Units And Measurement — Chapter 1

NCERT Examples

* Final answer should be rounded off to match the least no. of S.F in given data

Example 1

Each side of a cube - - - - - - - - significant figures?

Ans. Surface area of cube $= 6\,(\text{side})^2$

$= 6 \times (7.203)^2$

$= 6 \times 51.883209$

$= 311.299254$

$= 311.3\ m^2$ [S.F = 4]

Volume of cube $= (\text{side})^3$

$= (7.203)^3\ m^3$

$= 373.714754\ m^3$

$= 373.7\ m^3$ [upto 4 S.F]

Example 2

5.74 g of a substance - - - - - - - - figures in view.

Ans. Density $= \dfrac{\text{mass}}{\text{volume}}$

$= \dfrac{5.74}{1.2}\ g\,cm^{-3}$

$= 4.783\ g\,cm^{-3}$

$= 4.8\ g\,cm^{-3}$ [up to 2 S.F]

Example 3

Let us consider - - - - - - - - - dimensionally correct.

Ans. To check the correctness of

$$\frac{1}{2}mv^2 = mgh$$

Dimensions of L.H.S

$$\left[\frac{1}{2}mv^2\right] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$$

Dimensions of R.H.S

$[mgh] = [M][LT^{-2}][L]$

$= [ML^2T^{-2}]$

So, Dimensions of L.H.S = Dimensions of R.H.S

Hence the equation is correct.

Example 4

The SI unit of energy ——————— (c) $K = \frac{1}{2}mv^2 + ma$

Ans. SI unit of energy = J = $kg\,m^2s^{-2}$

Dimensions of energy, $K = [ML^2T^{-2}]$

Dimensions of velocity $[v] = [LT^{-1}]$

Dimensions of acceleration $[a] = [LT^{-2}]$

(a) $K = m^2v^3$

Dimensions of L.H.S $= [K] = [ML^2T^{-2}]$

Dimensions of R.H.S $= [m^2v^3] = [M^2][LT^{-1}]^3$

$= [M^2L^3T^{-3}]$

$[L.H.S] \neq [R.H.S]$

$\Rightarrow$ Equation is incorrect.

(b) $K = \frac{1}{2}mv^2$

$[L.H.S] = [K] = [ML^2T^{-2}]$

$[R.H.S] = [\frac{1}{2}mv^2] = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

$[L.H.S] = [R.H.S]$ *[$\frac{1}{2}$ is dimensionless]

$\Rightarrow$ Equation is correct.

(c) $K = ma$

$[L.H.S] = [ML^2T^{-2}]$

$[R.H.S] = [ma] = [M][LT^{-2}] = [MLT^{-2}]$

$[L.H.S] \neq [R.H.S]$

$\Rightarrow$ Equation is incorrect.

(d) $K = \frac{3}{16} mv^2$

$[L.H.S] = [K] = [ML^2T^{-2}]$

$R.H.S = \frac{3}{16} mv^2$

$= [M][LT^{-1}]^2$ $\quad$ [$\frac{3}{16}$ is dimensionless]

$= [ML^2T^{-2}]$

$[L.H.S] = [R.H.S]$

$\Rightarrow$ Equation is dimensionally correct.

(e) $K = \frac{1}{2} mv^2 + ma$

$[L.H.S] = [ML^2T^{-2}]$

$[R.H.S] = [\frac{1}{2} mv^2] + [ma]$

$[R.H.S] = [ML^2T^{-2}] + [MLT^{-2}]$

$[L.H.S] \neq [R.H.S]$

$\Rightarrow$ Equation is incorrect.

*[In R.H.S both terms should have same dimensions by principle of homogeniety]

Example 5

Consider a simple pendulum - - - - - - of dimensions.

Ans. Given that time period of simple pendulum depends on length, accn due to gravity and mass.

here $l \rightarrow$ length $\Rightarrow [l] = [L]$

$g \rightarrow$ accn due to gravity $\Rightarrow [g] = [LT^{-2}]$

$m \rightarrow$ mass $\Rightarrow [m] = [M]$

i.e. $T \propto l^a g^b m^c$

or $T = k l^a g^b m^c$ --(1) [$k \rightarrow$ dimensionless constant]

put dimensions on both sides

$[M^0L^0T] = [L]^a[LT^{-2}]^b[M]^c$

or $[M^0 L^0 T] = [M^c L^{a+b} T^{-2b}]$

On equating the dimensions on both sides, we get

For M

$$c = 0$$

For L

$$a + b = 0 \Rightarrow a = -b$$

For T

$$-2b = 1 \Rightarrow b = -\frac{1}{2}$$

and from $a = -b$

$$a = -\left(-\frac{1}{2}\right)$$

$$a = \frac{1}{2}$$

Put values of a, b and c in equation (1), we get

$$T = k\, l^{\frac{1}{2}} g^{-\frac{1}{2}} m^0$$

or $T = k\sqrt{\frac{l}{g}}$

here $k = 2\pi$

then $T = 2\pi\sqrt{\frac{l}{g}}$