left[-\cos \pi + 1 \right]$$

$$= \frac{I_0}{\omega} \left[1 + 1 \right] \quad [\because \cos \pi = -1]$$

$$\text{or } q = \frac{2 I_0}{\omega} = \frac{2 I_0}{2\pi/T}$$

$$\text{or } q = \frac{I_0 T}{\pi}$$

$$\text{Now } (I_{av})_{\text{half}} = \frac{q}{T/2} = \frac{2q}{T}$$

$$= \frac{2 I_0 T}{\pi T}$$

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$$= \frac{2}{\pi} I_0$$

$$\text{or } \boxed{(I_{av})_{\text{half}} = 0.637 I_0}$$

$$\text{also } \boxed{(E_{av})_{\text{half}} = 0.637 E_0}$$

i.e. the average value of ac current over half cycle is 0.637 time of peak value.

Root Mean Square Current (I_{rms})

Also known as effective current (I_{eff}) and virtual current ($I_{virtual}$).

The rms value of alternating current is equal to the value of direct current which gives the same heating effect in the same resistance, in the same time.

i.e.

$$H = I_{rms}^2 R T = \int_0^T I^2 R dt$$

or

$$I_{rms}^2 = \frac{1}{T} \int_0^T I^2 dt$$

$$= \frac{1}{T} \int_0^T (I_0 \sin \omega t)^2 dt \quad [I = I_0 \sin \omega t]$$

$$= \frac{I_0^2}{T} \int_0^T \sin^2 \omega t dt$$

$$= \frac{I_0^2}{T} \int_0^T \frac{(1 - \cos 2\omega t)}{2} dt \quad \left[\because \cos 2\omega t = 1 - 2\sin^2 \omega t \right]$$

$$= \frac{I_0^2}{2T} \int_0^T dt - \frac{I_0^2}{2T} \int_0^T \cos 2\omega t dt$$

$$= \frac{I_0^2}{2T} [t]_0^T - 0 \quad \left[\because \int_0^T \cos 2\omega t = 0 \right]$$

$$I_{rms}^2 = \frac{I_0^2}{2T} \cdot T$$

or

$$I_{rms} = \sqrt{\frac{I_0^2}{2}}$$

$$\begin{aligned} \int_0^T \cos 2\omega t &= \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T \\ &= \frac{1}{2\omega} [\sin 2 \times 2\pi \cdot T - 0] \\ &= \frac{1}{2\omega} [\sin 4\pi] = 0 \end{aligned}$$

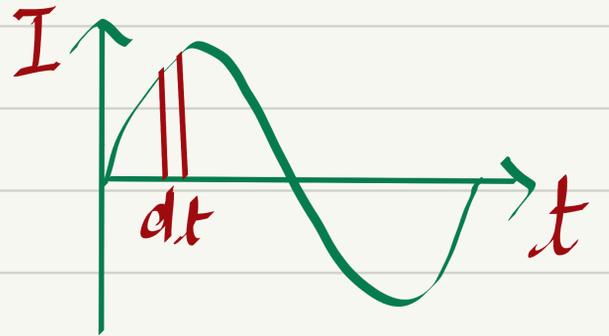
or
$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

How to calculate RMS current

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + \dots + I_n^2}{n}}$$

$$= \sum_{k=1}^n \sqrt{\frac{I_n^2}{n}}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T I^2 dt}$$



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Phasors: The rotating vectors which represent sinusoidal quantities are called phasors.

Phasor diagram: A diagram which represents AC current and voltage as rotating vectors, called phasor diagram.

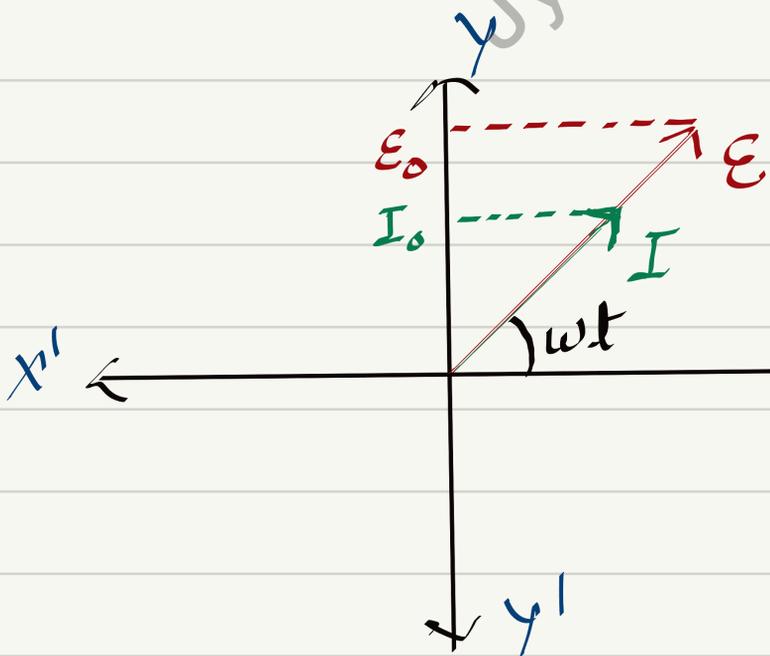


fig. 1.

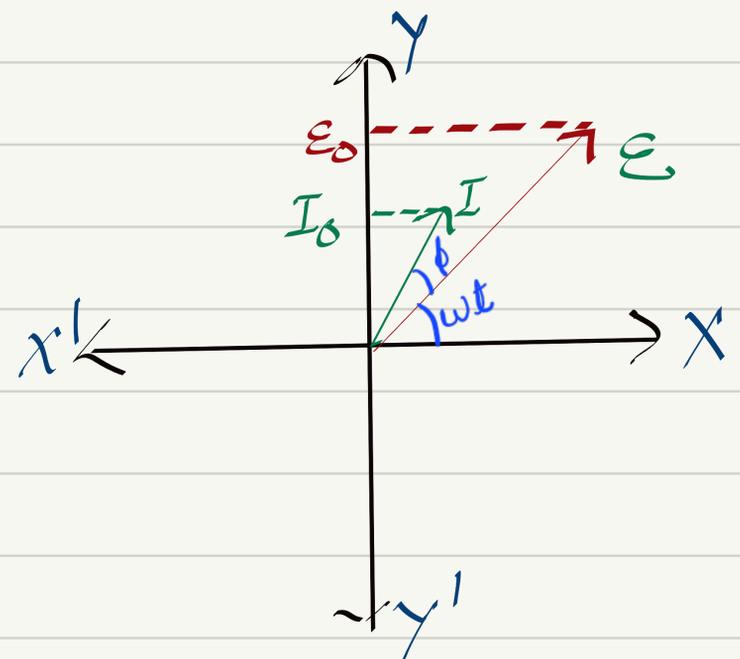


fig. 2

In fig. 1.

$$\left. \begin{aligned} \epsilon &= \epsilon_0 \sin \omega t \\ I &= I_0 \sin \omega t \end{aligned} \right\} \text{ same phase}$$

