Units And Measurements
Chapter 1
Handwritten NOTES
Class 11th PHYSICS
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Notes + Important Questions

Chapter - 1

UNITS AND MEASUREMENTS

Physical Quantities

Physical quantities are measurable properties of physical system.

Physical quantity = (Numerical value)(Units)

e.g

Length = 5 m
Mass = 10 Kg
Time = 3 sec

Unit: A unit is a standard reference used to measure physical quantities.

e.g. Units of length are - cm, m, km, ly etc.

Needs of Measurement: Measurement is needed to make the things clear, consistent and universal.

Definition of Measurement: Measurement is the comparision of a physical quantity with a standard unit.

Types of Units:

Units

Fundamental Units | Derived Units

Fundamental or base units: Fundamental units are the basic units which are independent and cannot be broken into simpler units. e.g length mass, time etc.

Derived Units: Derived units are formed by combination of fundamental units through multiplication or devision.

System of Units: A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units.

Some commonly used systems of units are:

(i) CGS → Centimetre, gram and second

(ii) FPS → Foot, pound and second

(iii) MKS → Metre, Kilogram and second

(iv) SI: The International System of Units: It is the modernised and extended form of metric system like CGS and MKS system.

It is based on the 7 basic units and 2 supplementary units :-

S.N.	Basic Physical Quantities	Basic Units
1	Length	Metre (m)
2	Mass	Kilogram (Kg)
3	Time	Second (S)
4	Temperature	Kelvin (K)
5	Electric Current	Ampere (A)
6	Luminous Intensity	Candela (Cd)
7	Quantity of matter	Mole (mol)

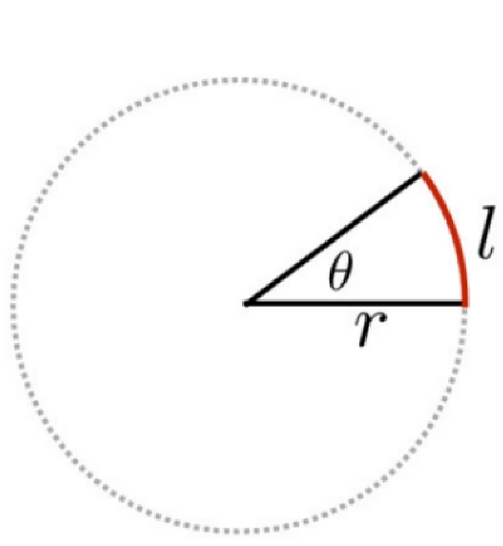
MKS & CGS are metric or decimal system but FPS is not.

S.N	Supplementary Quantities	Basic Units
1	Plane Angle	Radian
2	Solid Angle	Steradian

Angles

Angle: ratio of subtended arc length on circle to radius

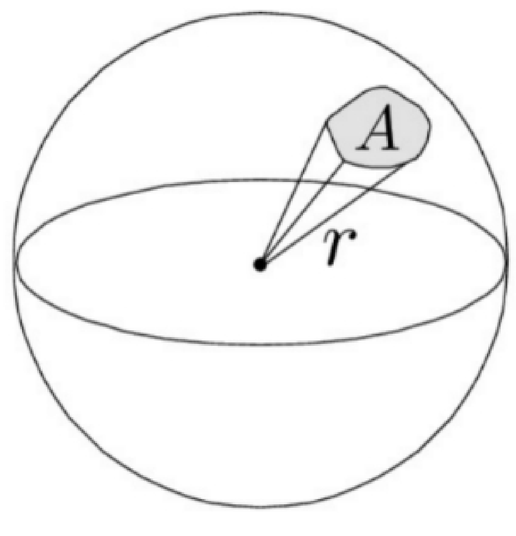
- $\theta = \frac{l}{r}$
- Circle has 2π radians



Solid Angles

Solid angle: ratio of subtended area on sphere to radius squared

- $\Omega = \frac{A}{r^2}$
- Sphere has 4π steradians



Units of Length

(a) For small distances

(i) **Fermi**: Used for measuring nuclear sizes. Also called femtometer. $\boxed{1 \text{ fermi} = 10^{-15} m}$

(ii) **Angstrom**: Used to express atomic dimensions.

$$\boxed{1 A^\circ = 10^{-10} m}$$

(iii) **Nanometer**: Used for measuring wavelength of light.

$$\boxed{1 \ nm = 10^{-9} m}$$

(iv) **Micron**: used to measure cell and bacteria.

$$\boxed{1 \mu = 10^{-6} m}$$

(b) For large distances:

(i) **Light Year**: It is the distance travelled by light in vacuum in one year.

$$\boxed{1 ly = 9.46 \times 10^{15} m}$$

(ii) **Astronomical Unit**: It is defined as the mean distance of earth from sun.

$$1 A.U = 1.496 \times 10^{11} m$$

or $\boxed{1 A.U = 1.5 \times 10^{11} m}$

Parsec (Parallactic Second): It is defined as the distance at which an arc of lenght 1 A.U. subtends an angle of 1 second of arc.

$$1 \text{ Parsec} = 3.08 \times 10^{16} m \qquad \left[\because r = \frac{l}{\theta}, \ l = 1 AU, \ \theta = 1''\right]$$

$$\boxed{1 \text{ Parsec} = 3.08 \times 10^{16} m = 3.26 \ ly}$$

Significant Figures: Significant figures are the digit in a number that show how accurate a measurement is. They include all certain digit and the first uncertain (doubtful) digit.

eg In a measurement of length of 15.3 all three digits are significant. (1 & 5 are certain and 3 is uncertain)

One Rule

Number

For non-decimal numbers	For decimal numbers
• Trailing zeros are not significant	• Zeros before the first non zero digit are not significant.

e.g.

