ELECTRIC CHARGES AND FIELDS

ELECTROMAGNETISM → ELECTRICITY, MAGNETISM

ELECTRICITY → ELECTROSTATICS, CURRENT ELECTRICITY

→ Electrostatics deal with the study of forces, electric fields and potentials arising due to static charges.

→ Static electricity is the imbalance of the charge on a surface of a material. It is different from dynamic electricity which involves moving charges (current)

e.g (i) We see spark when take off synthetic clothes or sweater;

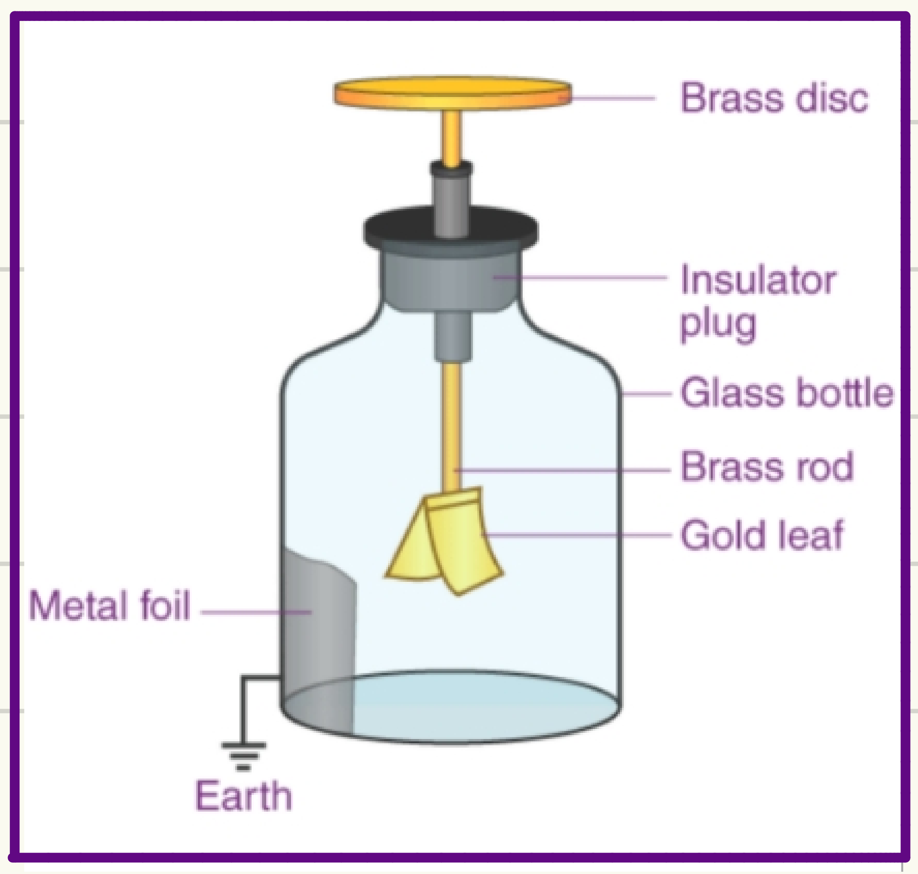
(ii) Experincing a shock while opening the door of a car.

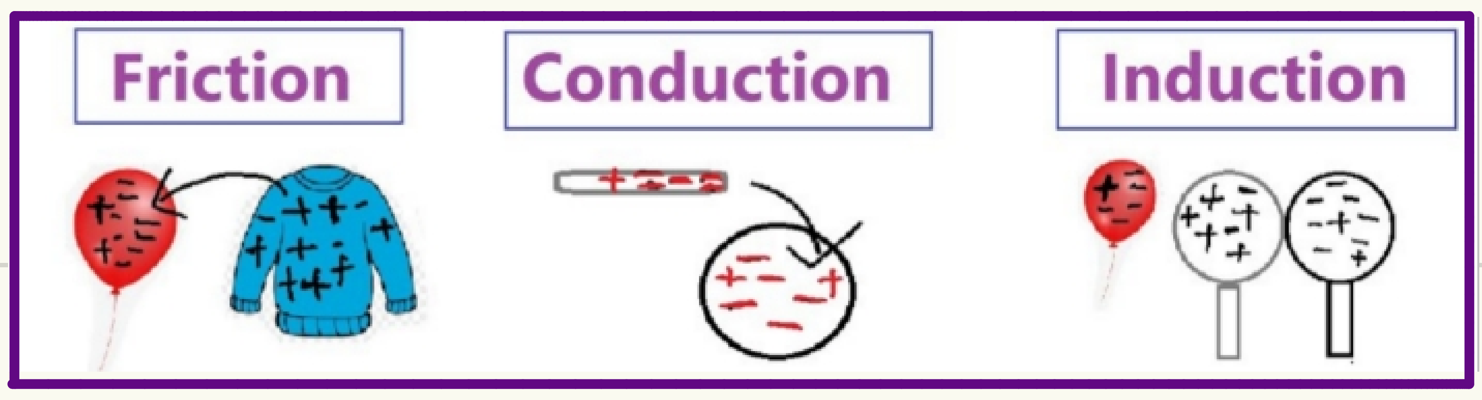
→ Electric charge is the physical property of a matter that causes electrostatic force.

- Two kinds of charges - positive and negative
- Like charges repel and unlike charges attract.
- Charge is conserved
- Charge is quantised
- SI unit of charge - coulomb (C)
- It is scalar quantity.
- Charged objects exert electric force on each other.

→ Gold Leaf Electroscope is a simple apparatus which is used to detect the presence of charge on a body. The degree of divergence of leaves is an indicator of of the amount of charge

- Its working is based on electrostatic repulsion.





→ Methods of Charging

(i) Friction (by rubbing) → Also called triboelectricity
→ Transfer of electrons between two bodies due to rubbing
(ii) Conduction (by touching) → Electrons transfer due to direct contact.
(iii) Induction (without contact) → Polarisation happens during induction, charges shift within the object

→

Conductors	Insulators
• Electricity can pass through them.	• Electricity can not pass through them.
• e.g. → metal, water, human body	• e.g. glass, plastic, rubber etc.
• Applications: grounding or earthing	• Applications: insulation

→ Basic Properties of Electric charge -

☆ Point charge - If the size of charged bodies is very small compared to the distance between them, they are treated as point charges.

1. Additivity of Charges

If a system contains n charges $q_1, q_2 \ldots q_n$ then the total charge of the system is given

$$\boxed{q_{net} = q_1 + q_2 + q_3 + \ldots q_n}$$

= Algebraic sum of all the individual charges

e.g. +1c +2c −3c +4c −5c → $q_{net} = 1+2-3+4-5$
$= -1C$

Charge and mass both are scalars, but charge can be positive and negative.

2. Conservation of Charges

Charge of an isolated system is always conserved.

It is not possible to creat or destroy net charge carried by an isolated system.

* Charge carrying particles may be created or destroyed in a process.

e.g

$$\underset{q=0}{\text{neutron}} \rightarrow \left.\begin{array}{l}\text{proton}(+e)\\ \text{electron}(-e)\end{array}\right] q=0$$

3. Quantisation of Charge -

The charge on a body is always given by

$$q = \pm ne \quad [n \rightarrow \text{integer } (+ve \text{ or } -ve)]$$

Electric charge is always integral multiple of e is termed as quantisation of charge.

* The value of basic unit of charge is

$$e = 1.602192 \times 10^{-19} C$$
$$\simeq 1.6 \times 10^{-19} C$$

Thus $-1C = 6.25 \times 10^{18}$ electrons $\left[n = \frac{q}{e}\right]$

* coulomb is very big unit of charge so we use $1 \mu C = 10^{-6} C$ or $1 mC = 10^{-3} C$.

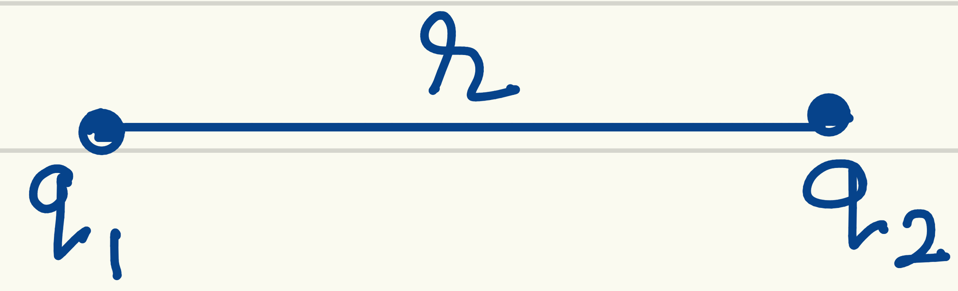
* At macroscopic level charged are enormous compared to e thus quantisation of charge can be ignored.

* Quarks have charges $\pm\frac{e}{3}$ or $\frac{2e}{3}$ but cannot in free state, hence not taken as elementry charge.

→ Coulomb's law -

This law states that the magnitude of electrostatic force between two point charges is directly proportional to the product of magnitude of the charges and inversely proportional to the square of the distance between them i.e

$$F \propto \frac{|q_1 q_2|}{r^2}$$



$$\boxed{F = k\frac{q_1 q_2}{r^2}} \quad -(1)$$

Where k is coulomb constant and $k = \frac{1}{4\pi\epsilon_0}$

In SI unit $k = 9\times10^9\,Nm^2C^{-2}$.

* If 1 C of charge when placed at a distance 1 m from another 1 C of charge in vacuum experiences an electrical force of repulsion of magnitude $9\times10^9\,N$.

* 1 C is very big unit of charge to be used. In practice 1 mC $(=10^{-3}C)$ or 1 $\mu C (=10^{-6}C)$, smaller units are used.

In eqn (1) $k = \frac{1}{4\pi\epsilon_0}$, then coulomb's law

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}$$

, ϵ_0 is permittivity of free space

$\epsilon_0 = 8.85\times10^{-12}N^{-1}m^{-2}C^2$

Characteristics of Coulomb's force

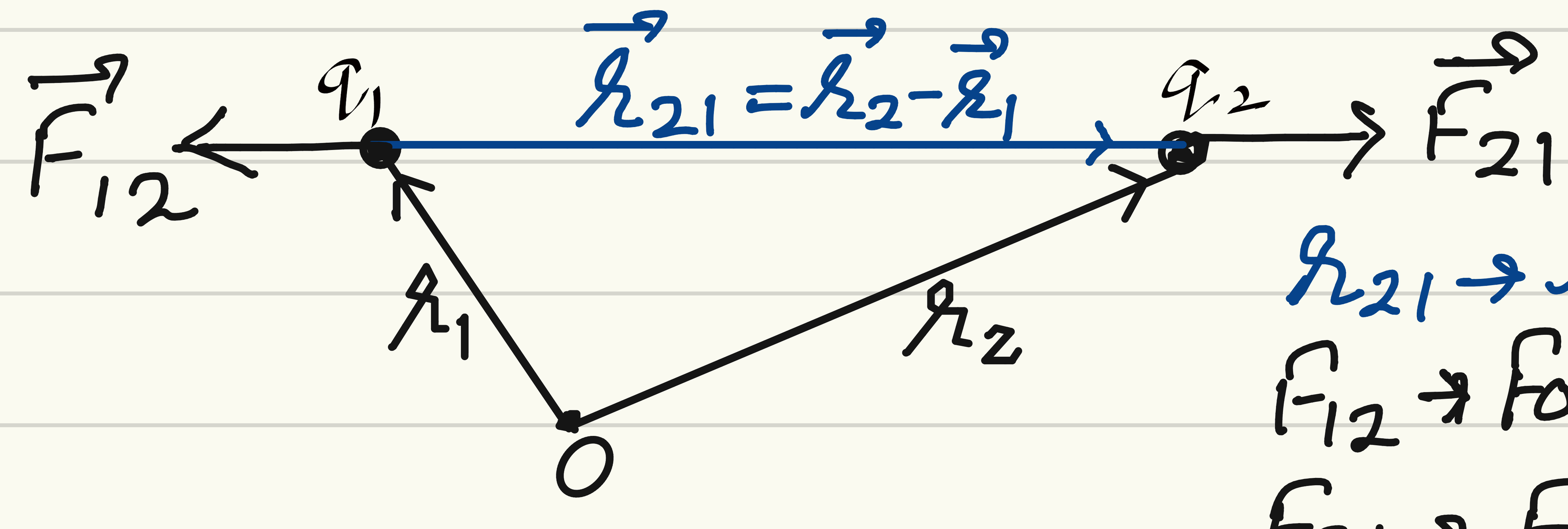
- It is a central force. i.e acts along a joining line.
- It obeys inverse square law i.e $F \propto \frac{1}{r^2}$

Limitations of Coulomb's law

- It hold good for point charges at rest.
- It is not applicable for distance less than 10^{-15} m
- It is not universal.