In fig. 2.

$$\left. \begin{aligned} \epsilon &= \epsilon_0 \sin \omega t \\ I &= I_0 \sin(\omega t + \phi) \end{aligned} \right\} \phi \rightarrow \text{phase difference}$$

AC circuit containing Resistance only

Suppose a resistor is connected to an AC source and $\epsilon = \epsilon_0 \sin \omega t$

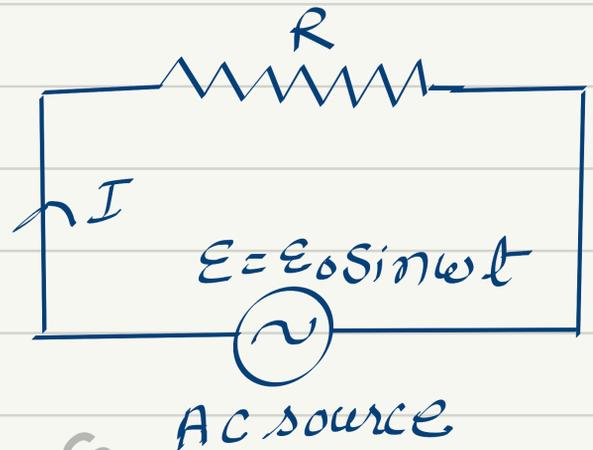
By KVL (Kirchhoff's voltage rule)

$$\epsilon - IR = 0$$

$$\Rightarrow \epsilon = IR$$

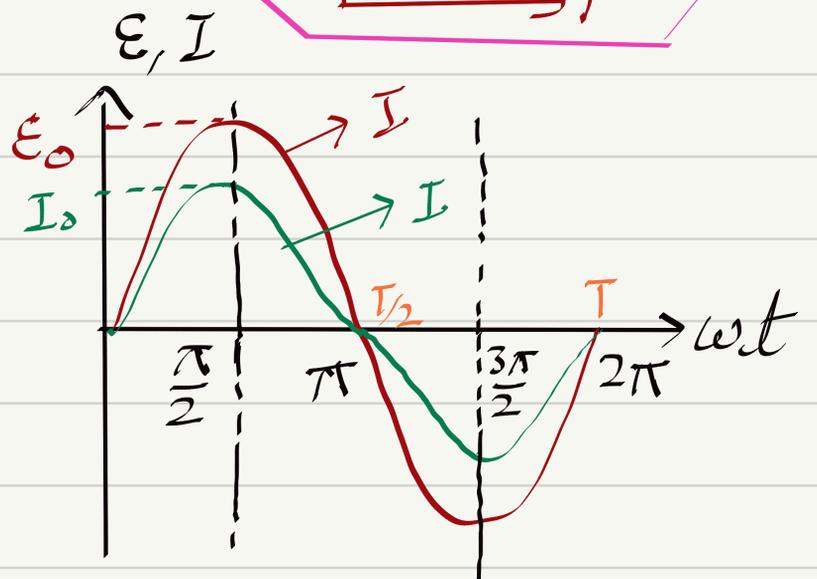
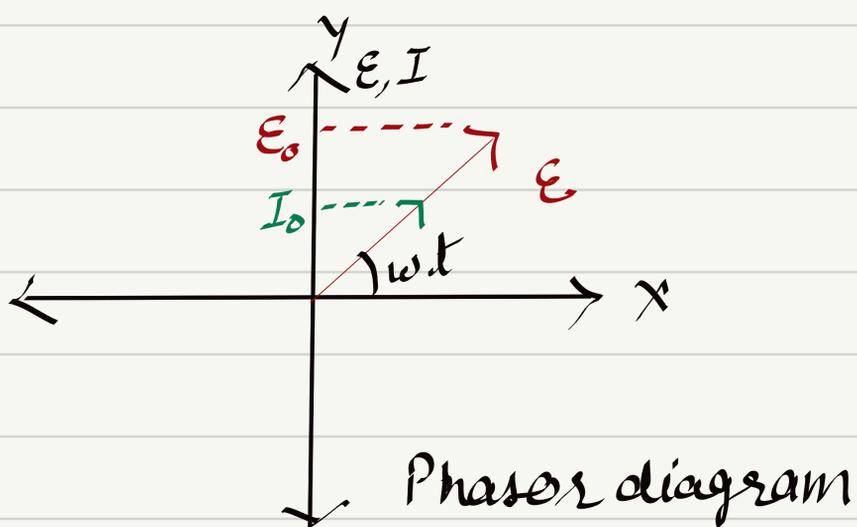
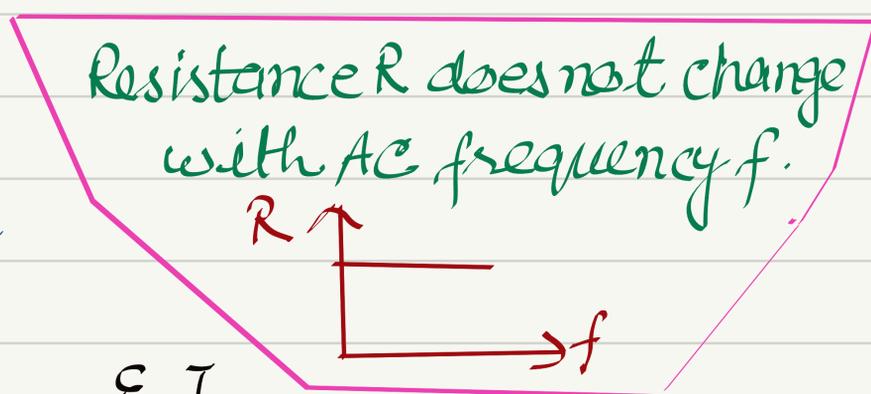
$$I = \frac{\epsilon}{R} = \frac{\epsilon_0 \sin \omega t}{R}$$

$$I = I_0 \sin \omega t \quad \left[I_0 = \frac{\epsilon_0}{R} \right]$$



As, $\epsilon = \epsilon_0 \sin \omega t$

and $I = I_0 \sin \omega t$
both have same phase.
* phase diff = 0



AC circuit containing Inductor only:

In fig an inductor is connected with AC source of emf $\epsilon = \epsilon_0 \sin \omega t$