Numbers	Significant figures (S.F)
4500 m	2
4500.0 m	5
0.00560 g	3
1.230 cm	4
100.0 kg	4
0.00089 s	2
670 N	2
6.070 ml	4
4500	∞

* Significant figure does not depend on the system of units. e.g 16.4 cm or 0.164 m both have 3 S.F.

* Power of 10 is not relatable to S.F. e.g 3.5×10^4 has 2 S.F.

* The numbers do not represent any measured values are exact and have infinite (∞) number of significant digit.

e.g. In $r = \frac{d}{2}$, the '2' is exact number and can be witten as 2.0, 2.00 or 2.000 as required.

Rounding off the uncertain digits

Case I: If last digit is less than 5 (i.e 0, 1, 2, 3, 4) leave the previous digid unchanged and drop the last digit.

e.g → Rounding off 7.43 to one decimal → 7.4

→ Rounding off 7.432 to one decimal → 7.4

→ Rounding off 7.558 to one decimal → 7.6

<u>Case II</u>: If last digit is greater than 5 (ie 6, 7, 8, 9)

Increase previous digit by 1 and drop the last digit.

e.g 7.47 → 7.5 (to one decimal place)

7.437 → 7.5 (to one decimal place)

<u>Case III</u>: If last digit is exactly 5

→ If digit before 5 is even, leave it unchanged and drop the 5. e.g 12.25 → 12.2

→ If digit before 5 is odd increase it by 1 and drop the 5. e.g 12.35 → 12.4

<u>Rules for uncertainty in Arithmetic calculations</u>

These rules help to decide how many significant figures should be there in final answer after arithmetic operations like addition, subtraction multiplication and division.

(1) <u>For addition and subtraction</u> The uncertainty in final result depends on the least precise value.

e.g if $l = 16.2$ cm (1 decimal place)

$b = 10.15$ cm (2 decimal place)

Perimeter of rectangle $= 2(l+b)$

$= 2(16.2 + 10.15)$

$= 52.70$ cm

Now apply S.F rule, the final answer = 52.7 cm (one decimal place)

<u>Multiplication and Division</u> The result should have the same number of S.F as the value with least S.F.

e.g. If $m = 5.74\ g$ (2 decimal place)
$V = 1.2\ cm$ (1 decimal place)

$$\text{Density} = \frac{m}{V} = \frac{5.74}{1.2} = 4.7833$$

Apply S.F rule, we get
Density $= 4.8\ g/cm^3$. (one decimal place)

Dimensions of Physical Quantities

The dimensions of a physical quantity are the power (exponents) to which the base quantities raised to represent that quantity. Like

$$\left.\begin{array}{l}\text{Mass } [M] \\ \text{Length } [L] \\ \text{Time } [T]\end{array}\right\} \longrightarrow [M^a L^b T^c]$$

→ If a quantity does not depend on a base quantity, its exponent is zero.

→ Derived quantities can be expressed using these base dimensions.

Quantity	Formula	Unit	Dimensions
Area	$A = l \times b$	m^2	$[L^2]$
Volume	$V = l \times b \times h$	m^3	$[L^3]$
Density	$\rho = m/V$	$Kg\, m^{-3}$	$[ML^{-3}]$
velocity	$v = d/t$	$m s^{-1}$	$[LT^{-1}]$
Acceleration	$a = \Delta v/\Delta t$	$m s^{-2}$	$[LT^{-2}]$
Force	$F = ma$	$Kg\text{-}m\text{-}s^{-2}$	$[MLT^{-2}]$
Work	$W = Fd$	$Kg\, m^2 s^{-2}$	$[ML^2T^{-2}]$
Energy	$K = \frac{1}{2}mv^2,\ U = mgh$	$Kg\, m^2 s^{-2}$	$[ML^2T^{-2}]$
Power	$P = W/t$	$Kg\, m^2 s^{-3}$	$[ML^2T^{-3}]$
Pressure	$P = F/A$	$Kg.\, m^{-1} s^{-2}$	$[ML^{-1}T^{-2}]$
Momentum	$p = mv$	$Kg\, m s^{-1}$	$[MLT^{-1}]$
Impulse	$I = F\Delta t$	$Kg\, m s^{-1}$	$[MLT^{-1}]$
Angle	$\theta = \text{Arc}/\text{Radius}$	Radian	[Unitless]

