MECHANICAL PROPERTIES OF SOLIDS

Chapter-8
(old ch. 9)

Deforming force: A force that causes a change in the structure of an object when applied.

Intermolecular force: In a solid atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to to the neighbouring molecules. These forces are known as intermolecular forces.

Elasticity: The property of a body by virtue of which, it tends to regain its original size and shape when the applied force is removed is called elasticity and such a body is called elastic body.

Perfectly elastic body: A body which regains it original configuration immediately and completely after removing the deforming force is called perfectly elastic body.
e.g. Quartz and phospher bronze are close to perfectly elastic bodies.

Plasticity: The inability of a body to return to its original size after removing the deforming force and such a body is called plastic body.

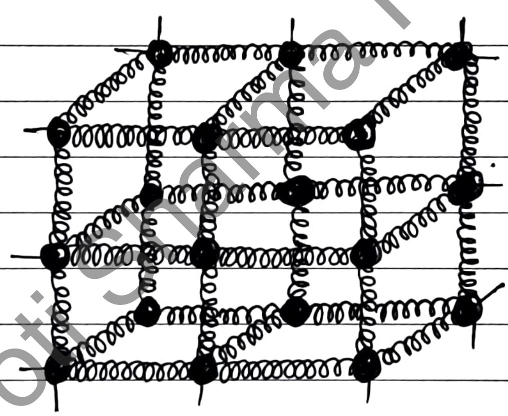
Perfectly plastic body: A body does not regain its original configuration at all after removing deforming force is called perfectly plastic body.
e.g. wax, putty etc are close to perfectly plastic

body.

* **Elastic behaviour of solids**: In a solid each atom or molecule is surrounded by neighbouring atoms or molecules. These are bounded together by interatomic or inter molecular forces and stay in a stable equilibrium position. A deforming force changes the intermolecular distances. When deforming force is removed the interatomic force tends to drive them back to their original positions. Thus the body regains it original shape and size.

Not in new syllabus

The restoring mechanism can be understood by spring-ball model as shown in fig.



When any ball is displaced from its equilibrium position, the spring system tries to restore the ball back to its original position.

* Robert Hooke performed experiments on spring and found that elongation (change in the length) produced in a body is proportional to the applied force or load.

* Elastic behaviour of solid can be explained in terms of microscopic nature of solid.

Stress and Strain

- Stress
  - Normal stress
    - Tensile stress
    - compressive stress
  - shearing stress
  - Hydraulic stress

Stress: The restoring force per unit area is known as stress.
If F is the force applied and A is the area of cross-section of the body,

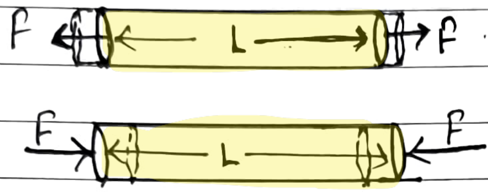
magnitude of the stress $= \frac{F}{A}$

SI unit - Nm^{-2} or pascal
Dimensional formula - $[ML^{-1}T^{-2}]$

The three types of stress are
1. Longitudinal or normal stress
2. Tengential or shearing stress
3. Hydraulic or volume stress

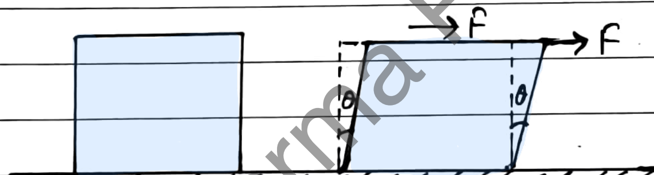
1. **Longitudnal or Normal stress:** It is defined as the restoring force per unit area perpendicular to the body.
Types - Tensile stress and compressive stress
Tensile stress: Tensile stress is the normal force per unit area that causes an object to increase in length. i.e the object is stretched out.

Compressive stress: Compressive stress is the normal force per unit area that causes an object to decrease in length. i.e object is compressed.



Tensile stress (L increases)
F → normal force
compressive stress (L decreases)

2. **Tangential or Shearing stress:** The restoring force per unit area developed due to the applied tangential force is known as tangential stress.