By KVL

$$\mathcal{E} - L \frac{dI}{dt} = 0$$

$$\Rightarrow \mathcal{E} = L \frac{dI}{dt}$$

$$\Rightarrow L dI = \mathcal{E} dt$$

$$\Rightarrow dI = \frac{\mathcal{E}}{L} dt$$

$$\Rightarrow dI = \frac{\mathcal{E}_0 \sin \omega t}{L} dt$$

$$\Rightarrow I = \int dI = \frac{\mathcal{E}_0}{L} \int \sin \omega t dt$$

$$\Rightarrow I = \frac{\mathcal{E}_0}{L} (-\cos \omega t)$$

$$\Rightarrow I = \frac{\mathcal{E}_0}{\omega L} (-\cos \omega t)$$

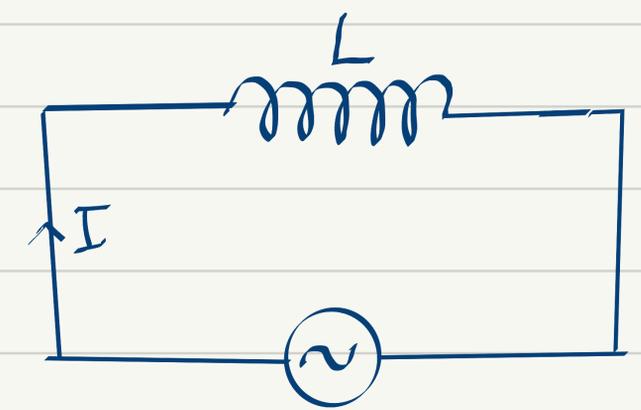
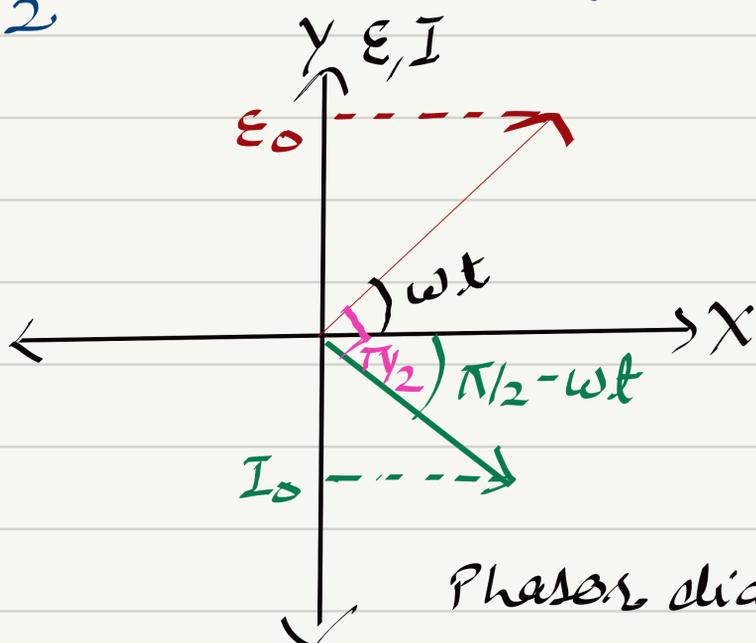
$$\Rightarrow I = \frac{\mathcal{E}_0}{\omega L} \left[\sin \left(\omega t - \frac{\pi}{2} \right) \right] \quad \left[\begin{array}{l} \sin \left(\frac{\pi}{2} - \theta \right) = \cos \theta \\ \sin \left(\theta - \frac{\pi}{2} \right) = -\cos \theta \end{array} \right]$$

$$\Rightarrow I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right), \text{ where } I_0 = \frac{\mathcal{E}_0}{\omega L}$$

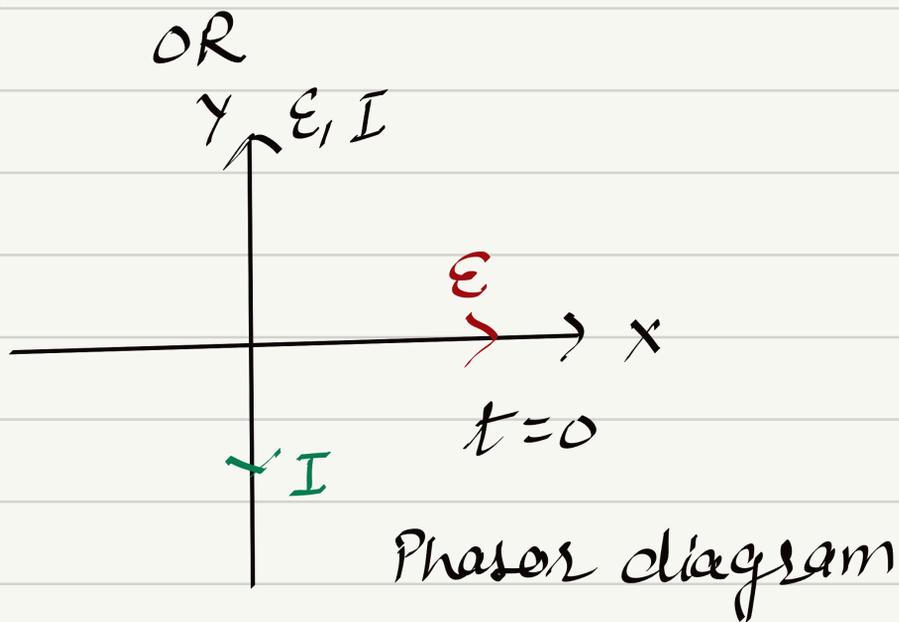
As $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

and $I = I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$

It is clear that current lags behind the emf by $\frac{\pi}{2}$. Thus voltage is ahead by a phase $\frac{\pi}{2}$.



Purely Inductive circuit



Inductive Reactance (X_L)

Here $I = \frac{\epsilon_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$ — (1)

$I = I_0 \sin(\omega t - \frac{\pi}{2})$ — (2)

on comparing (1) and (2), we get

$$I_0 = \frac{\epsilon_0}{\omega L}$$

but $I_0 = \frac{\epsilon_0}{R}$

so we find ωL same as R .
This $\omega L = X_L$ is called inductive reactance.

$$X_L = \omega L = 2\pi fL \quad f \rightarrow \text{frequency}$$

For AC,

$$X_L \propto f \quad , \quad f \uparrow \Rightarrow X_L \uparrow$$

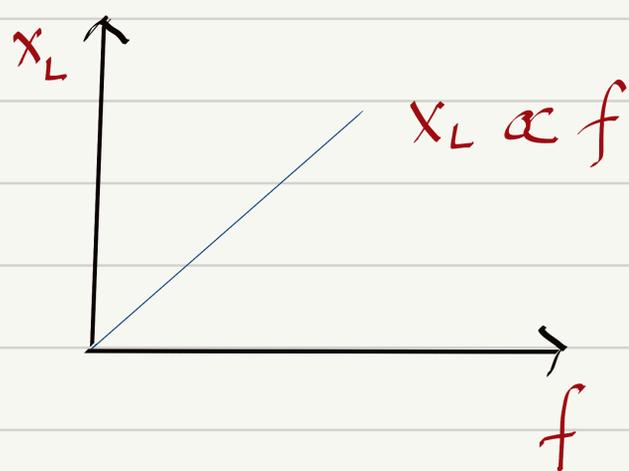
i.e. if frequency of AC increases, inductive reactance (resistance of circuit) also increases.

For DC, $f = 0 \Rightarrow X_L = 0$ (no resistance in circuit)

* Thus an inductor allows DC easily but not AC.

* S.I. unit of X_L is Ohm.

* Variation of X_L with f



AC circuit containing capacitor only

In fig. a capacitor is connected to an emf source.