Vector form of coulomb's law:

Let q_1 and q_2 are point charges with position vectors $\vec{r}_1$ and $\vec{r}_2$ respectively.



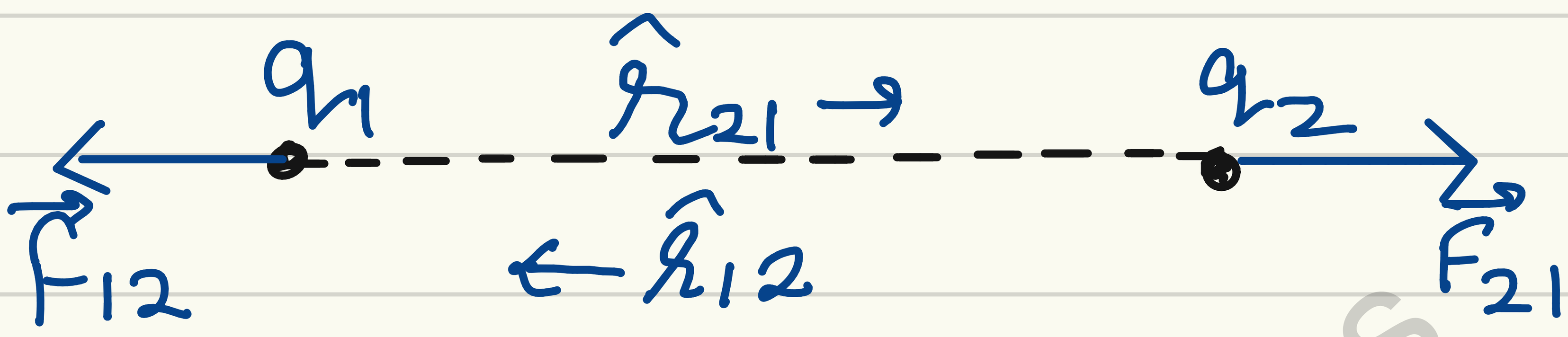
$r_{21} \rightarrow$ leading from 1 to 2

$F_{12} \rightarrow$ Force on q_1 due to q_2

$F_{21} \rightarrow$ Force on q_2 due to q_2

$q_1 q_2 > 0$

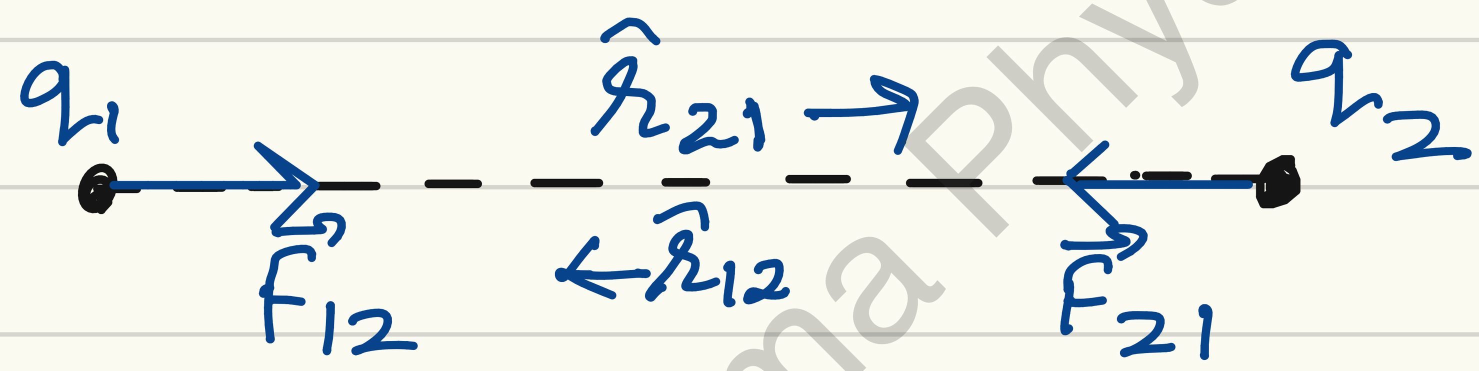
like charges



$$\hat{r}_{12} = \frac{\vec{r}_{12}}{r_{12}}$$

$q_1 q_2 < 0$

unlike charges



$$\hat{r}_{21} = \frac{\vec{r}_{21}}{r_{21}}$$

The coulomb force between q_1 and q_2 is given as

$$\boxed{\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}} \quad \text{———(1)}$$

and

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}} \quad \text{———(2)}$$

Equations (1) and (2) are the vector forms of coulomb's law.

Here $\vec{r}_{12} = -\vec{r}_{21} \Rightarrow \hat{r}_{12} = -\hat{r}_{21}$

Therefore from (1) and (2)

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Thus Coulomb's law agrees with Newton's third law of motion.

* Equations (1) and (2) are valid for any sign of q_1 and q_2.
* Coulomb's law gives the forces between two charges q_1 and q_2 in vacuum.

Permittivity is the measure of how much electric field gets reduced or allowed in a medium.

Permittivity (ϵ) - A property of a material that measures the opposition it offers against an electric field.

It is an ability of a substance to store electrical energy in an electric field.

Relative Permittivity (ϵ_r)

By coulomb's law, force between charges q_1 and q_2 in free space

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \quad -(1)$$

When charges q_1 and q_2 are placed in a medium of permittivity ϵ

$$F_m = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \quad -(2)$$

$F_m \rightarrow$ force in medium

Divide (1) ÷ (2), we get

$$\boxed{\frac{F}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r}$$

Here ϵ_r is called relative permittivity.

It is defined as the ratio of permittivity of the medium to the permittivity of vacuum.

* It has no unit.
* $\epsilon_r = K$ <u>K is dielectric constant</u>

Dielectric constant K a substance is defined as the ratio of substance's permittivity to the permittivity of free space.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0} = K}$$

or $K = \epsilon_0 \epsilon_r$

* K has no unit.

ϵ_r and K has no unit

Forces between multiple charges

Principle of Superposition - Force on any charge due to a number of other charges is the vector sum of all the forces on the charge.
The individual forces are unaffected due to the presence of other charges.

e.g. For a system of three charges q_1, q_2 and q_3 force on q_1 due to q_2 and q_3

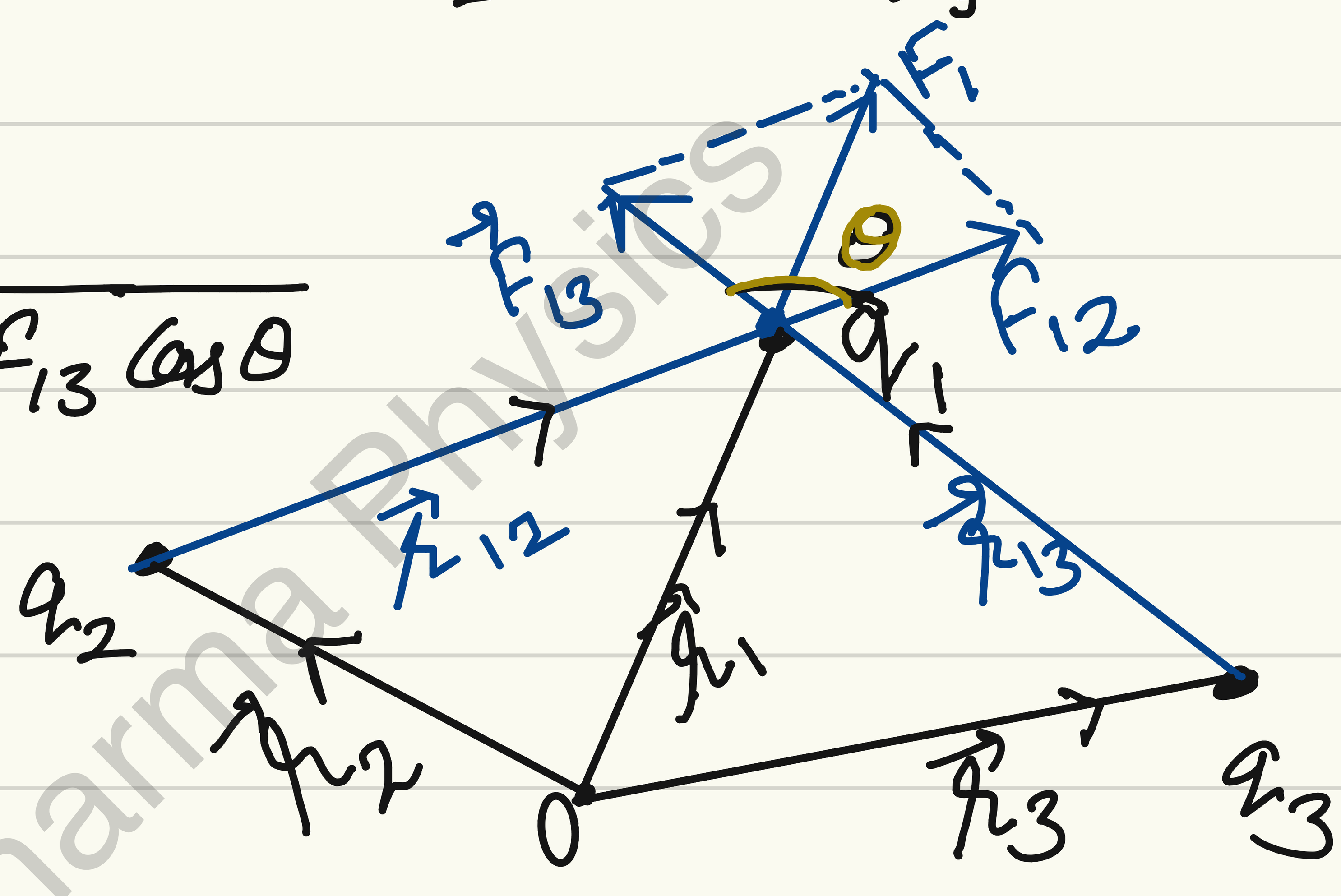
$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}}$$

$$|\vec{F_1}| = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}F_{13}\cos\theta}$$

here,

$$F_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$F_{13} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13}$$