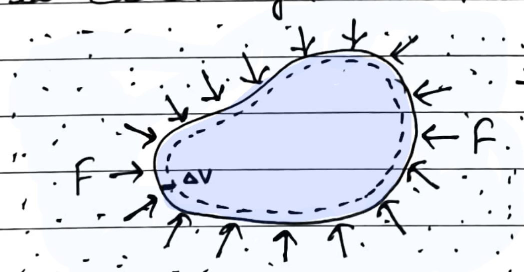


F → tangential force

before applying force

After applying force body deformed by an angle θ

3. **Hydraulic or Volume stress:** If a body is subjected to a uniform force from all sides, then corresponding stress is called hydraulic stress or hydrostatic stress.



→ Fluid

* Hydraulic stress is the measure of internal force per unit area acting on the liquid.
It is the restoring force per unit area when force is applied by the fluid on the body.

Strain: When a deforming force is applied on a body, the shape of ~~a b~~ the body changes and the body is said to be strained.
Strain is the ratio of change in configuration to the original configuration of the body.

$$\text{strain} = \frac{\text{Change in dimension}}{\text{original dimension}}$$

It has no unit and dimensions.

Strain
- Longitudinal strain
- Shear strain
- Volumetric strain

Types of strain are

1. **Longitudinal strain**: It is defined as the increase in length per unit original length.

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{\Delta l}{l}$$

2. **Volumetric strain**: It is defined as the change in volume per unit original volume.

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

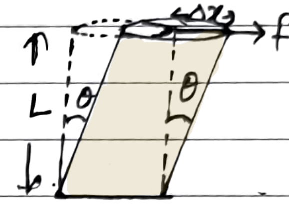
$$= \frac{\Delta V}{V}$$

3. **Shear strain**: It is define as the angle θ (in radian)

through which the fixed perpendicular face gets turned on applying tangential deforming force.

OR

The ratio of relative displacement of the faces Δx to the length L of the body



$$\text{Shearing strain} = \frac{\Delta x}{L} = \tan\theta$$

where θ is the angular displacement

Usually θ is very small, so

$$\tan\theta \simeq \theta = \frac{\Delta x}{L}$$

Hooke's Law: For small deformation the stress and strain are proportional to each other. This is known as Hooke's law.

Thus,

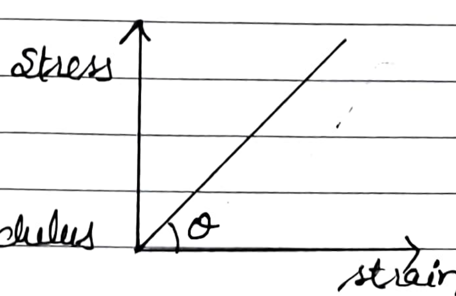
$$\text{Stress} \propto \text{strain}$$

$$\text{strain} = k \times \text{strain}$$

Where k is modulus of elasticity.

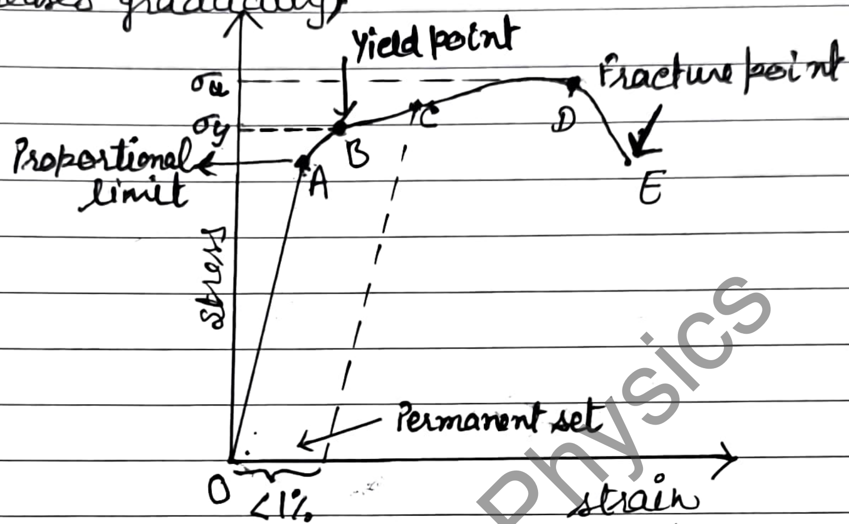
* **Hooke's law is empirical law.** This law is valid for most of the materials but some materials does not obey this law linearly. For example rubber (non-hookean material) and for aluminium, Hooke's law is only valid for a portion of elastic range.