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

By KVL

$$\mathcal{E} - \frac{q}{C} = 0$$

$$\text{or } \mathcal{E} = \frac{q}{C}$$

$$q = C\mathcal{E}$$

$$q = C\mathcal{E}_0 \sin \omega t$$

$$I = \frac{dq}{dt} = \frac{d}{dt} (C\mathcal{E}_0 \sin \omega t)$$

$$= C\mathcal{E}_0 \cos \omega t \cdot \omega$$

$$= \omega C \mathcal{E}_0 \cos \omega t$$

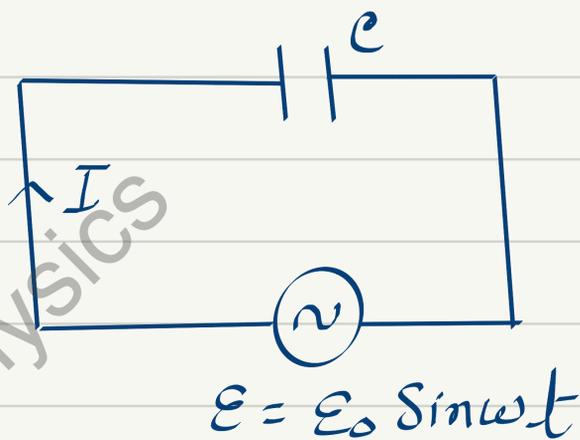
$$I = \frac{\mathcal{E}_0}{\frac{1}{\omega C}} \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right)$$

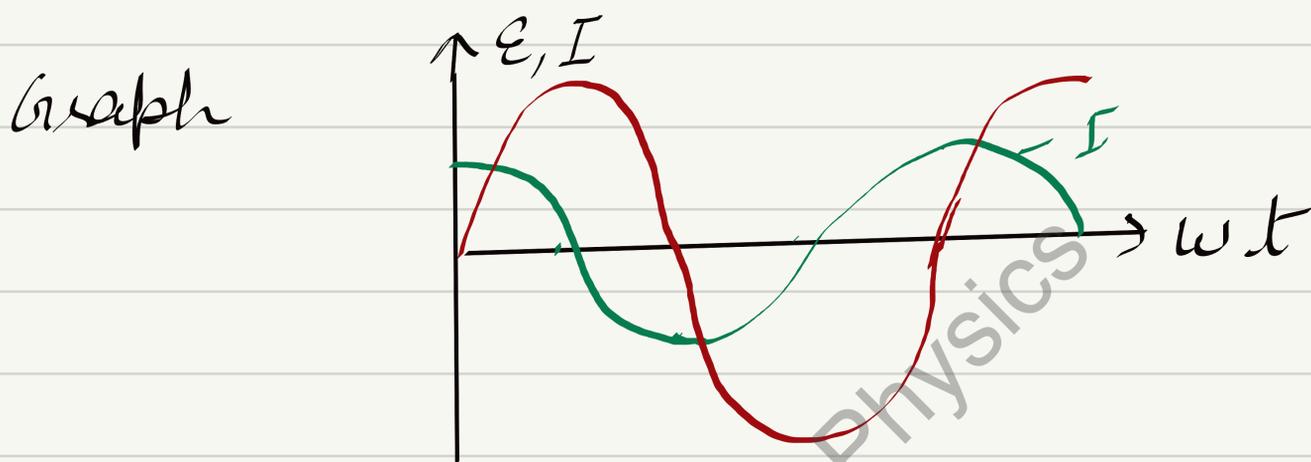
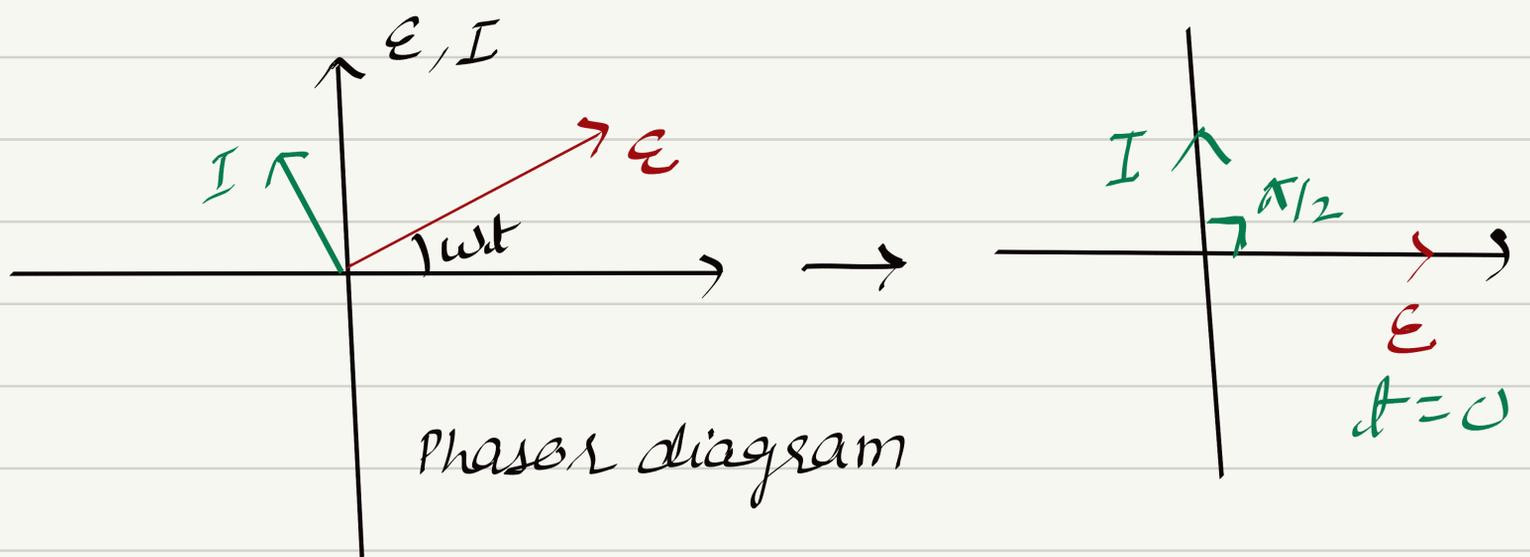
As $\mathcal{E} = \mathcal{E}_0 \sin \omega t$

and $I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right)$

There is a phase difference of $\pi/2$ between



ϵ and I . It is clear that current is leading emf by $\frac{\pi}{2}$.