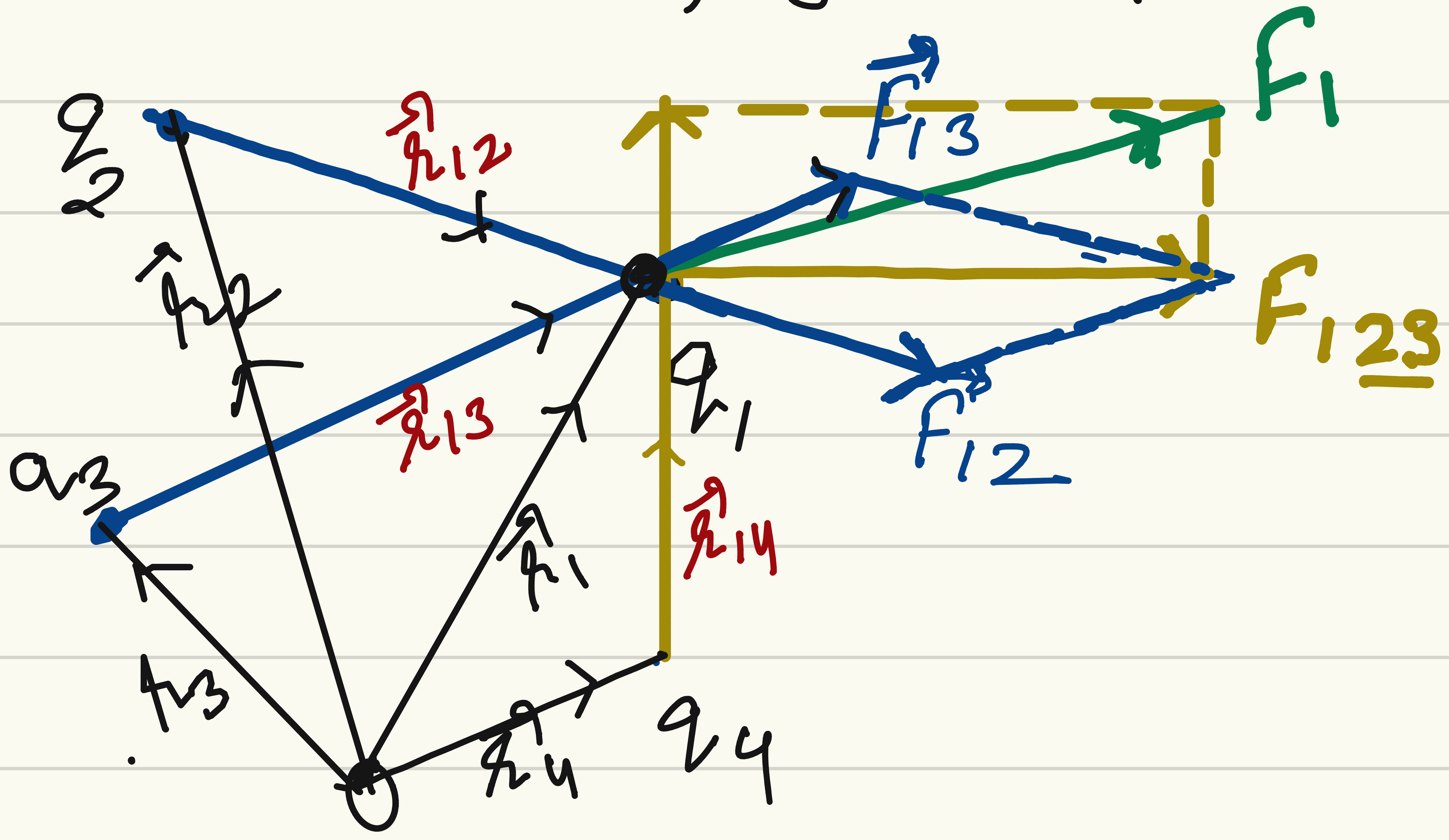
For four system of charges q_1, q_2, q_3 and q_4 net force on q_1 due to q_1, q_2, q_3 and q_4

$$\vec{F_{123}} = \vec{F_{12}} + \vec{F_{13}}$$

net force on q_1

$$\vec{F_1} = \vec{F_{123}} + \vec{F_{14}}$$

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}}$$



For n system of charges, net force on q_1

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \cdots \vec{F_{1n}}$$

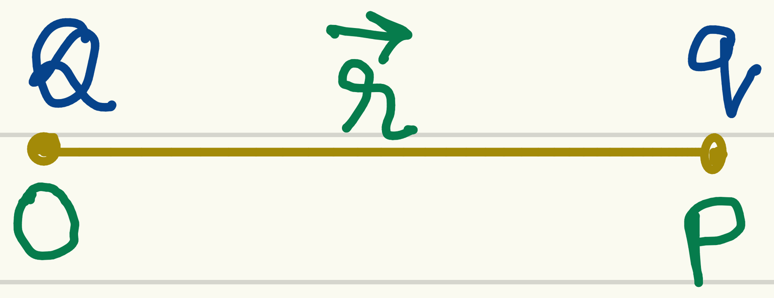
$$F_1 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} \cdots \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

* Vector sum is obtained by $\parallel^{gm}$ law.

Electric field - Electric field due to a charge Q at a point in space may be defined as the force experienced by unit charge at that point.

Electric field – The space around a charge in which any other charge experiences a force.

$$\vec{E} = \frac{\vec{F}}{q}$$



$$= \frac{\frac{1}{4\pi\epsilon_0}\frac{Qq\,\hat{r}}{r^2}}{q} \qquad \left[\because F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\right]$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\cdot\hat{r}} \qquad \hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

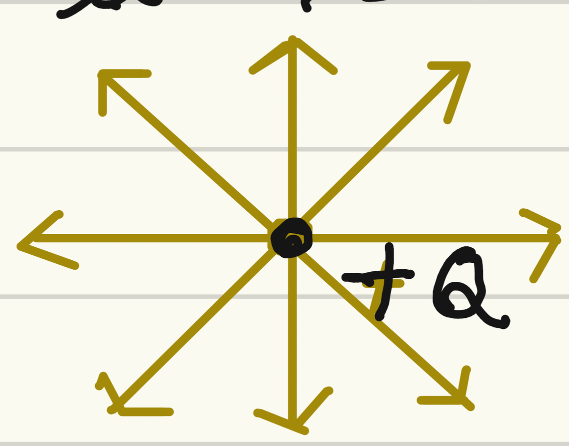
* Direction of electric field is +ve charge to -ve charge.

* Charge 'Q' which produces electric field is called source charge and the charge 'q' which tests the electric field is called test charge.

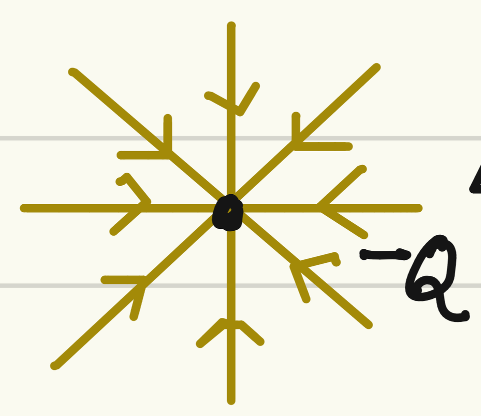
* Test charge means a negligibly small charge which does not apply any force on source charge but experiences a force due to the source charge.

$$E = \lim_{q \to 0} \frac{F}{q}$$

* Electric field E is independent of q. It depends on source charge Q and r.
* The field exists at every point in three-dim^n space.
* For +ve charge E will be directed radially outwards from the charge and for -ve charge it is radially inwards.



$\vec{E}$ radially outwards

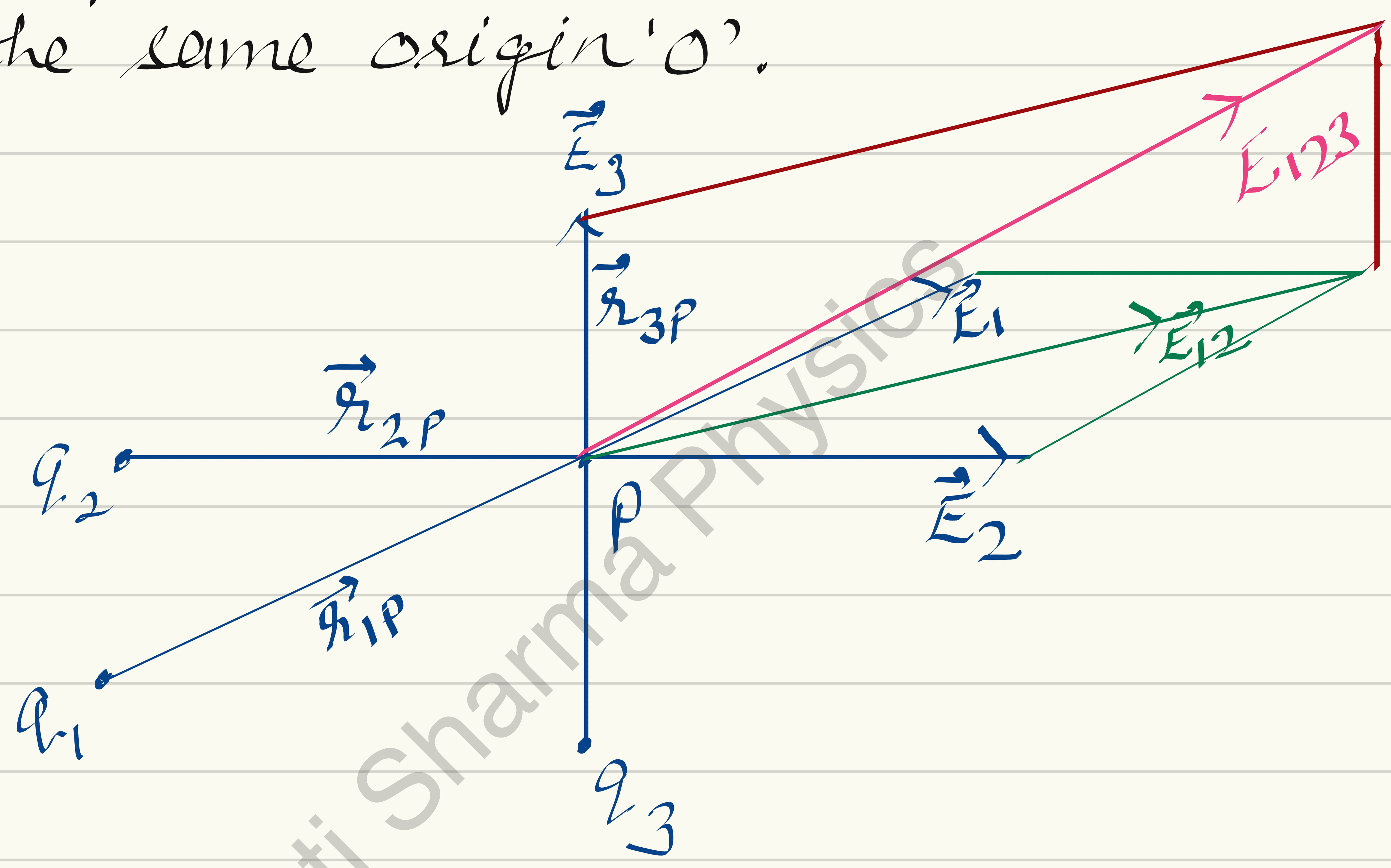


$\vec{E}$ radially inwards

* Like force, electric field E also depends on the distance r. Thus at equal distance from Q, the magnitude of $\vec{E}$ is also same. E has spherical symmetry.

Electric field due to a system of charges

consider a system of charges $q_1, q_2 \dots q_n$ with position vector $r_1, r_2 \dots r_n$ relative to the same origin 'O'.



By the principle of superposition the electric field $\vec{E}$ at $\vec{r}$ due to the system of charges is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots \vec{E}_n$$

$$= k\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + k\frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \dots k\frac{q_n}{r_{nP}^2}\hat{r}_{nP}$$

or $$\boxed{E = k\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}}$$

Physical Signification of Electric field

1. Under Static condition

- Electric field is an elegent way of characterising electrical environment of a system of charges.
- Electric field at a point in the space around a system of charges, tell the force on unit positive test charge placed at that point.
- Electric field is independent of test charge.

The term field refers to a quantity that is defined at every point in space and may vary from point to point.

* The true physical significance of the concept of electric field deals with time dependent electromagnetic phenomena.