Stress-strain graph for Hooke's law is straight line.
Slope of graph determines modulus of elasticity.



Stress - Strain Curve

Fig shows a typical stress - strain graph for a metal such as copper or soft iron. (Weight on wire increases gradually)



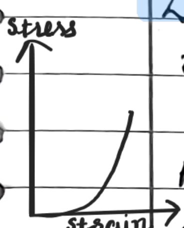
O to A - • Stress $\propto$ strain.
- Curve is linear
- Hooke's law is obeyed.

A to B - • Stress is not proportional to strain
- B represents the yield point. i.e body regains original dimensions when load is removed. σ_y is yield strength or elastic limit.

B to D - • The strain increases rapidly even for small change in stress.
- At any point b/w BD (say C) when ~~load~~ load is removed, body does not regain original dimensions.
- Strain is not zero for zero stress.
- Material said to have permanent set.
- Deformation is called plastic deformation.
- Point D is called ultimate tensile strength...

D to E - • Beyond D additional strain is produced even by reduced load.
• Fracture point occurs at point E.

* If D and E are close, body is brittle.
* If D and E are far apart, body is ductile.



Graph for Elastomers

Elastomers: Substances like aorta, rubber etc. do not obey Hooke's law over most of the region are called elastomers.
An elastomer has extremely weak inter-molecular force and a low Young's modulus.
* Elastomers are viscoelastic (elasticity + viscosity).

Young's Modulus (Y): The ratio of tensile or compressive stress to longitudinal strain is defined as Young's modulus.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

for circular wire $A = \pi r^2$

$$Y = \frac{FL}{\pi r^2 \Delta L}$$

$\Delta L \rightarrow l \rightarrow$ change

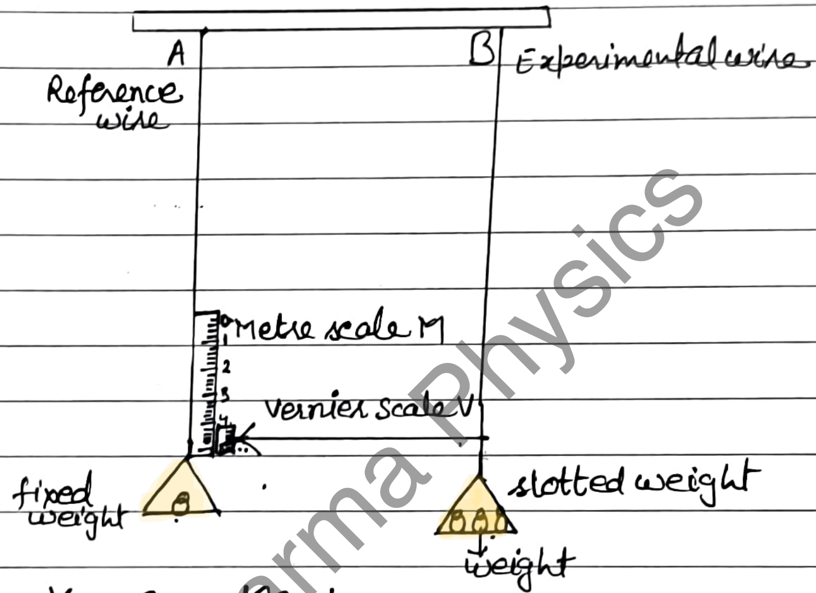
also

$$Y = \frac{mgL}{\pi r^2 \Delta L} \qquad [\because F = mg]$$

SI unit - $N\text{-}m^{-2}$ or Pa
Dim'al formula = $[ML^{-1}T^{-2}]$

Determination of Young's modulus of the material of a wire:

* A typical experimental arrangement to determine the Young's modulus is shown in fig.