Capacitive Reactance (X_c)

$$I = \frac{\epsilon_0}{\frac{1}{\omega c}} \left(\sin \omega t + \frac{\pi}{2} \right)$$

or $I = I_0 \left(\sin \omega t + \frac{\pi}{2} \right)$

where $I_0 = \frac{\epsilon_0}{\frac{1}{\omega c}}$

on comparing $I_0 = \frac{\epsilon}{R}$

we find $\frac{1}{\omega c}$ and R , both has same role.

i.e. $\frac{1}{\omega c}$ is effective resistance and called

capacitive reactance X_c

$$X_c = \frac{1}{\omega c} = \frac{1}{2\pi f c}$$

SI unit of X_c is ohm (Ω)

For AC $X_c \propto \frac{1}{f}$

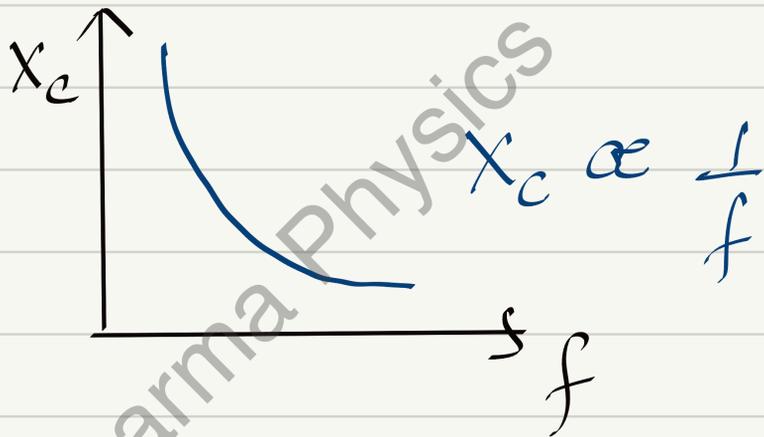
i.e. if frequency of AC current increases, capacitive reactance decreases.

For DC $X_c = \infty$ as $f = 0$

* Thus a capacitor AC to flow easily but blocks DC

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{\sqrt{2} \cdot \frac{1}{\omega C}} = \frac{E_{rms}}{X_c}$$

graph b/w X_c and f



R-L Series Circuit

Fig shows a resistor and an inductor connected in series with AC emf source $E = E_0 \sin \omega t$ here.

$$V_R = IR$$

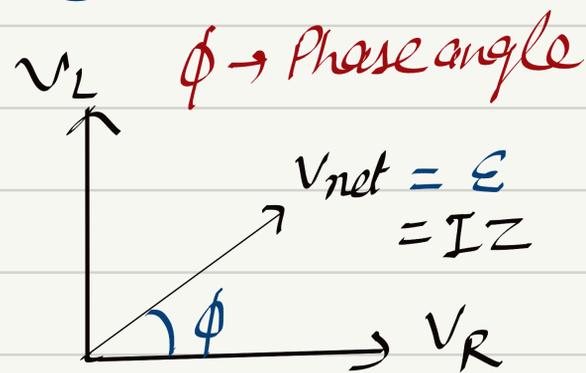
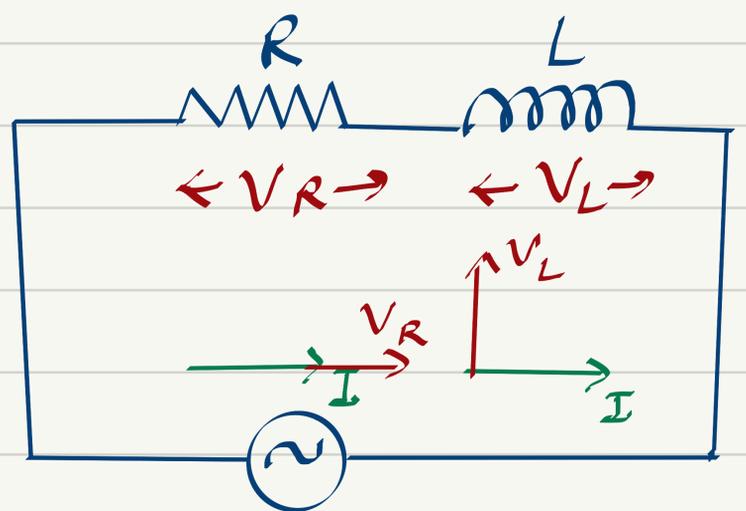
and $V_L = IX_L$

$$V_{net} = E = \sqrt{(IR)^2 + (IX_L)^2}$$

$$E = I \sqrt{R^2 + X_L^2}$$

OR $\frac{E}{I} = \sqrt{R^2 + X_L^2}$

$$\tan \phi = \frac{V_L}{V_R} \Rightarrow \phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$



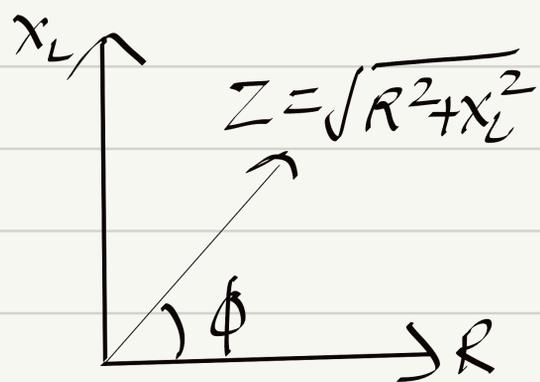
Phasor diagram

$$\text{or } \boxed{Z = \sqrt{R^2 + X_L^2}}$$

where $Z = \frac{\mathcal{E}}{I} = \text{Impedance}$

So, for I ,

$$I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + X_L^2}}$$



$$\tan \phi = \frac{X_L}{R}$$

Impedance - Total resistance of an AC circuit.
SI unit - ohm (Ω)

R-C Series circuit:

$$V_R = IR$$

$$V_C = IX_C$$

$$V_{\text{net}} = \mathcal{E} = \sqrt{V_R^2 + V_C^2}$$

$$\mathcal{E} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\mathcal{E} = I \sqrt{R^2 + X_C^2}$$