2. Electromagnetic Non-Static Condition

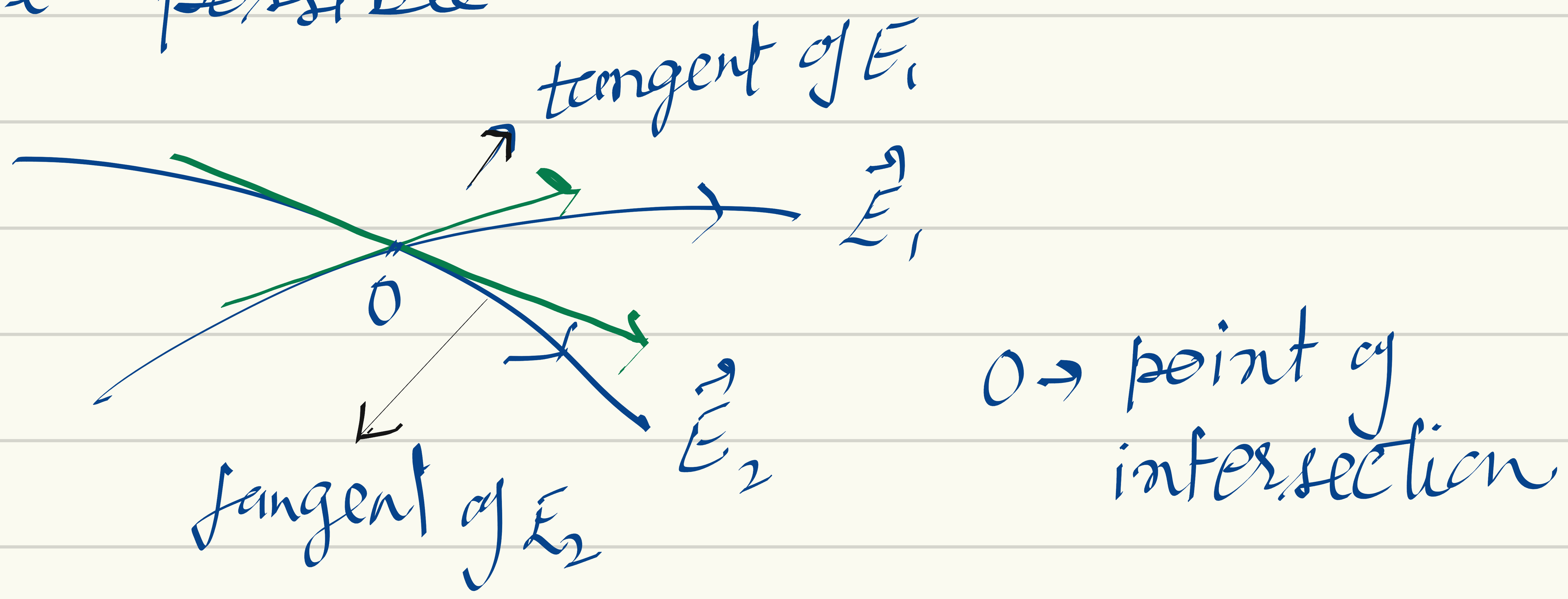
- Even though electric and magnetic field can be detected only by their effects (forces) on charges, they are regarded as physical entites, not merely methematical construct.
- Concept of field was first introduced by Faraday and is now among the central concept of physics.

Electric Field lines: These are the imaginary lines which represent electric field pictorially and give the direction of electric field at any point.

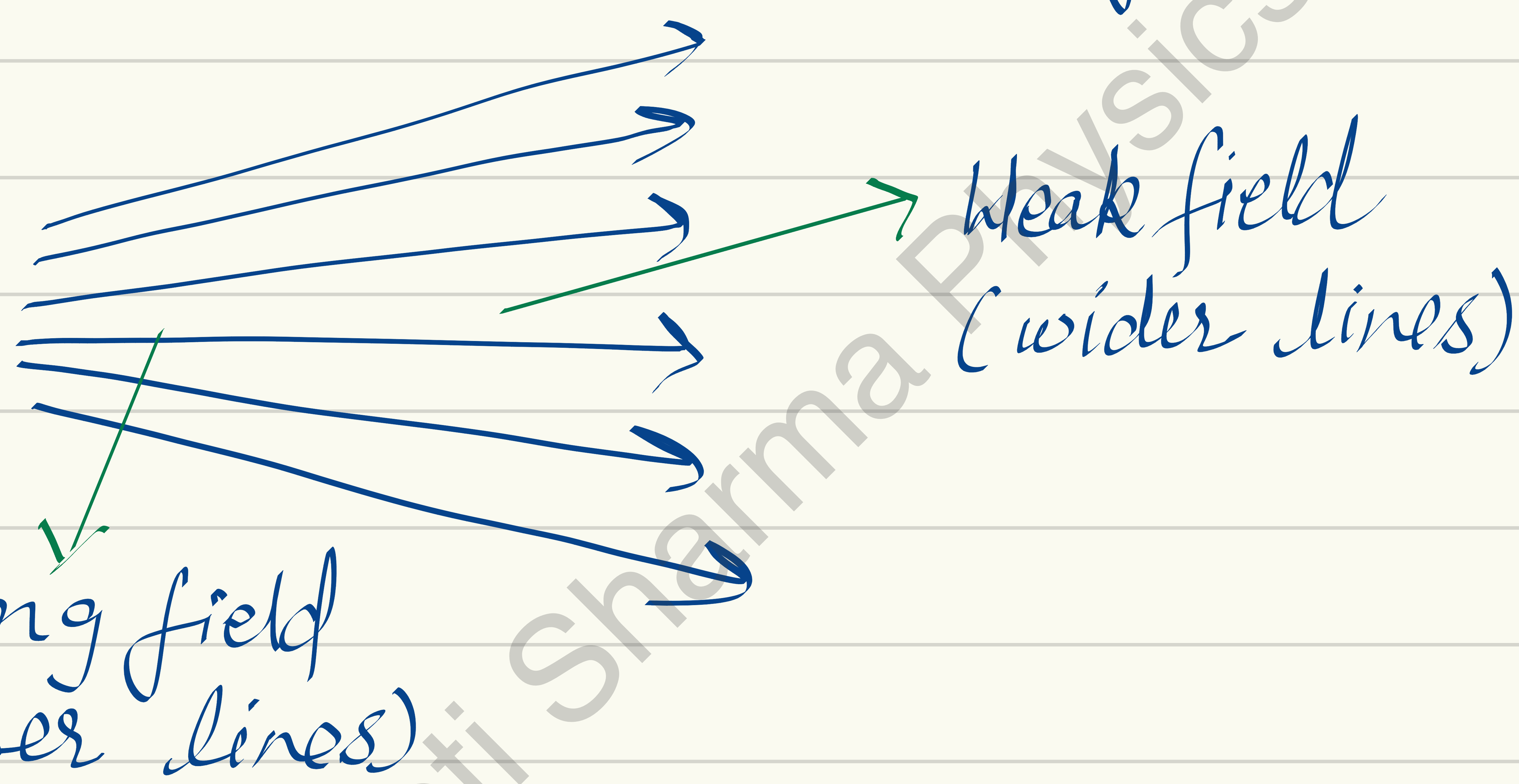
Properties of Electric field lines

1. Start from +ve charge, end at -ve charge.
2. Do not form closed loop because starting and ending point are not same.

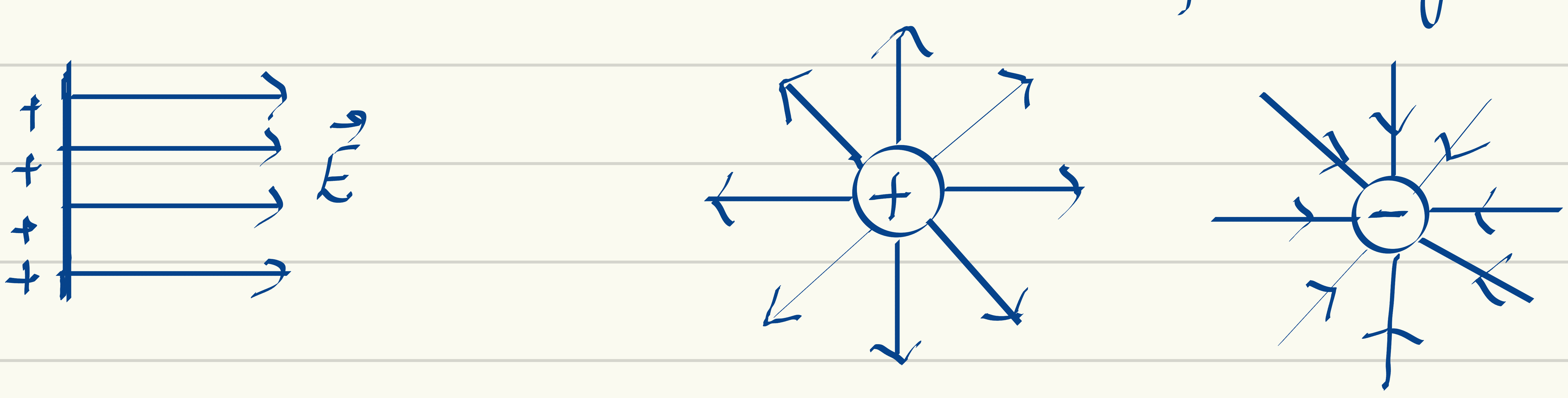
3. Two field lines from one source never intersect each other because if they do so there will be two directions of electric field at one point which is not possible.



4. Closer the lines, stronger the field and wider the lines weaker the field.



5. Always perpendicular to the surface of charge.



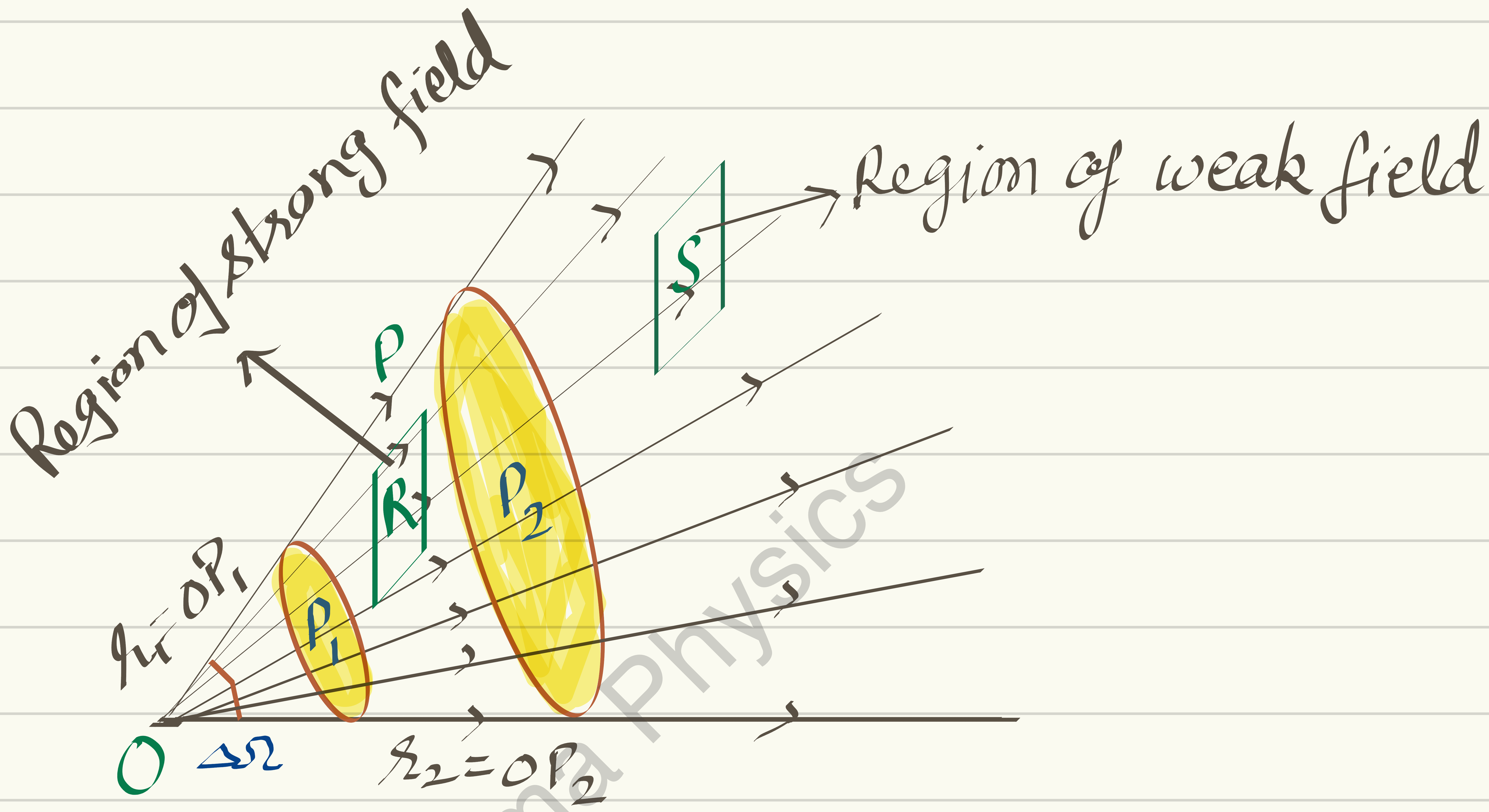
6. Do not pass through a conductor, as electric field inside a conductor is zero.