DIMENSIONAL FORMULAE OF PHYSICAL QUANTITIES

S.No	Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
1.	Area	Length × breadth	$[L^2]$	$[M^0 L^2 T^0]$
2.	Volume	Length × breadth × height	$[L^3]$	$[M^0 L^3 T^0]$
3.	Mass density	Mass/volume	$[M]/[L^3]$ or $[M L^{-3}]$	$[M L^{-3} T^0]$
4.	Frequency	1/time period	$1/[T]$	$[M^0 L^0 T^{-1}]$
5.	Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0 L T^{-1}]$
6.	Acceleration	Velocity /time	$[LT^{-1}]/[T]$	$[M^0 L T^{-2}]$
7.	Force	Mass × acceleration	$[M][LT^{-2}]$	$[M L T^{-2}]$
8.	Impulse	Force × time	$[M L T^{-2}][T]$	$[M L T^{-1}]$
9.	Work, Energy	Force × distance	$[MLT^{-2}]\,[L]$	$[M L^2 T^{-2}]$
10.	Power	Work/time	$[ML^2 T^{-2}]/[T]$	$[M L^2 T^{-3}]$
11.	Momentum	Mass × velocity	$[M]\,[LT^{-1}]$	$[M L T^{-1}]$
12.	Pressure, stress	Force/area	$[M L T^{-2}]/[L^2]$	$[M L^{-1} T^{-2}]$
13.	Strain	$\dfrac{\text{Change in dimension}}{\text{Original dimension}}$	$[L]/[L]$ or $[L^3]/[L^3]$	$[M^0 L^0 T^0]$
14.	Modulus of elasticity	Stress/strain	$\dfrac{[M L^{-1} T^{-2}]}{[M^0 L^0 T^0]}$	$[M L^{-1} T^{-2}]$
15	Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[M L^0 T^{-2}]$
16.	Surface energy	Energy/area	$[ML^2 T^{-2}]/[L^2]$	$[M L^0 T^{-2}]$
17.	Velocity gradient	Velocity/distance	$[LT^{-1}]/[L]$	$[M^0 L^0 T^{-1}]$
18.	Pressure gradient	Pressure/distance	$[ML^{-1} T^{-2}]/[L]$	$[M L^{-2} T^{-2}]$
19.	Pressure energy	Pressure × volume	$[ML^{-1} T^{-2}]\,[L^3]$	$[M L^2 T^{-2}]$
20.	Coefficient of viscosity	Force/area × velocity gradient	$\dfrac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[M L^{-1} T^{-1}]$
21.	Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0 L^0 T^0]$
22.	Trigonometric ratio (sinθ, cosθ, tanθ, etc.)	Length/length	$[L]/[L]$	$[M^0 L^0 T^0]$
23.	Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0 L^0 T^{-1}]$

24.	Angular acceleration	Angular velocity/time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
25.	Radius of gyration	Distance	$[L]$	$[M^0LT^0]$
26.	Moment of inertia	Mass × (radius of gyration)^2	$[M]\,[L^2]$	$[ML^2\,T^0]$
27.	Angular momentum	Moment of inertia × angular velocity	$[ML^2]\,[T^{-1}]$	$[ML^2\,T^{-1}]$
28.	Moment of force, moment of couple	Force x distance	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
29.	Torque	Angular momentum/time, Or Force × distance	$[ML^2\,T^{-1}]\,/\,[T]$ or $[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
30.	Angular frequency	2π × Frequency	$[T^{-1}]$	$[M^0L^0T^{-1}]$
31.	Wavelength	Distance	$[L]$	$[M^0LT^0]$
32.	Hubble constant	Recession speed/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
33.	Intensity of wave	(Energy/time)/area	$[ML^2\,T^{-2}/T]/[L^2]$	$[ML^0T^{-3}]$
34.	Radiation pressure	$\frac{\text{Intensity of wave}}{\text{Speed of light}}$	$[MT^{-3}]/[LT^{-1}]$	$[ML^{-1}\,T^{-2}]$
35.	Energy density	Energy/volume	$[ML^2T^{-2}]/\,[L^3]$	$[ML^{-1}\,T^{-2}]$
36.	Critical velocity	$\frac{\text{Reynold's number} \times \text{coefficient of viscocity}}{\text{Mass density} \times \text{radius}}$	$\frac{[M^0L^0T^0][ML^{-1}\,T^{-1}]}{[ML^{-3}][L]}$	$[M^0LT^{-1}]$
37.	Escape velocity	(2 × acceleration due to gravity × earth's radius)$^{1/2}$	$[LT^{-2}]^{1/2}$ x $[L]^{1/2}$	$[M^0LT^{-1}]$
38.	Heat energy, internal energy	Work (= Force × distance)	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
39.	Kinetic energy	(1/2) mass × (velocity)^2	$[M]\,[LT^{-1}]^2$	$[ML^2T^{-2}]$
40	Potential energy	Mass × acceleration due to gravity × height	$[M]\,[LT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
41.	Rotational kinetic energy	½ × moment of inertia × (angular velocity)^2	$[M^0L^0T^0]\,[ML^2]$x$[T^{-1}]^2$	$[M\,L^2\,T^{-2}]$
42.	Efficiency	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$	$\frac{[ML^2\,T^{-2}]}{[ML^2\,T^{-2}]}$	$[M^0L^0T^0]$
43.	Angular impulse	Torque × time	$[ML^2\,T^{-2}]\,[T]$	$[M\,L^2\,T^{-1}]$
44.	Gravitational constant	$\frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{[M]\,[M]}$	$[M^{-1}L^3T^{-2}]$
45.	Planck constant	Energy/frequency	$[ML^2\,T^{-2}]\,/[T^{-1}]$	$[ML^2T^{-1}]$