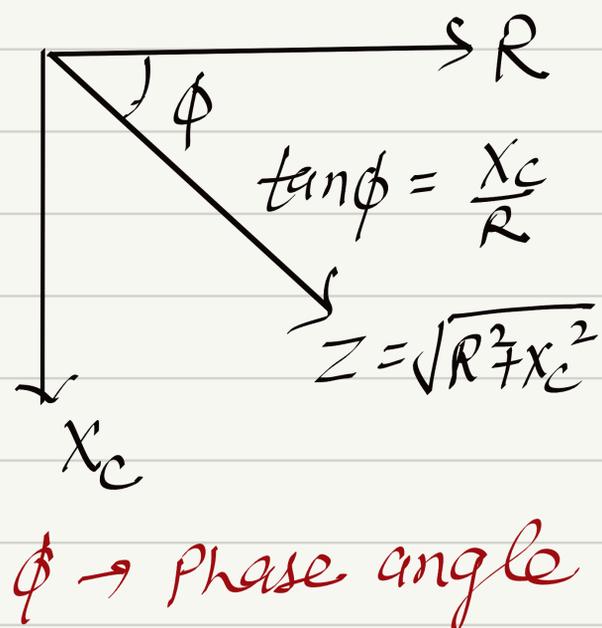
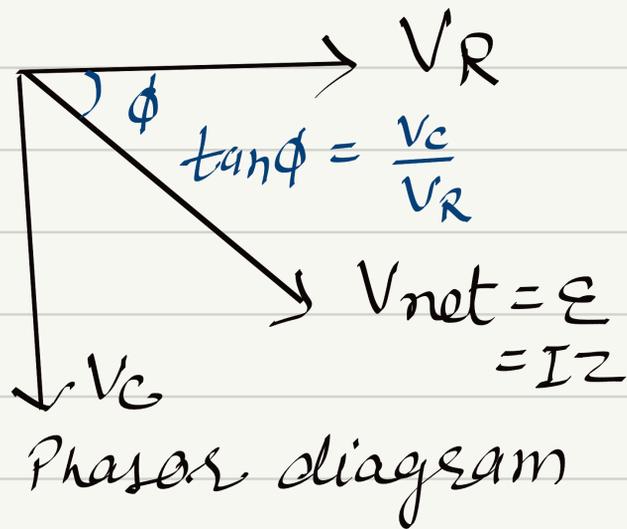
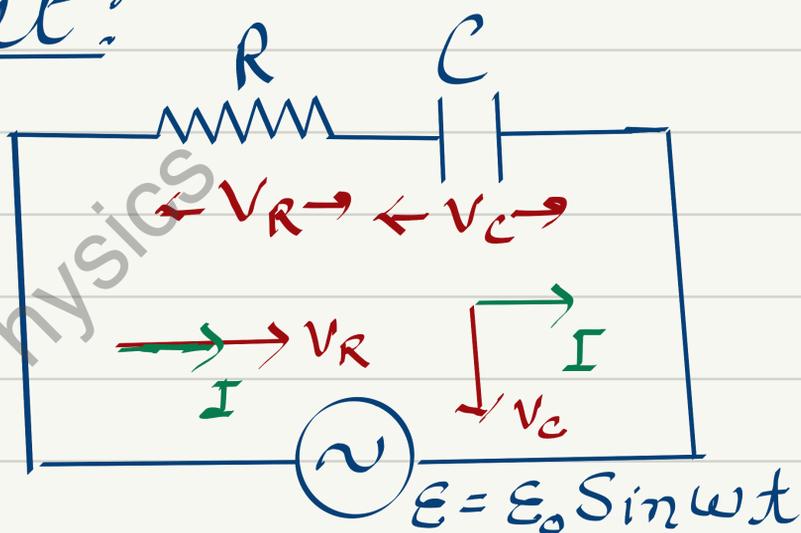
$$\text{or } \frac{\mathcal{E}}{I} = \sqrt{R^2 + X_C^2}$$

$$\frac{\mathcal{E}}{I} = Z = \sqrt{R^2 + X_C^2}$$

$Z =$ Total resistance in an AC circuit.

Expression for I

$$\boxed{I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + X_C^2}}}$$



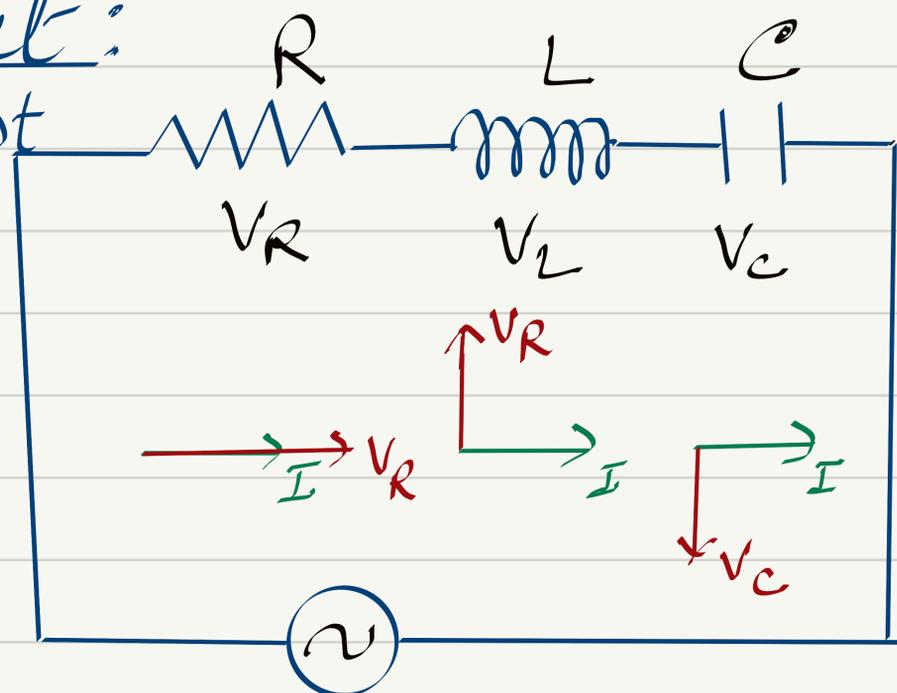
R-L-C Series circuit:

Emf of AC circuit $\mathcal{E} = \mathcal{E}_0 \sin \omega t$
 Case I when $X_L > X_C \Rightarrow V_L > V_C$

$$V_R = IR$$

$$V_L = IX_L$$

$$V_C = IX_C$$



As V_L and V_C are in opposite direction

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

$$\mathcal{E} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

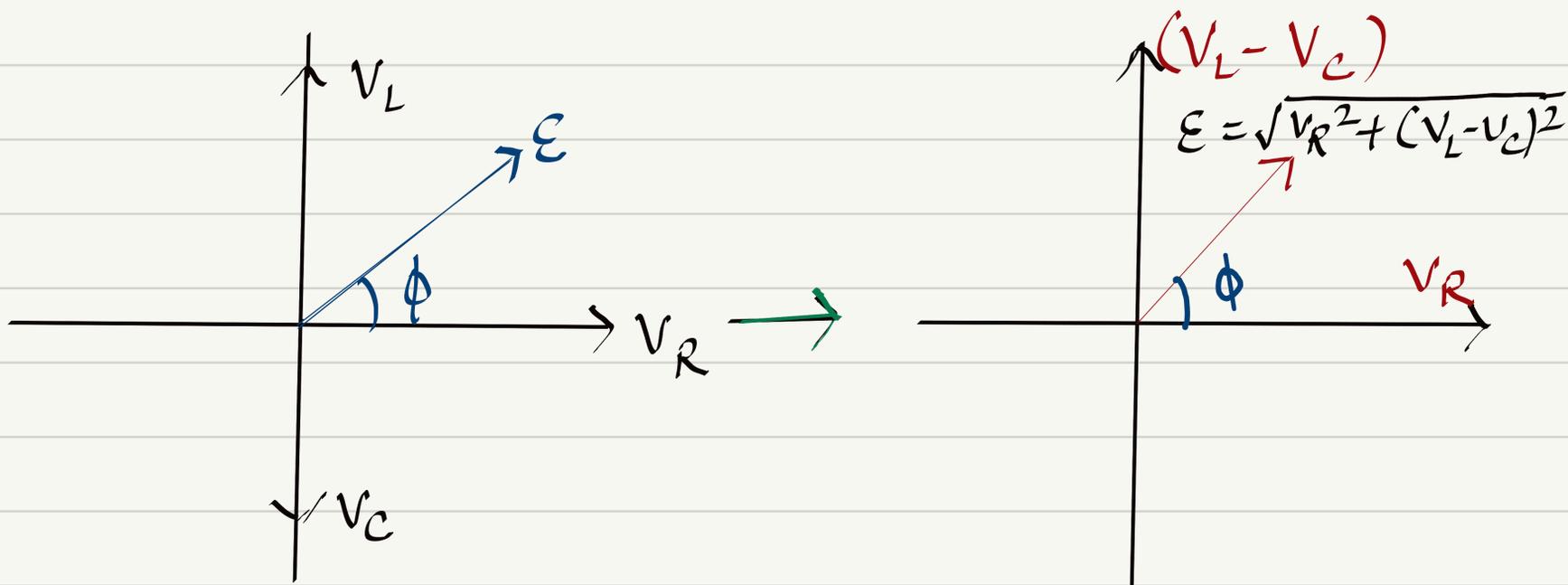
$$\mathcal{E} = \sqrt{IR^2 + (IX_L - IX_C)^2}$$