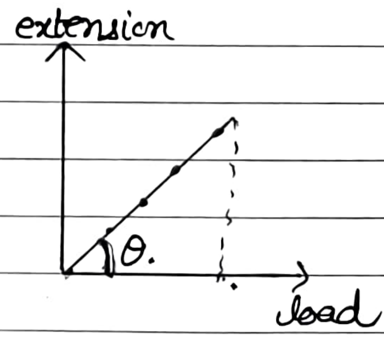


$$Y = \frac{\sigma}{\varepsilon} = \frac{Mg \cdot L}{\pi r^2 \Delta L}$$

By using this formula Young's modulus of the material is determined.

A graph is plotted b/w load and extension produced. Slope of the load-extension line

$$\tan\theta = \frac{\Delta L}{Mg} \Rightarrow \Delta L = Mg\tan\theta$$



$$Y = \frac{\text{stress}}{\text{strain}} = \frac{MgL}{\pi r^2 \Delta L}$$

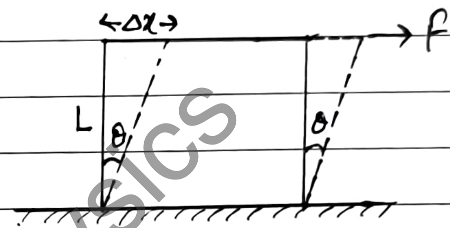
$$= \frac{MgL}{\pi r^2 Mg\tan\theta}$$

$$Y = \frac{L}{\pi r^2 \tan\theta}$$

Shear Modulus (G): The ratio of shearing stress to the corresponding shearing strain is called the shear modulus of the material and is represented by G. It is also called modulus of rigidity.

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$= \frac{F/A}{\Delta x/L}$$



or $$G = \frac{FL}{A\Delta x}$$

also
$$G = \frac{F/A}{\theta} = \frac{F}{A\theta}$$

$$G = \frac{F}{A\theta}$$

SI unit - Nm^{-2} or Pa

* Shear modulus is generally less than Young's modulus. For most materials $G = \frac{Y}{3}$

Bulk Modulus (B): The ratio of hydraulic stress to hydraulic strain is called Bulk modulus.

$$B = \frac{-p}{\Delta V/V}$$

$$B = \frac{-PV}{\Delta V}$$

-ve sign shows that when pressure P increased the volume V.

SI unit - Nm^{-2} or Pascal (Pa)

Compressibility (k): The reciprocal of Bulk modulus is called compressibility.

$$k = \frac{1}{B} = \frac{1}{-\Delta p / \frac{\Delta V}{V}}$$

$$k = -\frac{\Delta V}{pV}$$

SI unit - $N^{-1} m^2$
Dim^n - $[M^{-1} L T^2]$

* Bulk modulus of solids are about 50 times larger than that of water.
* Since $B_{solid} > B_{liquid} > B_{gas}$, thus solids are least compressible and gases are most.

Applications of Elastic Behaviour of Materials:

1 **Thickness of metallic ropes** - The thickness of metal ropes used in cranes to lift heavy loads is decided from the knowledge of the elastic limit of the material and the factor of safty.
Suppose we want to make a crane of lifting capacity of 10 tonnes ($=10^4$ kg). Let the safty factor is 10.

$$\underset{\text{(elastic limit)}}{\text{Ultimate stress}} = \frac{F}{A} = \frac{Mg}{\pi r^2}$$

For steel ultimate stress $= 30 \times 10^7 \ Nm^{-2}$

$$30\times10^{7} = \frac{(10^{4}\times10)\times9.8}{\pi r^{2}}$$ [10 is safty facter]

or $$r^{2} = \frac{10^{5}\times 9.8}{30\times10^{7}\times3.14}$$

$$r^{2} = \frac{9.8\times10^{-2}}{94.2} = 0.1040\times10^{-2}$$

or $r = 0.322\times10^{-1}$

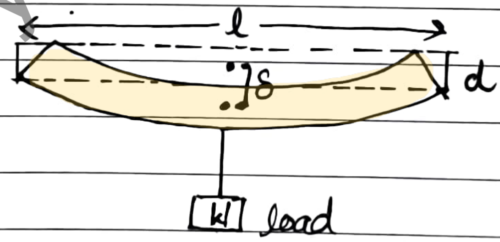
$r = 0.0322\ m$

or $r = 3.2\ cm$

* To increase the flexibility and strength of the rope it is always made of large number of thin wires braided together.