46.	Heat capacity, entropy	Heat energy / temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
47.	Specific heat capacity	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{temperature}}$	$[ML^2T^{-2}]/[M][K]$	$[M^0L^2T^{-2}K^{-1}]$
48.	Latent heat	Heat energy/mass	$[ML^2T^{-2}]/[M]$	$[M^0L^2T^{-2}]$
49.	Thermal expansion coefficient or Thermal expansivity	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	$[L]/[L][K]$	$[M^0L^0K^{-1}]$
50.	Thermal conductivity	$\frac{\text{Heat energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$\frac{[ML^2T^{-2}][L]}{[L^2][K][T]}$	$[MLT^{-3}K^{-1}]$
51.	Bulk modulus or $(\text{compressibility})^{-1}$	$\frac{\text{Volume} \times (\text{change in pressure})}{(\text{change in volume})}$	$\frac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	$[ML^{-1}T^{-2}]$
52.	Centripetal acceleration	$(\text{Velocity})^2/\text{radius}$	$[LT^{-1}]^2/[L]$	$[M^0LT^{-2}]$
53.	Stefan constant	$\frac{(\text{Energy} / \text{area} \times \text{time})}{(\text{Temperature})^4}$	$\frac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	$[ML^0T^{-3}K^{-4}]$
54.	Wien constant	Wavelength × temperature	$[L][K]$	$[M^0LT^0K]$
55.	Boltzmann constant	Energy/temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
56.	Universal gas constant	$\frac{\text{Pressure} \times \text{volume}}{\text{mole} \times \text{temperature}}$	$\frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$
57.	Charge	Current × time	$[A][T]$	$[M^0L^0TA]$
58.	Current density	Current /area	$[A]/[L^2]$	$[M^0L^{-2}T^0A]$
59.	Voltage, electric potential, electromotive force	Work/charge	$[ML^2T^{-2}]/[AT]$	$[ML^2T^{-3}A^{-1}]$
60.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$
61.	Capacitance	Charge/potential difference	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^4A^2]$
62.	Electrical resistivity or $(\text{electrical conductivity})^{-1}$	$\frac{\text{Resistance} \times \text{area}}{\text{length}}$	$[ML^2T^{-3}A^{-2}]$ $[L^2]/[L]$	$[ML^3T^{-3}A^{-2}]$
63.	Electric field	Electrical force/charge	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
64.	Electric flux	Electric field × area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$

Dimensional Equation: Equation formed by equating a physical quantity with its dimensional formula.
e.g

$[F] = [M^1L^1T^{-2}]$ $F \rightarrow$ Force

$[\rho] = [M^1L^{-3}T^0]$ $\rho \rightarrow$ Density

Principle of Homgeneity: According to this principle All terms in a physical equation must have the same dimensions.
It helps to check the correctness of an equation or formula.
e.g. • If deriving formula for speed, all terms should reduce to $[LT^{-1}]$
• In $v = u + at$, all three terms have same dimensions $[LT^{-1}]$

Applications of Dimensions: Uses of dimensions

1. **Deducing relation among the physical quantities**

For this we should know the dependence of physical quantity on other quantities (upto 3 only) and consider it as a product type of the dependence.

Example

Time period 'T' of a simple pendulum depends on the quantities mass of pendulum 'm', length of string 'l' and acceleration due to gravity.
i.e

$$T \propto m^a l^b g^c \quad [a, b \text{ and } c \text{ are exponents}]$$

or $$T = k\, m^a l^b g^c \quad \text{—— (1)}$$

where k is dimensional constant

put dimensions on both sides, we get

$$[M^0L^0T^1] = [M]^a[L]^b[LT^{-2}]^c$$

or $$[M^0L^0T^1] = [M^aL^{b+c}T^{-2c}]$$

On equating the dimensions on both sides -

For M $\boxed{a = 0}$

For L $\quad b + c = 0 \Rightarrow b = -c$

For T $\quad -2c = 1 \Rightarrow \boxed{c = -\frac{1}{2}}$

So $\boxed{b = \frac{1}{2}}$ $\quad [\because b = -c]$

$a = 0$ shows that Time period is independent of mass

Now from eqn (1)

$$T = k\, m^a l^b T^c$$

$$T = k\, m^0 l^{1/2} T^{-1/2}$$

or $\quad T = k\sqrt{\frac{l}{g}}$

here $k = 2\pi$, then

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

2. Checking the dimensional consitency of equations (To check the correctness of formula)

We can add or subtract similar physical quantities only. This simple priciple is very useful to check the correctness of an equation.

Example

Check whether the given equation is dimensionally correct $\quad \frac{1}{2}mv^2 = mgh$

In equation $\frac{1}{2}mv^2 = mgh$

$[m] = [M]$, $[v] = [LT^{-1}]$, $[g] = [LT^{-2}]$ and $[h] = [L]$

put dimensions on both sides, we get

$$[M][LT^{-1}]^2 = [M][LT^{-2}][L]$$

$$\Rightarrow [ML^2T^{-2}] = [ML^2T^{-2}]$$

i.e. $\quad$ L.H.S = R.H.S

Hence equation is dimensionally correct.