$$\mathcal{E} = I \sqrt{R^2 + (X_L - X_C)^2}$$

or $\frac{\mathcal{E}}{I} = \sqrt{R^2 + (X_L - X_C)^2} = Z =$ Total resistance of an AC circuit, called impedance.

Expression for I

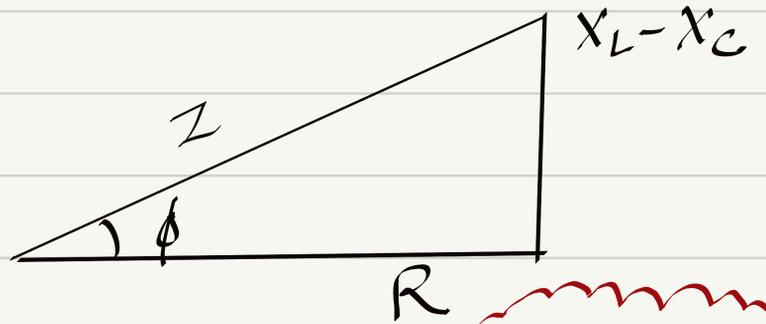
$$I = \frac{\mathcal{E}}{Z} = \frac{\mathcal{E}}{\sqrt{R^2 + (X_L - X_C)^2}}$$



Phasor diagram

Impedance Triangle: The relationship b/w R, Z, X_L and X_C is shown by right angled triangle, called impedance triangle.

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$

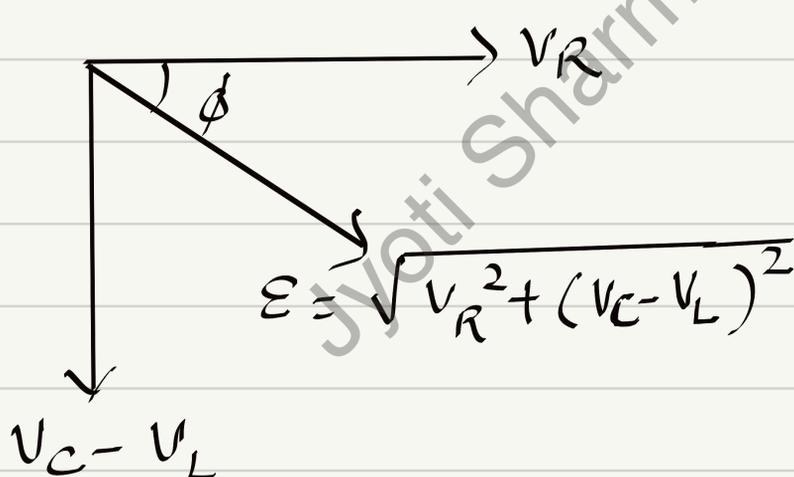


$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

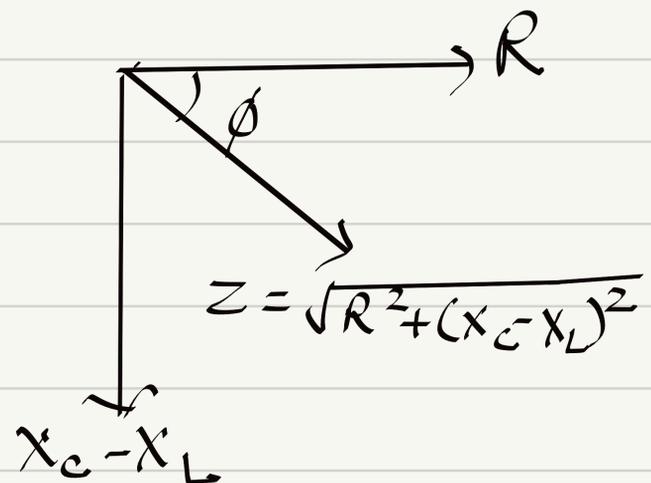
$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The LCR circuit is said to be inductive when $X_L > X_C$

Case II when $X_C > X_L \Rightarrow V_C > V_L$



$$\tan \phi = \frac{V_C - V_L}{V_R} = \frac{X_C - X_L}{R}$$



$$\tan \phi = \frac{X_C - X_L}{R}$$

$$I = \frac{\epsilon}{Z} = \frac{\epsilon}{\sqrt{R^2 + (X_C - X_L)^2}}$$

i.e. $Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

The LCR circuit is said to be capacitive when $X_C > X_L$

Case III $X_L = X_C \Rightarrow V_L = V_C$, Resonance Condition

The current in the circuit

$$I = \frac{\epsilon}{Z} = \frac{\epsilon}{\sqrt{R^2 + (X_C - X_L)^2}}$$

when $X_L = X_C$

$$I = \frac{\epsilon}{\sqrt{R^2 + 0}} = \frac{\epsilon}{R}$$

ie $Z = R$ [min^m Z]

I becomes maximum when

$$X_L = X_C$$

$$\text{or } \omega L = \frac{1}{\omega C}$$

$$\text{or } \omega^2 = \frac{1}{LC}$$

$$\text{or } \omega = \frac{1}{\sqrt{LC}}$$

The LCR circuit is said to be purely resistive when $X_L = X_C$

By

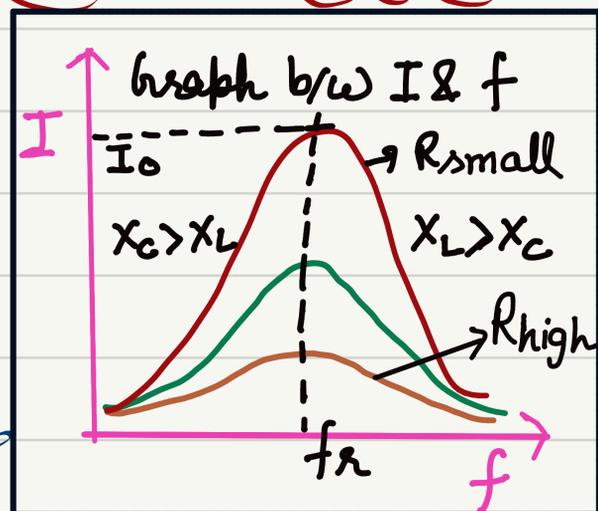
$$\omega R = 2\pi f R = \frac{1}{\sqrt{LC}}$$

where f_R is resonance frequency

$$f_R = \frac{1}{2\pi\sqrt{LC}}$$

and

$$I_0 = \frac{\epsilon_0}{R}$$



Power factor $\cos \phi$ for RLC circuit

$$\cos \phi = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Power Factor ($\cos \phi$)