2. A bridge is designed such that it does not bend too much under the load of traffic.

Consider a rectangular bar of length l, breadth b and thickness d supported at both ends as shown in fig



When a load W is suspended at its middle, the bar gets depressed by an amount given by

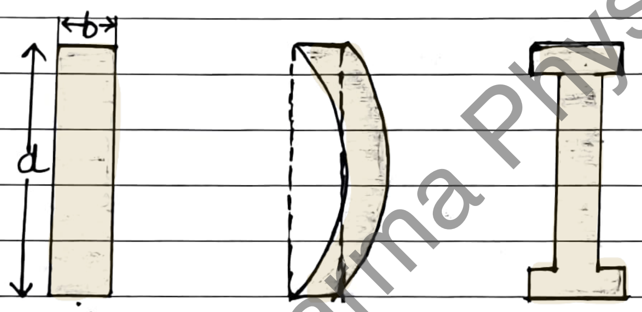
$$\delta = \frac{Wl^{3}}{4Ybd^{3}}$$

Bending can be reduced by using a material of large Young's modulus Y.

As δ is proportional to d^{-3} and only to b^{-1}, so depression can be decreased more effectively by increasing the depth d rather than the broadth b.

3. To avoid buckling beams are made of I shaped.

A beam or bar has a tendency to bend under a weight. This bending is called buckling.
To avoid buckling beams are made of I shaped.
I shaped cross-section provides a large load bearing surface.



Rectangular bar Buckling of bar I shaped cross-section of a bar.

This is the reason of making I shaphed railway track.

Maximum height of a mountain: The maximum height of a mountain on earth depends upon shear modulus of rock. The stress due to all the material on the top should be less than critical shearing stress at which the rocks flow.
The elastic limit of a typical rock is about $3\times10^{8}\,N/m^{2}$ and tangential shear is of order $h\rho g$.

$$h_{max}\rho g = 3\times10^{8}$$

$$h_{max} = \frac{3\times10^{8}}{\rho g} = \frac{3\times10^{8}}{3\times10^{3}\times9.8}$$

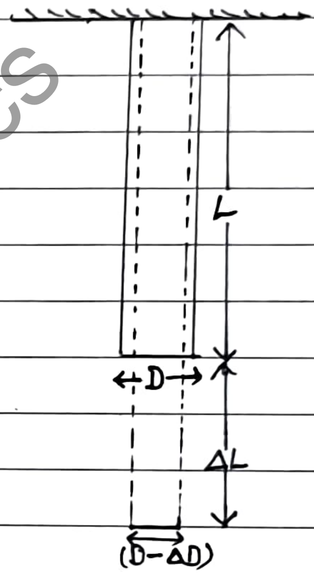
$$\simeq 10{,}000\ m = 10\ Km$$

This is nearly the height of the mount Everest. Height greater than this will not be able to stand due to the weight of mountain.

Poisson's Ratio: Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

Longitudinal strain $= \frac{\Delta L}{L}$

Lateral strain $= \frac{-\Delta D}{D}$



Poisson's ratio $\sigma = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$

$= \frac{-\Delta D/D}{\Delta L/L}$

or $\sigma = -\frac{L}{D} \cdot \frac{\Delta D}{\Delta L}$

−ve sign indicates that longitudinal and lateral strains are opposite.

Unit − unitless and dimensionless.

Elastic Potential Energy: When a wire is stretched, work has to be done against these restoring forces. This work done is stored in the form of elastic potential energy.

$$W = \int dW = \int_0^l F\,dx$$

$$= \int_0^l \frac{YAx}{L}\,dx \qquad \left[\because Y = \frac{FL}{Al} \quad l \to x\right]$$

$$= \frac{YA}{L}\int_0^l x\,dx$$

or $W = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l$

$= \frac{1}{2}\frac{YA}{L} . l^2$

$= \frac{1}{2} Y . \frac{l^2}{L^2} (AL)$

or $U = W = \frac{1}{2} \times$ Young's modulus $\times (\text{strain})^2 \times$ Volum

where U is elastic potential energy.

Energy density - Elastic energy density is elastic potential energy per unit volume.

$u = \frac{U}{\text{Volume}} = \frac{1}{2} \times \text{Young's modulus} \times (\text{strain})^2$

$u = \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times (\text{strain})^2$

$u = \frac{1}{2} \times \text{stress} \times \text{strain}$