Example The SI unit of energy is Joule (J) which has demensions : $[ML^2T^{-2}]$
Which of the following formulas for kinetic energy K are dimensionally incorrect -

(a) $K = m^2 v^3$

(b) $K = \frac{1}{2} mv^2$

(c) $K = ma$

(d) $K = \frac{3}{16} mv^2$

(e) $K = \frac{1}{2}mv^2 + ma$

Solⁿ:

(a) Given, $K = m^2 v^3$

$[ML^2T^{-2}] = [M]^2 [LT^{-1}]^2$

$[ML^2T^{-2}] = [M^2L^2T^{-2}]$

L.H.S ≠ R.HS

So it is incorrect

(b) Given, $K = \frac{1}{2} mv^2$

$[ML^2T^{-2}] = [M][LT^{-1}]^2$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

LHS = R.HS

It is correct.

(c) Given

$K = ma$

$[ML^2T^{-2}] = [MLT^{-2}]$

LHS ≠ R.H.S

It is incorrect.

(d) $K = \frac{3}{16} mv^2$

$[ML^2T^{-2}] = [M][LT^{-1}]^2$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

LHS = R.H.S

So it is dimensionally correct.

(e) Given,

$$K = \frac{1}{2}mv^2 + ma$$

$$[ML^2T^{-2}] = [M][LT^{-1}]^2 + [M][LT^{-2}]$$

$$[ML^2T^{-2}] = [ML^2T^{-2}] + [MLT^{-2}]$$

$$L.H.S \neq R.H.S$$

Different dimensions being added which is not possible so it is incorrect.

→ Formulas (a), (c) and (e) are ruled out using dimensional analysis.
Only (b) and (d) are dimensionally correct.
However (d) is not actually correct. This is one of the limitations of dimensions.

3. Conversion of system of units

To convert the numerical value of a physical quantity from one system of units to another without changing the actual quantity.

$$\boxed{n_1 u_1 = n_2 u_2}$$

n_1 → numerical value in system 1
n_2 → numerical value in system 2
u_1 → unit in system 1
u_2 → unit in system 2

So

$$n_2 = n_1 \left(\frac{u_1}{u_2}\right)$$

Using dimensional analysis

$$\boxed{n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c}$$

<u>Example:</u> Convert 1 J into erg

Sol^n^- Joule is unit of energy

$[\text{energy}] = [ML^2T^{-2}]$

$n_1 = 1$, $n_2 = ?$ $a = 1$, $b = 2$, $c = -2$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]$$

$$n_2 = 1 \times \left[\frac{Kg}{g}\right]^1 \left[\frac{m}{cm}\right]^2 \left[\frac{s}{s}\right]^{-2}$$

$$n_2 = \left[\frac{1000g}{g}\right]\left[\frac{100cm}{cm}\right] \times 1$$

or $n_2 = 10^5$

Now $n_2 u_2 = \underline{10^5 \text{ erg}}$

Limitations of Dimensional Analysis

- The value of dimensional constant cannot be determined.
- This method cannot be applied to equations involving exponential and trigonometric functions (e^x, $\sin\theta$, $\cos\theta$, log etc)
- It can not be applied to an equation involving more than 3 physical quantities.
- It can check the correctness of a formula dimensionally only. It cannot check the formula absolutely correct or not.
- It cannot detect dimensionless quantities like refractive index, angles etc.

* Dimensionless quantities still have units

→ Plane angle (dimensionless) unit → Radian (rad)

→ Solid angle (dimensionless) unit → Steradian (sr)

* Dimensions are same but physical meaning is different →

→ Work (scalar) and torque (vector)

S No.	Physical Quantity	Relation with Other Quantity	Dimensional formula	SI unit
54.	Specific resistance or resistivity	$\rho = \frac{RA}{l}$	$[ML^3T^{-3}A^{-2}]$	Ωm
55.	conductivity	$1/\rho$	$[M^{-1}L^{-3}T^3A^2]$	$\Omega^{-1}m^{-1}$
56.	Electric dipole moment	$q \times 2l$	$[M^0LTA]$	Cm
57.	Magnetic field	$B = \frac{F}{qv \sin\theta}$	$[ML^0T^{-2}A^{-1}]$	T (tesla)
58.	Magnetic Flux	$\phi = BA$	$[ML^2T^{-2}A^{-1}]$	Wb (weber)
59.	Permeability of free space	$\mu_0 = \frac{4\pi r \cdot F}{I_1 I_2 l}$	$[MLT^{-2}A^{-2}]$	
60.	Magnetic moment	Current × Area	$[M^0L^2T^0A]$	Am^2
61.	Pole strength	$\frac{\text{Magnetic moment}}{\text{Magnetic length}}$	$[M^0LT^0A]$	Am

Important Questions for Practice

* Kindly mind the square brackets [ ] if they are missing.

APPLICATIONS OF DIMENSIONS:

(i). To check the correctness of given formula.

Eg:- given,

$v = u + at$, where.

v = final velocity
u = initial "
a = acceleration
t = time

Q. check if given equation is correct or not.

$v = u + at$

Put all the dimension;

$[LT^{-1}] = [LT^{-1}] + [LT^{-2}][T]$

$[LT^{-1}] = [LT^{-1}] + [LT^{-1}]$

LHS = RHS

Therefore given eqn is correct

(ii). Check whether the equation is correct or not

$t_1 \Rightarrow s = ut + \frac{1}{2} at$ s = displacement

<u>Soln.</u> $s = ut + \frac{1}{2} at$

Put all the dimension

$[L] = [LT^{-1}][T] + [LT^{-2}][T]$

$[L] = [L] + [LT^{-1}]$

LHS $\neq$ RHS

Hence, the equation is not correct.