7. Pass through a dielectric. (Electric field polarises the molecules or atoms in a material)

* Dielectrics are insulators.
* Separation of centre of +ve and -ve charge is called polarisation.

Dependence Of Electric Field Strength On The Distance And Its Relation To The Number Of Field Lines (Inverse Square Law)

Field lines carry information about the direction of electric field at different points in space.



In fig.
consider a point charge (+ve) at 'O'

$r_1 = OP_1$

$r_2 = OP_2$

Area of R element = Area of S element

$\Delta\Omega$ = solid angle

From fig it is clear that equal number of field lines are passing through P_1 and P_2 surfaces.

Now by Area = $r^2 \times$ solid angle

Area of surface P_1

$$A_1 = r_1^2 \Delta\Omega$$

Area of surface P_2

$$A_2 = r_2^2 \Delta\Omega$$

Let n no. of lines are passing through each area P_1 and P_2.

So,
number of field lines cutting unit area element at P_1

$$E_1 = \frac{n}{r_1^2 \Delta\Omega}$$

also at P_2,

$$E_2 = \frac{n}{r_2^2 \Delta\Omega}$$

Now $\frac{E_1}{E_2} = \frac{\text{strength of field at } P_1}{\text{strength of field at } P_2} = \frac{n / r_1^2 \Delta\Omega}{n / r_2^2 \Delta\Omega}$

or $\frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$

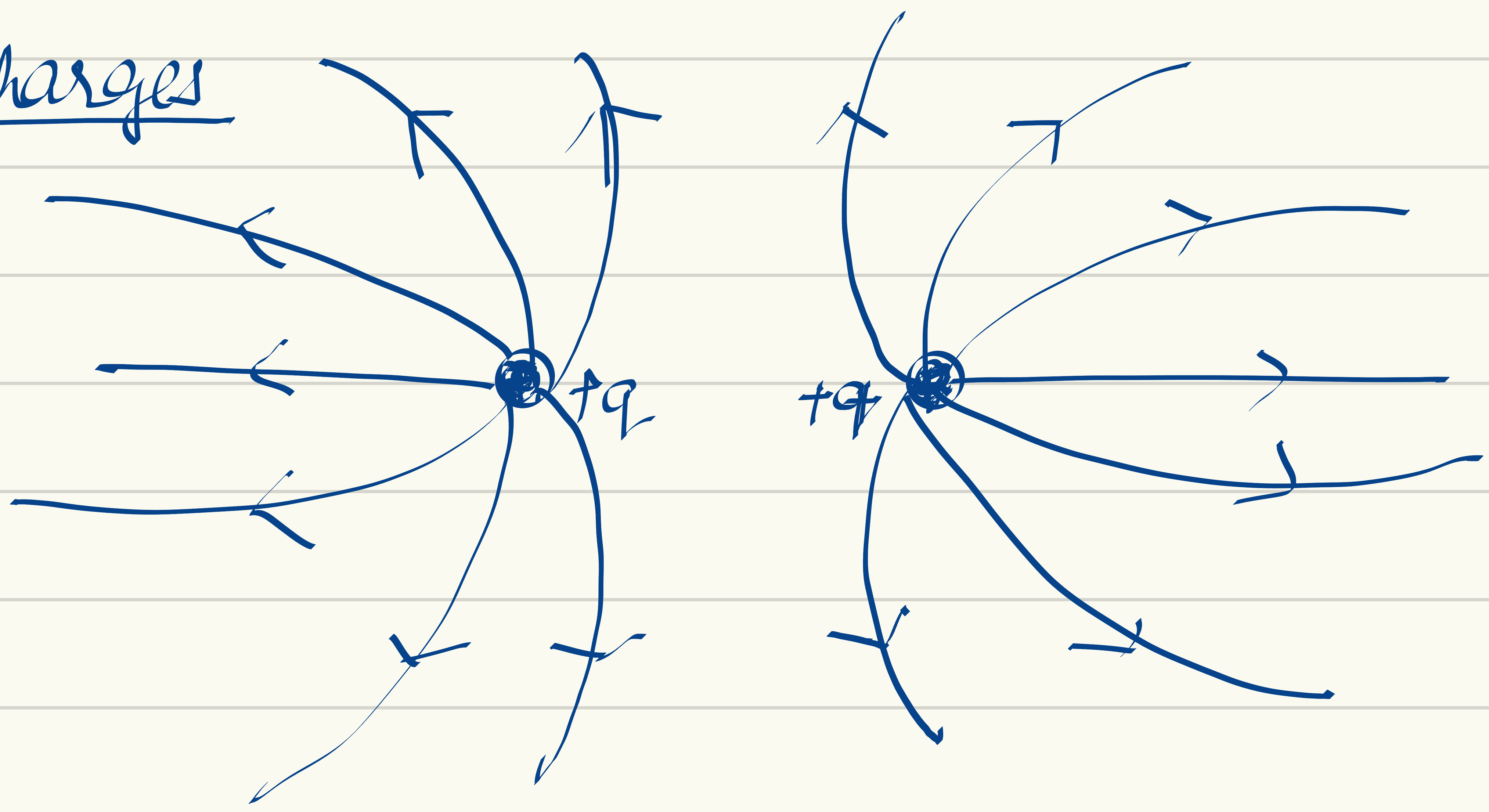
or $\boxed{E \propto \frac{1}{r^2}}$

This is the inverse square dependency of electric field.

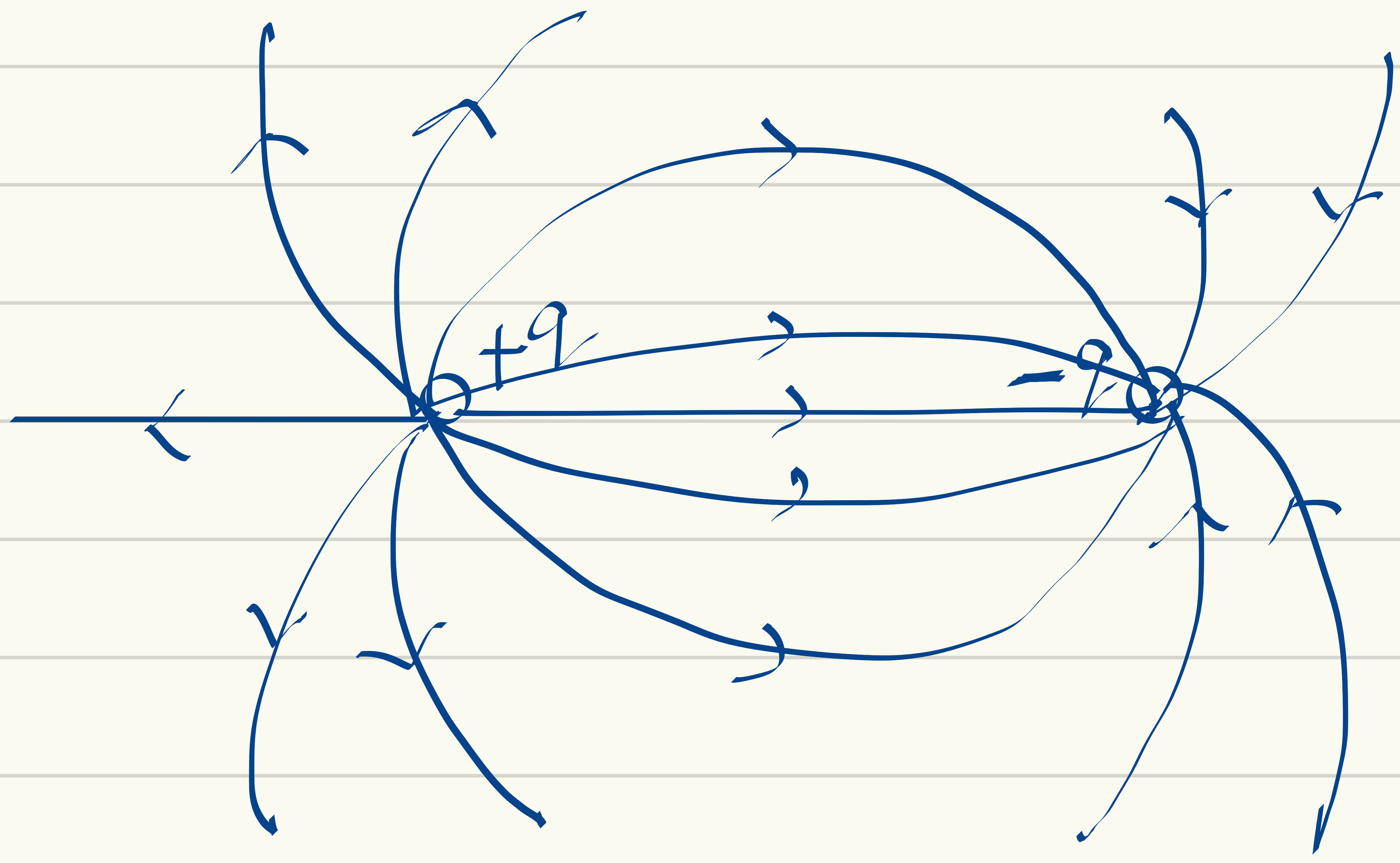
* The picture of field lines was invented by Faraday.
* Electric field lines are also known as lines of force.

Field lines due to some simple charge configuration

(1) Like charges



(ii) Unlike charge (Electric dipole)



Electric Flux (ϕ): The rate of flow of electric field through an area is called electric flux.

No. of electric field lines intersecting a given area

Electric flux $\Delta\phi$ through an area ΔS is given as

$$\Delta\phi = \vec{E}\cdot\vec{\Delta S} = E\Delta S\cos\theta \qquad [\vec{\Delta S} = \Delta S\cdot\hat{n}]$$

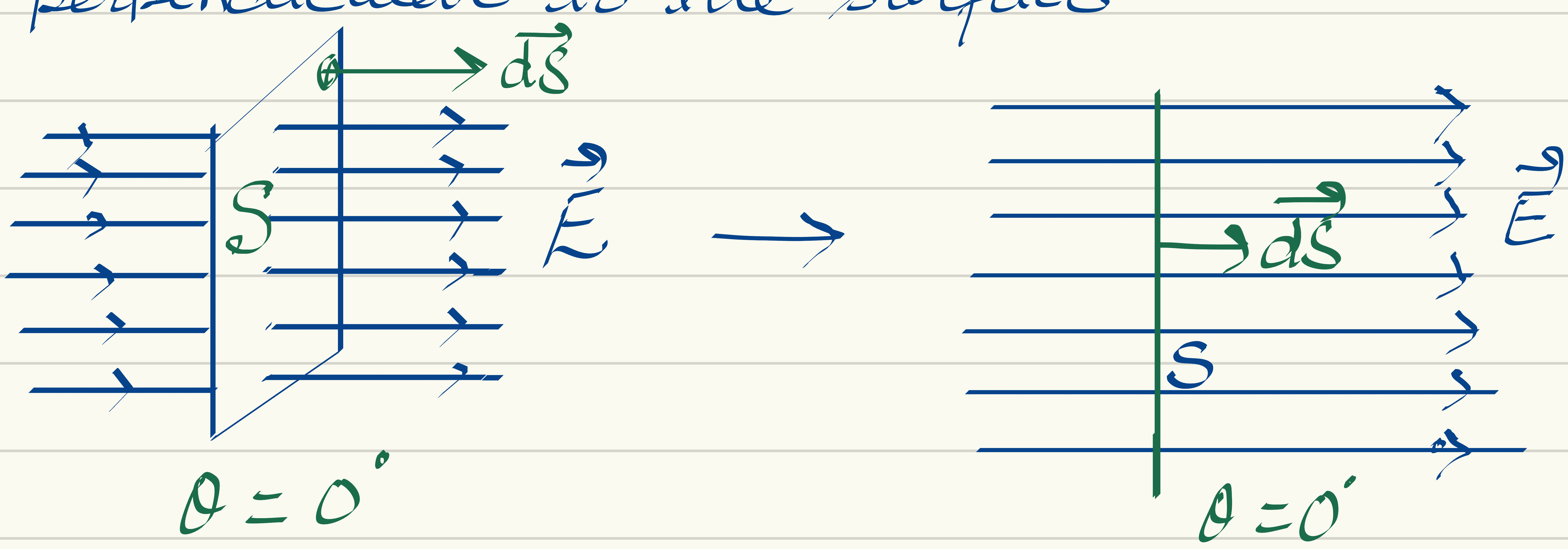
or $$d\phi = (E\cos\theta)dS = EdS\cos\theta = \vec{E}\cdot\vec{dS}$$

$$\boxed{\phi = \int d\phi = \int \vec{E}\cdot\vec{dS} = \int EdS\cos\theta}$$

Here θ is the angle between $\vec{E}$ and $\vec{dS}$

* Area vector ($\vec{dS}$) – A vector whose magnitude is equal to the area of a surface and whose direction is perpendicular to the surface.