Power factor is defined as the ratio of true power and virtual power.

$$\cos \phi = \frac{P}{V_{rms} I_{rms}} = \frac{P}{\epsilon_{rms} I_{rms}}$$

* Power factor is always +ve and not more than 1

(i) For pure resistive circuit

$$\cos \phi = \frac{R}{Z} = \frac{R}{R} = 1$$

$$\cos \phi = 1 \quad [\because \epsilon \text{ \& } I \text{ are in phase, i.e. } \phi = 0]$$

(ii) For pure inductive or pure capacitive circuit

$$\cos \phi = 0 \quad [\because \epsilon \text{ \& } I \text{ are out of phase, } \phi = 90]$$

(iii) For RC circuit, $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$

(iv) For RL circuit, $\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}}$

Power in AC circuit: The rate at which electrical energy is consumed is called power.

Let emf $\epsilon = \epsilon_0 \sin \omega t$

and current $I = I_0 \sin(\omega t + \phi)$

Power $P = \epsilon I$

$= \epsilon_0 \sin \omega t \cdot I_0 \sin(\omega t + \phi)$

$= \frac{\epsilon_0 I_0}{2} [2 \sin \omega t \cdot \sin(\omega t + \phi)]$ [Multiply and divide by 2]

$= \frac{\epsilon_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)]$

[$\because 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$
and $\cos(-\phi) = \cos \phi$]

For average power

$P_{av} = \frac{\epsilon_0 I_0}{2} \cos \phi$

[\because average value of $\cos 2\omega t = 0$ and $\cos \phi$ is independent of t]

$P_{av} = \frac{\epsilon_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$

or $P_{av} = \epsilon_{rms} I_{rms} \cos \phi \Rightarrow P_{av} = I_{rms}^2 R$

Here $\cos \phi$ is power factor.

P is called true power.

$\epsilon_{rms} I_{rms}$ is called virtual power.

Special Cases -

(i) AC circuit having resistor only
in such case $\phi = 0$, then

$P_{av} = \epsilon_{rms} I_{rms} \cos 0'$

i.e. $P_{av} = \epsilon_{rms} I_{rms}$

Thus True Power = Virtual Power.

(ii) and (iii) AC circuit having inductor only or capacitor only
in such case $\phi = 90^\circ$

$$P_{av} = E_{rms} I_{rms} \cos 90^\circ$$

$$P_{av} = 0 \quad [\cos 90^\circ = 0]$$

Thus no power loss takes place for pure inductive and for pure capacitive circuit.

$P_{av} = 0$ for pure inductive and pure capacitive circuit.

@jyotisharmaphysics

Wattless Current The current in an AC circuit is said to be wattless current when the average power consumed or dissipated in the circuit is zero.

OR

If power consumed in an AC circuit is zero, the current in the circuit is called wattless current.

For wattless current

$$\cos \phi = 0$$

i.e. the current in purely inductive and purely capacitive circuit is wattless current.

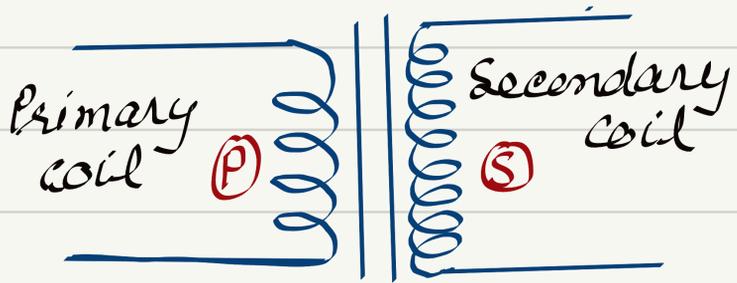
Transformers The electrical device which is used to alter voltage is called transformer.

Transformer converts low AC voltage to high AC voltage and vice-versa.

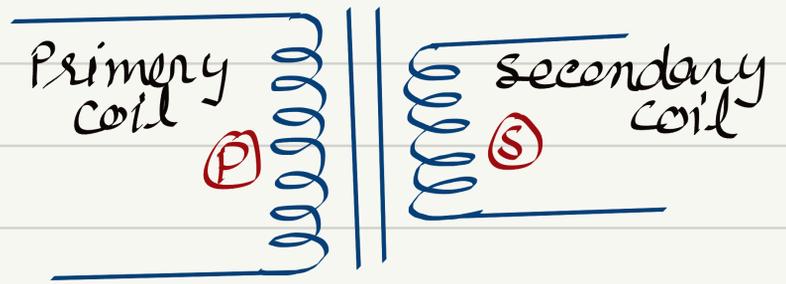
Two types of transformers are -

1. Step-up Transformer. The transformer which convert low voltage to high voltage.
i.e. output increases

2. Step-down Transformer. The transformer which convert high voltage to low voltage.
i.e. output decreases.



Step-up transformer



Step-down transformer

Principle: Transformer works on the principle of 'mutual induction'. i.e. an emf is induced in a coil when a changing current flows through its nearby coil.

Theory and Working

According to Faraday's law for primary coil induced emf

$$\mathcal{E}_p = -N_p \frac{d\phi}{dt} \quad \text{--- (1)}$$

for secondary coil,

$$\mathcal{E}_s = -N_s \frac{d\phi}{dt} \quad \text{--- (2)}$$

from eqⁿ (1) and (2)

$$\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s} \quad \text{[on dividing]}$$

$$\frac{N_p}{N_s} = k \quad (\text{transformation constant})$$

$$\text{so, } \boxed{\frac{\mathcal{E}_p}{\mathcal{E}_s} = \frac{N_p}{N_s} = k}$$

* $k < 1$ for step-down transformers
i.e. $N_s < N_p$

* $k > 1$ for step-up transformers
i.e. $N_s > N_p$

For an ideal transformer there is no loss on energy.

Output Power = Input Power

$$E_s I_s = E_p I_p \quad [\because P = VI = EI]$$

$$\text{OR} \quad \frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} \quad [\because \frac{E_s}{E_p} = \frac{N_s}{N_p}]$$

$$\text{OR} \quad \boxed{E \propto \frac{1}{I}}$$

Efficiency of a transformer (η)

$$\boxed{\eta = \frac{\text{Output Power} \times 100}{\text{Input Power}}}$$

No transformer is 100% efficient due to some energy loss.

- e.g.
1. Flux leakage
 2. Eddy current
 3. Resistance of windings
 4. Hysteresis loss