(iii) $x = x_0 + v_0 t + \frac{1}{2} at^2$

x_0 = distance travelled
t = time
v_0 = initial velocity
a = acceleration

The dimensions of various terms:

$[L] = [L] + [LT^{-1}][T] + [LT^{-2}][T^2]$

$[L] = [L] + [L] + [L]$

Since the dimensions of all terms are same, hence the given eq^n is dimensionally correct.

(iv) $FS = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$,

F = force
m = mass
S = distance
u, v = velocity $[LT^{-1}]^2$

Put all the dimensions

$MLT^{-2} . L = [M][LT^{-1}]^2 - [M][LT^{-1}]^2$

$ML^2T^{-2} = ML^2T^{-2} - ML^2T^{-2}$

Hence, the equation is correct.

(v) $\lambda = \frac{h}{mv}$ h- Planck's constant

mv = mass x velocity

λ - wavelength

$\lambda = [L]$

$$\frac{h}{mv} = \frac{ML^2T^{-1}}{[M][LT^{-1}]} = [L]$$

LHS = RHS

Hence, the equation is correct.

(vi). $v = \sqrt{\frac{2GM}{R}}$

$[v] = LT^{-1}$

$$\left[\frac{2GM}{R}\right]^{\frac{1}{2}} = \left[\frac{M^{-1}L^3T^{-2}.M}{L}\right]^{\frac{1}{2}}$$

$$= [L^2 T^{-2}]^{\frac{1}{2}}$$

$$= LT^{-1}$$

LHS = RHS

Hence, the equation is correct.

(vii) $E = mc^2$ E = Energy

$[E] = ML^2T^{-2}$

$[mc^2] = M[LT^{-1}]^2 = ML^2T^{-2}$

LHS = RHS

Q. Find the dimension of a/b in the equation $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

By principle of homogeneity,

$a\sqrt{x} = F$

$$a = \frac{F}{\sqrt{x}} = \frac{[MLT^{-2}]}{\sqrt{L}}$$

$$= \frac{MLT^{-2}}{L^{1/2}} = [ML^{1/2}T^{-2}]$$

$$bt^2 = F$$

$$b = \frac{F}{t^2} = \frac{[MLT^{-2}]}{T^2} = MLT^{-4}$$

$$\boxed{\frac{a}{b} = \frac{ML^{1/2}T^{-2}}{MLT^{-4}}}$$

Q. find the dimension of $a \times b$ in:

$P = \frac{b - x^2}{at}$, P = Power, x = distance, t = time, V = volume

By principle of homogeneity,

$[b] = [x^2] = L^2$

$[b] = [L^2]$

Now,

$$[P] = \left[\frac{b}{at}\right]$$

$$[a] = \frac{[b]}{[P][t]} = \frac{L^2}{[ML^2T^{-3}][T]}$$

$$= M^{-1}T^2$$

$$\boxed{\begin{aligned} a \times b &= L^2 \times M^{-1}T^2 \\ &= M^{-1}L^2T^2 \end{aligned}}$$

Q vander wall's equation for gas is:

$$\left[P + \frac{a}{V^2}\right](V - b) = RT. \quad a = ? \quad b = ?$$

Hence write SI unit of a and b.

By principle of homogeneity,

$$P = \frac{a}{V^2}$$

$PV^2 = a$

$[ML^{-1}T^{-2}][L^3]^2 = a$

$ML^5T^{-2} = a$

SI unit of a is $kg\,m^5 s^{-2}$.

$b = V = L^3$.

SI unit of b is m^3

Q. $\mu = A + \frac{B}{\lambda^2}$. find SI unit of A and B

Here,

$\mu = \frac{\text{velocity of light in air}}{\text{velocity of light in glass}}$

By POH, = a dimensionless number

$[A] = \mu$ = dimensionless

As $\left[\frac{B}{\lambda^2}\right] = \mu$

$[B] = [\lambda^2][\mu] = L^2 \cdot 1 = L^2$

SI unit of B is m^2.

Q. $y = a \sin(\omega t - kx)$, t and x is time and distance respectively. obtain dimension for ω and k.

An angle is dimensionless.

$[\omega t] = 1$ or $[\omega] = \frac{1}{[t]} = \frac{1}{T} = T^{-1}$

$[kx] = 1$ or $[k] = \frac{1}{[x]} = \frac{1}{L} = L^{-1}$

CONVERSION OF UNITS (one system to another)

$$n_1 u_1 = n_2 u_2$$

n_1 → numerical value, u_1 → unit

$$\underset{n_1}{1}\ \underset{u_1}{km} = \underset{n_2}{1000}\ \underset{u_2}{m}$$

$n_2 = ?$

$$n_2 = n_1 \left[\frac{u_1}{u_2}\right]$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Example! Change 36 km/hr into m/~~hr~~s.