(1)

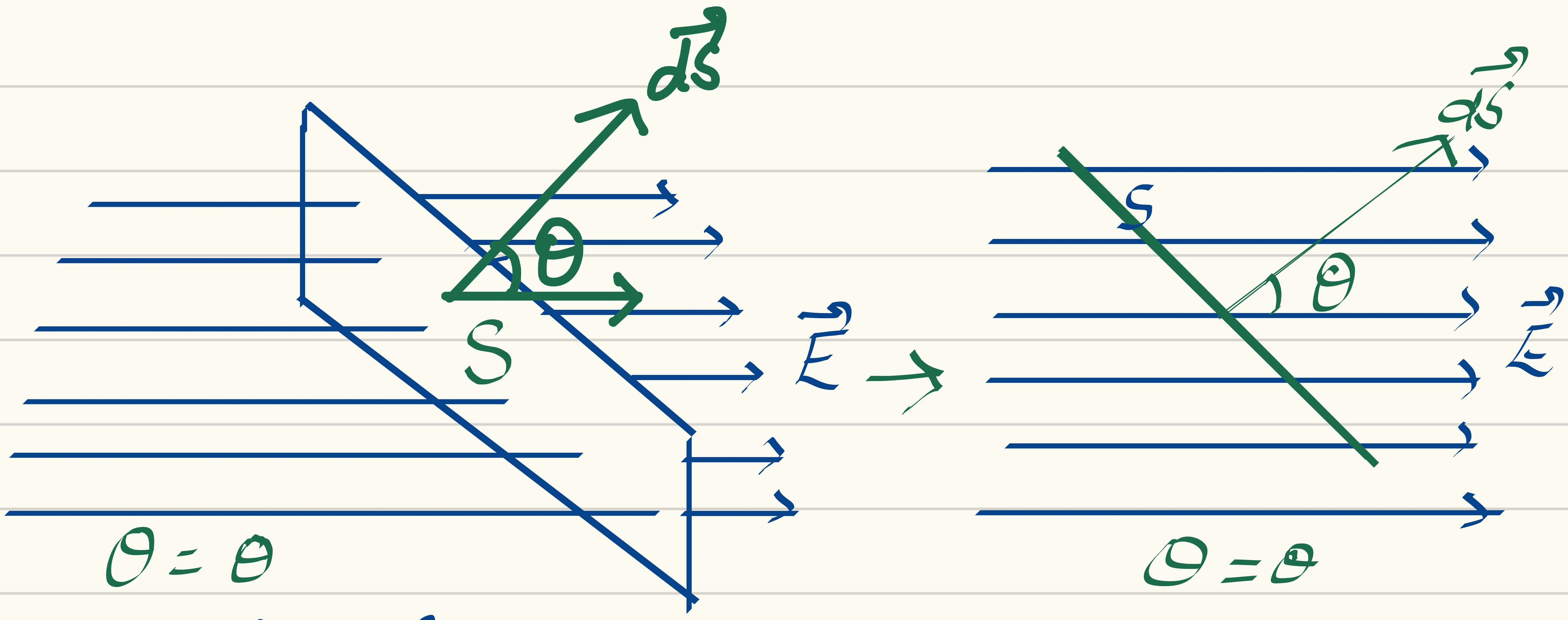


When $\theta = 0°$

$$\phi = \int EdS\cos 0° = \int EdS = ES$$

or $\boxed{\phi_m = ES}$ $\qquad \phi_{max} \rightarrow max^m$ flux

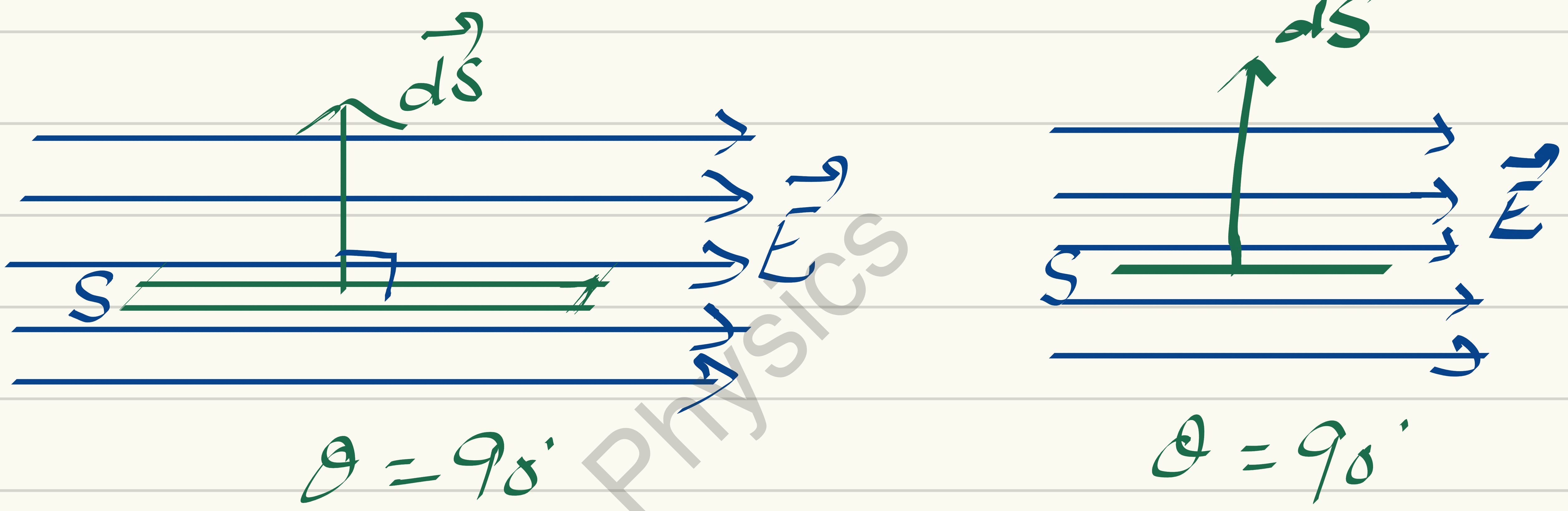
(ii)



$$\phi = \int E\,ds\cos\theta$$

$$\boxed{\phi = ES\cos\theta}$$

(iii)

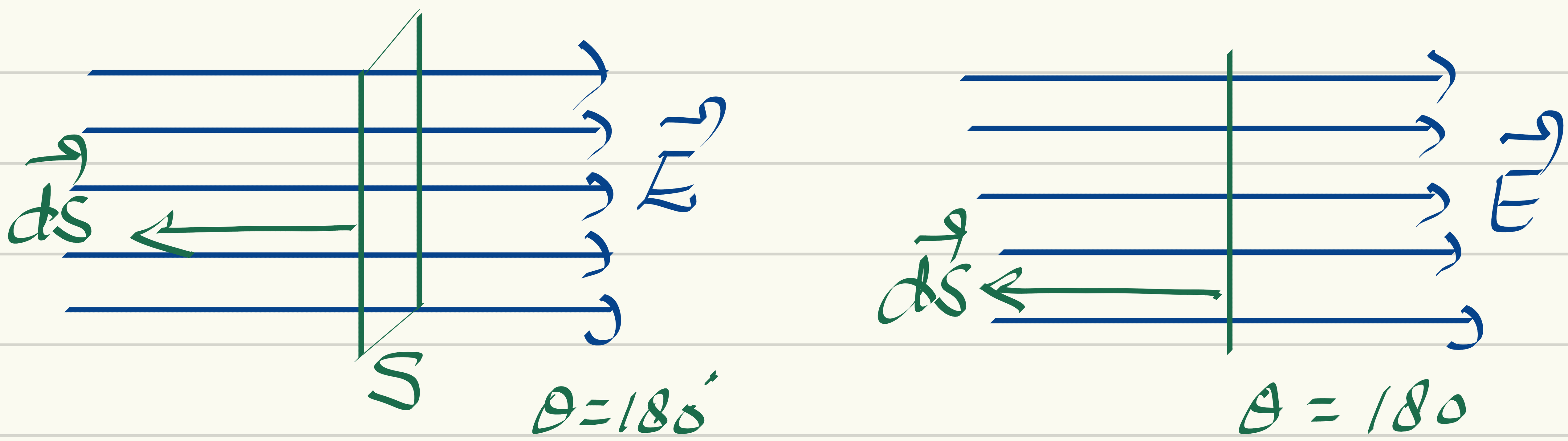


$$\phi = \int E\,ds\cos 90°$$

$\phi = 0$ [no flux] [$\cos 90° = 0$]

No field lines passes through the surface S.

(iv)



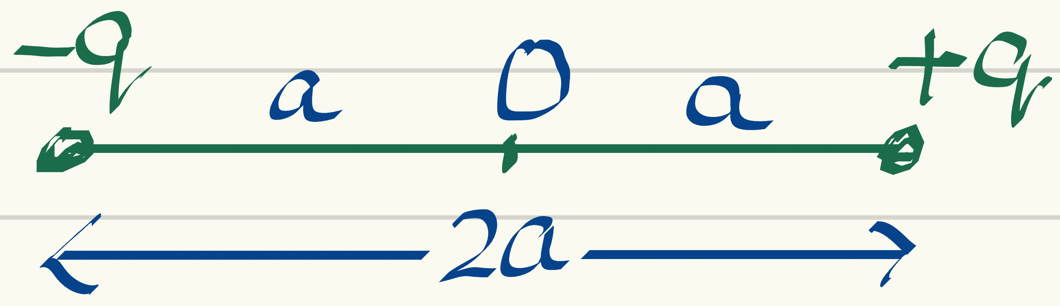
$$\phi = \int E\,ds\cos 180°$$

$\phi = -ES$ [-ve flux] [$\because \cos 180 = -1$]

* SI units - Volt-meters (V-m)
  - $N\,m^2\,C^{-1}$
* Flux is a scalar quantity.

* For closed surface the direction of area vector is taken outwards $\perp$ to the surface by convention.

<u>Electric Dipole</u>: It is a pair of equal and opposite point charges q and $-q$ separated by a distance $2a$. e.g. H_2O, HCl etc.