$n_1 = 36$
$u_1 = km/hr \quad [M^0 L T^{-1}]$
$u_2 = m/s \quad [M^0 L T^{-1}]$
$n_2 = ?$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^0 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-1}$$

$$= 36 \left[\frac{km}{m}\right]^1 \left[\frac{hr}{sec}\right]^{-1}$$

$$= 36 \left[\frac{1000\cancel{m}}{\cancel{m}}\right] \left[\frac{\cancel{60 \times 60s}}{\cancel{60s}}\right]^{-1} \left[\frac{60 \times 60s}{sec}\right]^{-1}$$

$$= 36 \,[1000] \left[\frac{1}{60 \times 60}\right]$$

$$= \frac{\cancel{36000}^{10}}{\cancel{3600}}$$

$$= 10\ m/s$$

Q. Convert 100 N into Dyne.

$F = [MLT^{-2}]$

$a = 1,\ b = 1,\ c = -2$

$n_1 = 100$
$u_1 = N = [kg\,m\,s^{-2}]$
$u_2 = dyne = [g\,cm\,s^{-2}]$

$$n_2 = n_1 \left[\frac{u_1}{u_2}\right]$$

$$n_2 = 100 \left[\frac{M_1}{M_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^{-2}$$

$$n_2 = 100 \left[\frac{kg}{g}\right]^1 \left[\frac{m}{cm}\right]^1 \left[\frac{s}{s}\right]^{-2}$$

$$= 100 \left[\frac{1000g}{g}\right]^1 \left[\frac{100cm}{cm}\right]^1 [1]^{-2}$$

$$= 100 [1000] [100]$$

$$= 100 \times 10^5$$

$$100N = 10^7 \text{ dyne.}$$

NCERT

Q1.3)

$n_1 = 4.2$

$u_1 = J = kg\,m^2/s^2$

$n_2 = ?$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$4.2 \left[\frac{kg}{\alpha kg}\right]^1 \left[\frac{m}{\beta m}\right]^2 \left[\frac{s}{\gamma s}\right]^{-2}$$

$$= 4.2 \left[\frac{1}{\alpha}\right]^1 \left[\frac{1}{\beta}\right]^2 \left[\frac{1}{\gamma}\right]^{-2}$$

$$= 4.2 \left[\alpha^{-1} \beta^{-2} \gamma^2\right]$$

Q. Convert 1 Joule into erg

Joule is SI unit of energy

$$\text{energy} = [ML^2T^{-2}]$$

$$a = 1, \; b = 2, \; c = -2$$

$$J = kg\,m^2 s^{-2}$$

$$\text{Erg} = g\,cm^2 s^{-2}$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 1 \left[\frac{kg}{g}\right]^1 \left[\frac{m}{cm}\right]^2 \left[\frac{s}{s}\right]^{-2}$$

$$= 1 \left[\frac{1000\cancel{g}}{\cancel{g}}\right]^1 \left[\frac{100\cancel{cm}}{\cancel{cm}}\right]^2 [1]^{-2}$$

$$= 1 [1000][10000]$$

$$1 J = 10^7 \text{ erg.}$$

Q. When 1 m, 1 kg, and 1 min are taken as fundamental units, the magnitude of force is 36 units. What will be the value of this force in CGS system?

Energy $= [MLT^{-2}]$

$a = 1, b = 1, c = -2$

Force $= N = kg\,m/s^2$ CGS unit $= g\,cm/s^2$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 36 \left[\frac{kg}{g}\right]^1 \left[\frac{m}{cm}\right]^1 \left[\frac{60s}{s}\right]^{-2}$$

$$= 36 \left[\frac{1000\cancel{g}}{\cancel{g}}\right]^1 \left[\frac{100\cancel{cm}}{\cancel{cm}}\right]^1 \left[\frac{60\cancel{s}}{\cancel{s}}\right]^{-2}$$

$$= 36 [1000] [100] [60]^{-2}$$

$$= \cancel{36} [1000] [1\cancel{00}] \left[\frac{1}{\cancel{36}\cancel{00}}\right]$$

$$= 10^3 \text{ dyne.}$$

Q The value of G in CGS system is 6.67×10^{-8} dyne $cm^2\, g^{-2}$. Calculate it in SI units

$$F = \frac{G m_1 m_2}{R^2}$$

$$G = \frac{FR^2}{m_1 m_2}$$

$$[G] = \frac{[MLT^{-2} \cdot L^2]}{MM} = M^{-1} L^3 T^{-2}$$

$a = -1, \; b = 3, \; c = -2$

$G =$ dyne $cm^2 g^{-2}$ $\qquad$ SI $=$ $N m^2 kg^{-2}$

$n_1 = 6.67 \times 10^{-8}$ $\qquad$ $n_2 = ?$

$M_1 = 1g$ $\qquad$ $M_2 = 1kg = 1000\,g$

$L_1 = 1cm$ $\qquad$ $L_2 = 1m = 100\,cm$

$T_1 = 1s$ $\qquad$ $T_2 = 1\,sec$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$6.67 \times 10^{-8} \left[\frac{1g}{1000g}\right]^{-1} \left[\frac{1cm}{100cm}\right]^3 \left[\frac{1}{1}\right]^{-2}$$

$$= 6.67 \times 10^{-11}\ Nm^2 kg^{-2}.$$

Q. The surface tension of water is 72 dyne cm^{-1}. Express it in SI units.

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

surface tension $= [M L^0 T^{-2}]$

$$= 72 \left[\frac{dyne}{kg}\right]^1 \left[\frac{s}{s}\right]^{-2}$$

$$= 72 \left[\frac{1g}{1000g}\right]$$

$$= 72 \times 10^{-3}$$

$$= 0.072\ Nm^{-1}$$

RELATION AMONG THE PHYSICAL QUANTITIES

To derive the relationship among physical quantities. By making use of the homogeneity of dimensions, we can derive an expression for a physical quantity if we know the various factors on which it depends.