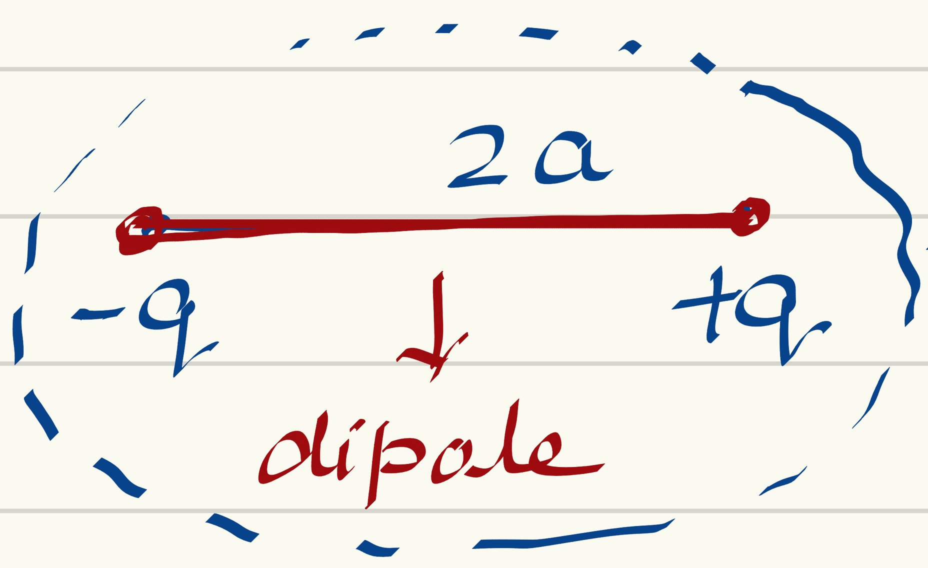


<u>Electric Dipole moment</u> (p) It is equal to the product of magnitude of either charge and length of dipole.

$$p = |q| \times 2a$$

$$\boxed{p = 2qa}$$

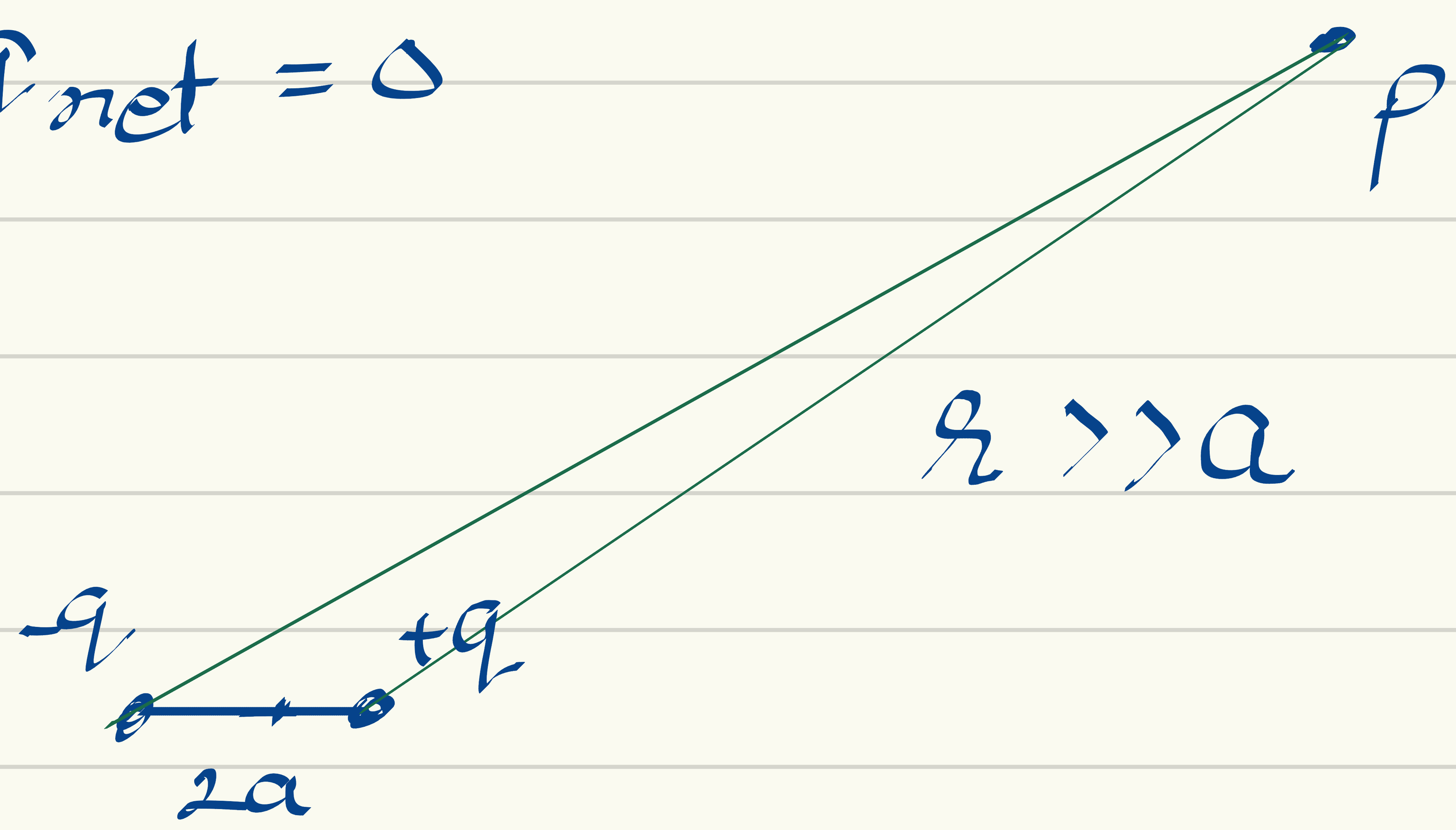
* Dipole moment is a vector quantity.
* Direction of dipole moment is $-ve$ to $+ve$ by convention.
* The net charge on an electric dipole is zero. This does not mean that the field of dipole is zero.
* Electric field near the dipole is not zero as the two charges are separated by some distance but for $r >> 2a$ the fields due to $+q$ and $-q$ nearly cancel out.

(i) 

$\rightarrow q_{net} = 0$

(ii) $r >> 2a$

$E_P \simeq 0$

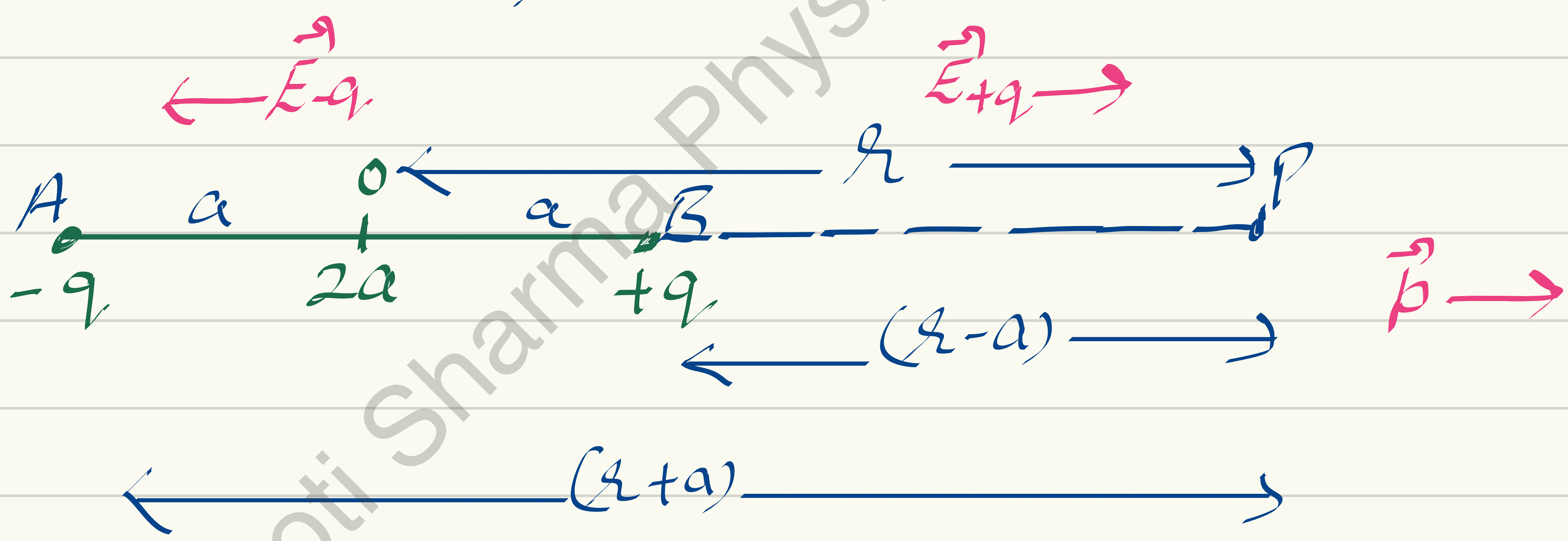


The Field At An Electric Dipole:

Electric field due to a dipole at any point can be found out from coulomb's law and principle of superposition.
The two cases are -

(1) **Electric Field due to a dipole, at any point P on the Axial line**
(For the point on the axis)

Consider an electric dipole AB of length 2a. At a point 'P' on the axis of dipole electric field is to be found.



In fig - $OP = r$, $AB = 2a$
$AP = (r+a)$
$BP = (r-a)$

Field at 'P' due to $-q$ charge

$$\vec{E}_{-q} = -k \frac{q}{AP^2} \hat{p}$$

[-ve sign shows the opposite dir^n of $\vec{E}_{-q}$ and $\vec{p}$]

So $$\vec{E}_{-q} = -k \frac{q}{(r+a)^2} \hat{p} \quad \text{— (1)}$$

and $\vec{E}_{+q} = k\dfrac{q}{BP^2}\hat{p}$ [$\vec{E}$ & $\vec{p}$ have same direction]

$$E_{+q} = k\frac{q}{(r-a)^2}\hat{p} \quad \text{—(2)}$$

Now the net field on 'P'

$$\vec{E}_{axial} = \vec{E}_{net} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \left[k\frac{q}{(r-a)^2} - k\frac{q}{(r+a)^2}\right]\hat{p}$$

$$= kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= kq\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2(r+a)^2}\right]\hat{p}$$

$$= kq\left[\frac{\cancel{r^2}+\cancel{a^2}+2ra-\cancel{r^2}-\cancel{a^2}+2ra}{(r^2-a^2)^2}\right]\hat{p}$$

$$= kq\left[\frac{4ra}{(r^2-a^2)^2}\right]\hat{p}$$

$$\boxed{\vec{E}_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}\,r}{(r^2-a^2)^2}}$$

For $r >> a$

$$\vec{E}_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2)^2}$$

$$\boxed{\vec{E}_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}}$$

*[θ b/w $\vec{E}$ & $\vec{p} = 0°$]

i.e $\quad E \propto \frac{1}{r^3}$ [for dipole]

and $\quad E \propto \frac{1}{r^2}$ [for point charge]

* Direction of $\vec{E}_{axial}$ is same as of $\vec{p}$.
i.e. angle between $\vec{E}_{axial}$ and $\vec{p}$ is zero.