Example: Let us derive an expression for the centripetal force F acting on particle for mass m moving with velocity v in a circle of radius r.

Let $F \propto m^a v^b r^c$ or $F = k m^a v^b r^c$ — (1)

→ k is dimensionless,

$[MLT^{-2}] = 1 [M]^a [L]^b [T]^c$

$MLT^{-2} = M^a L^{b+c} T^{-b}$

on comparing,

$a = 1$

$b + c = 1$ or $c = -1$

$-2 = -b$ or $b = 2$

from eq (1)

$$F = kmv^2 r^{-1} = k\frac{mv^2}{r}$$

Q The frequency 'ν' of vibration of a stretched string depends upon:

(i) its length l,

(ii) its mass per unit length 'm'

(iii) the tension T in the string.

obtain dimensionally an expression for 'ν'.

$\nu = k\, l^a m^b T^c$ — (1)

k = dimensionless constant

$[\nu] = [T^{-1}]$

$[l] = [L]$

$[T] = [MLT^{-2}]$

$[m] = [ML^{-1}]$

Put in eq(1)

$[T^{-1}] = [L]^a \, [ML^{-1}]^b \, [MLT^{-2}]^c$

$M^0 L^0 T^{-1} = M^{b+c} \, L^{a+c} \, T^{-2c}$

On comparing,

$b + c = 0$
$a - b + c = 0$
$-2c = -1$

so, $a = -1$
$b = -\frac{1}{2}$
$c = \frac{1}{2}$

$\Rightarrow \boxed{V = k l^{-1} m^{-1/2} T^{1/2} \quad \text{or} \quad V = \frac{k}{l}\sqrt{\frac{T}{m}}}$

Q. A planet moves around the sun in nearby circular orbit. It period of revolution 'T' depends on:

(i) radius 'r' (ii) Mass 'M'
(iii) gravitational constant 'G'.

$T \propto r M G$

$T = k r^a M^b G^c$ —(1)

$T = [T]$
$r = [L]$
$M = [M]$

$G = [M^{-1} L^3 T^{-2}] = \frac{Fr^2}{m_1 m_2}$

Put in (1)

$[T] = [L]^a \, [M]^b \, [M^{-1} L^3 T^{-2}]^c$

$M^0 L^0 T = M^{b-c} \, L^{a+3c} \, T^{-2c}$

for M,
$b - c = 0$
$b = -\frac{1}{2}$

for L,
$a + 3c = 0$
$a = \cancel{3} \; \frac{3}{2}$

for T,
$-2c = 1$
$c = \cancel{3} -\frac{1}{2}$

from eq (1)

$\cancel{T \propto M^{3/2} L}$

$$\boxed{T = r^{3/2} M^{-1/2} G^{-1/2} \quad \text{or} \quad T^2 = \frac{k^2 r^3}{MG}}$$

<u>Question</u> [HOTS] If the velocity of light c, the constant of gravitation G and Plank's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G. and h

$[c] = LT^{-1}$

$[G] = M^{-1}L^3T^{-2}$

$[h] = ML^2T^{-1}$

for Mass,

$m \propto cGh$

$m = k\, c^a G^b h^c \quad \text{—(1)}$

Putting dimensions in (1)

$[M] = [LT^{-1}]^a [M^{-1}L^3T^{-2}]^b [ML^2T^{-1}]^c$

$[ML^0T^0] = M^{-b+c} L^{a+3b+2c} T^{-a-2b-c}$

<u>for M,</u>

$-b + c = 1$

$c = 1 + b \quad \text{—(2)}$

put $b = \frac{-1}{2}$

$c = \frac{1}{2}$

<u>for L,</u>

$a + 3b + 2c = 0$

put $c = 1 + b$

$a + 3b + 2(1+b) = 0$

$a + 3b + 2 + 2b = 0$

$\cancel{a} + 5b = -2$

$-\cancel{a} - 3b = 1$

$2b = -1$

$b = \frac{-1}{2}$

for T,

$-a - 2b - c = 0$

put $c = 1 + b$

$-a - 2b - (1+b) = 0$

$-a - 2b - 1 - b = 0$

$-a - 3b - 1 = 0$

$-a - 3b = 1$

$a = \frac{1}{2}$

$\Rightarrow c^{1/2} G^{-1/2} h^{1/2}$

<u>Q.</u> If force, velocity and time are taken as fundamental units then find the dimension of mass.

$[F] = [MLT^{-2}]$

$[V] = [LT^{-1}]$

$[T] = [T]$

$m \propto F^a v^b T^c$

$m = k F^a v^b T^c$

$[M] = [MLT^{-2}]^a [LT^{-1}]^b [T]^c$

$ML^0T^0 = M^a L^{a+b} T^{-2a-b+c}$

for M

$a = 1$

$L = a + b = 0$

$b = -1$

$T = -2a - b + c = 0$

$-2(1) - (-1) + c = 0$

$\boxed{c = 1}$

$m = k F^1 v^{-1} T^1$