(II) Electric field due to a dipole on the Equatorial Plane

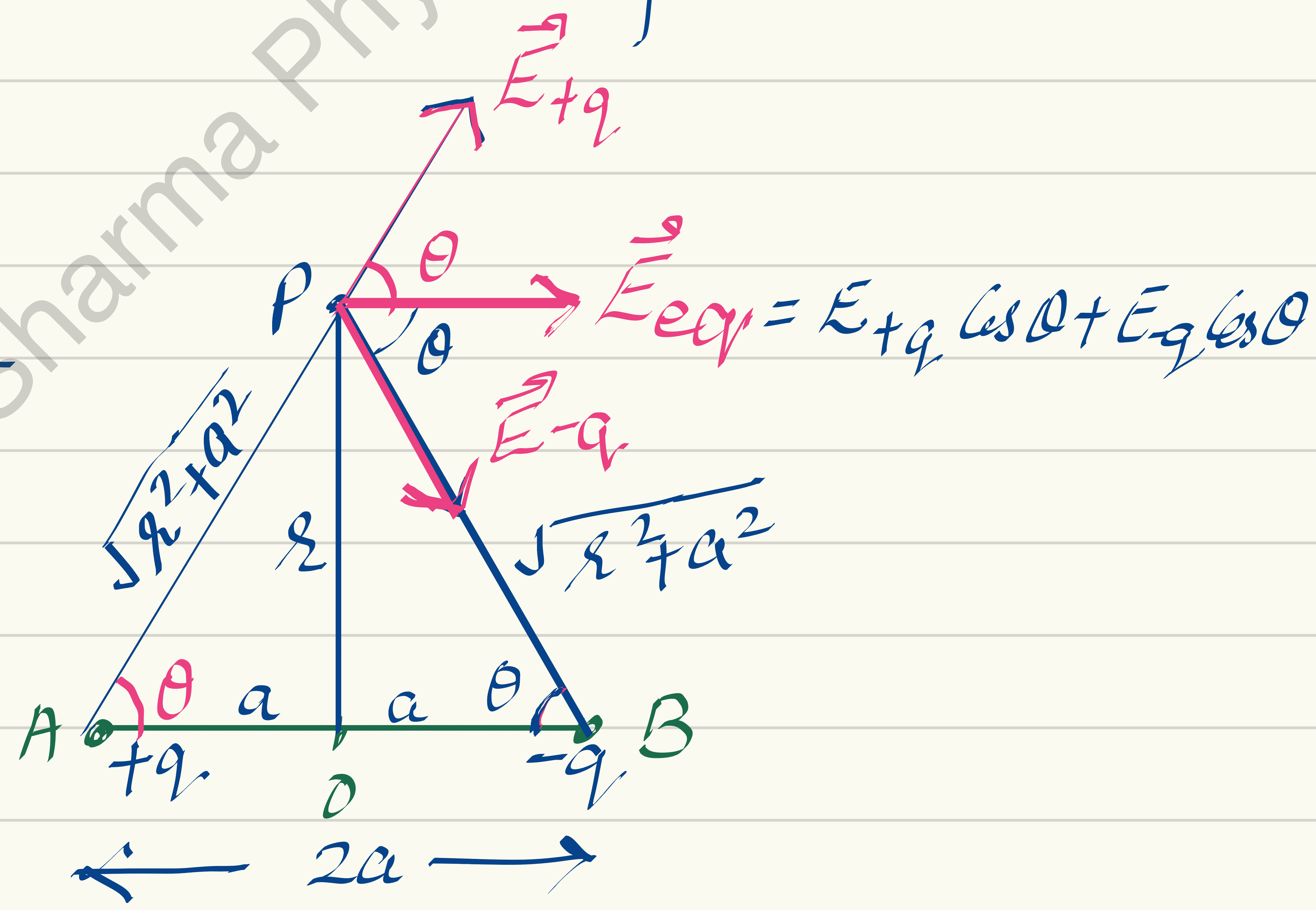
Let 'P' is the point on the equatorial plane of a dipole where electric field is to be determined.

In fig

$AP = PB = \sqrt{r^2+a^2}$

$OP = r$

$\cos\theta = \frac{a}{\sqrt{r^2+a^2}}$



Magnitudes of the electric fields due to the charges +q and -q are equal.

$$E_{+q} = E_{-q} = k\frac{q}{(\sqrt{r^2+a^2})^2}$$

$$= k\frac{q}{(r^2+a^2)}$$

clearly the components of E_{+q} and E_{-q}, normal to the dipole cancel away.

The components along the dipole axis add up.

So $$E_{eq} = E_{net} = E_{+q}\cos\theta + E_{-q}\cos\theta$$

$$= 2k\frac{q}{(r^2+a^2)}\cos\theta \quad [\because E_{+q}=E_{-q}]$$

$$= \frac{2kq}{(r^2+a^2)}\cdot\frac{a}{(r^2+a^2)^{1/2}} \quad \left[\because \cos\theta = \frac{a}{\sqrt{r^2+a^2}}\right]$$

$$E_{eq} = k\frac{p}{(r^2+a^2)^{3/2}}$$

In vector form
$$\vec{E}_{eq} = -\frac{k\vec{p}}{(r^2+a^2)^{3/2}}$$

$-ve$ sign shows the opposite direction of $\vec{E}_{eq}$ and $\vec{p}$.

For $r >> a$
$$\vec{E}_{eq} = -\frac{k\vec{p}}{r^3}$$

or $$\boxed{\vec{E}_{eq} = -\frac{1}{4\pi\epsilon_0}\cdot\frac{\vec{p}}{r^3}}$$

* Angle b/w $\vec{E}_{equatorial}$ and $\vec{p}$ is $180°$.

* $\vec{E}_{ax}$ and $\vec{E}_{eq}$ depend on product of q and a ie on qa but independent of q and a separately.

* $\boxed{E_{axial} = 2\times E_{equatorial}}$ $\boxed{\vec{E}_{ax} = -2\vec{E}_{eq}}$

* Magnitude and direction of the dipole field depends on distance r and angle b/w $\vec{r}$ and $\vec{p}$.

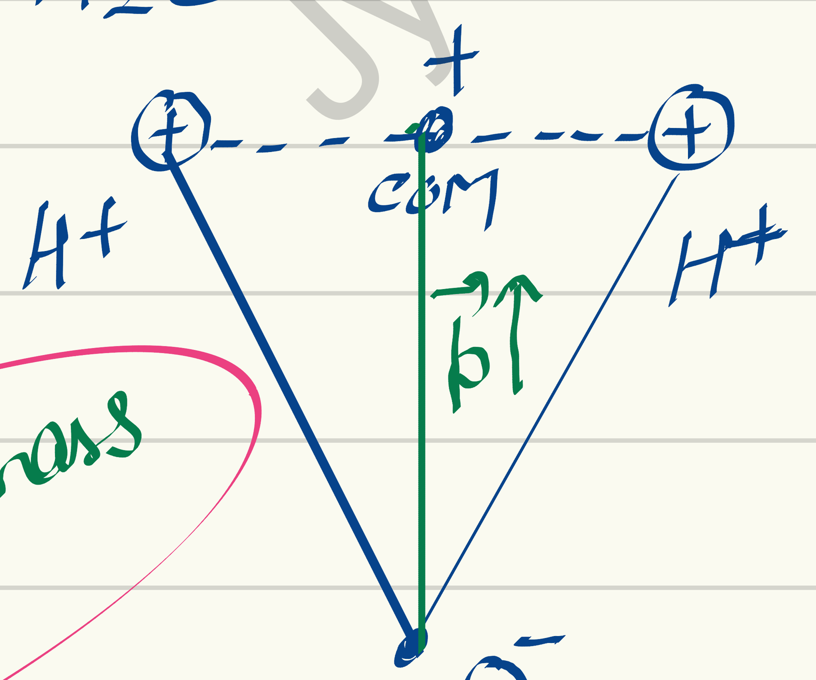
* <u>Point dipole</u> : An ideal dipole in which size $2a \rightarrow 0$ and charge $q \rightarrow \infty$ in such a way that dipole moment $p = q \times 2a$ has a finite value. Such a dipole is called point dipole.
(A dipole of negligibly small size is called point dipole or an ideal dipole.)

<u>Physical Significance of dipoles</u> :
Polar and Non polar molecules -

Polar Malecules

↓

COM of +ve and -ve charge does not concide.
e.g. H_2O



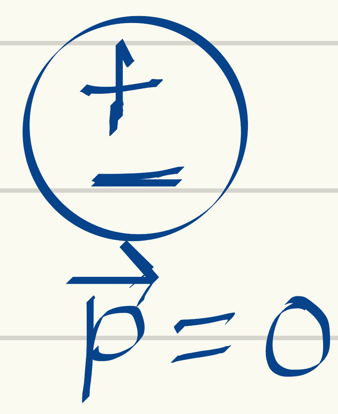
COM → centre of mass

* Polar molecules have permanent dipole moment.

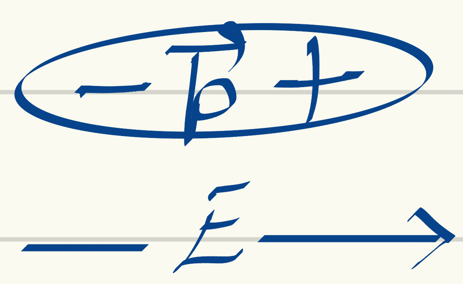
Non polar molecules

↓

COM of +ve and -ve charge concides.
e.g. CO_2, CH_4, H_2, N_2, O_2 etc.



* Under influence of uniform electric field it becomes polar molecules.



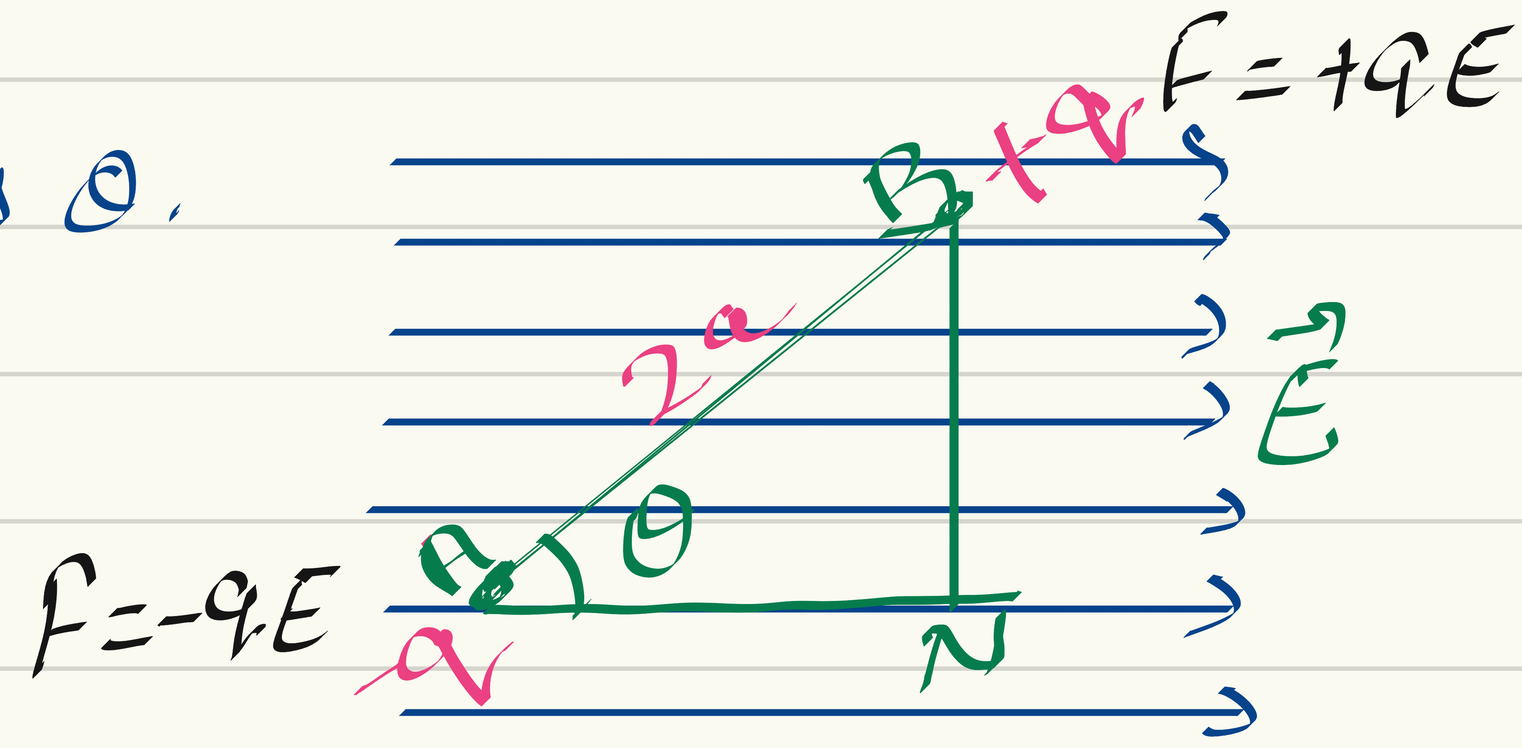
Various materials show interesting properties and important application in the present or absence of electric field.

Dipole in Uniform External Field; Torque on Dipole

Consider a dipole AB placed in uniform electric field as shown in fig

Let angle b/w $\vec{p}$ and $\vec{E}$ is θ.

here



Net force on the dipole

$$F_{net} = qE - qE$$
$$= 0$$

But a couple of force acting on dipole develops a torque -

$$\tau = |\vec{F}| \times \perp \text{ distance } (BN)$$

$$\tau = qE \times 2a\sin\theta \qquad \left[\because \sin\theta = \frac{BN}{2a}\right]$$

$$= (2aq)E\sin\theta$$

$$\boxed{\tau = pE\sin\theta}$$

OR $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

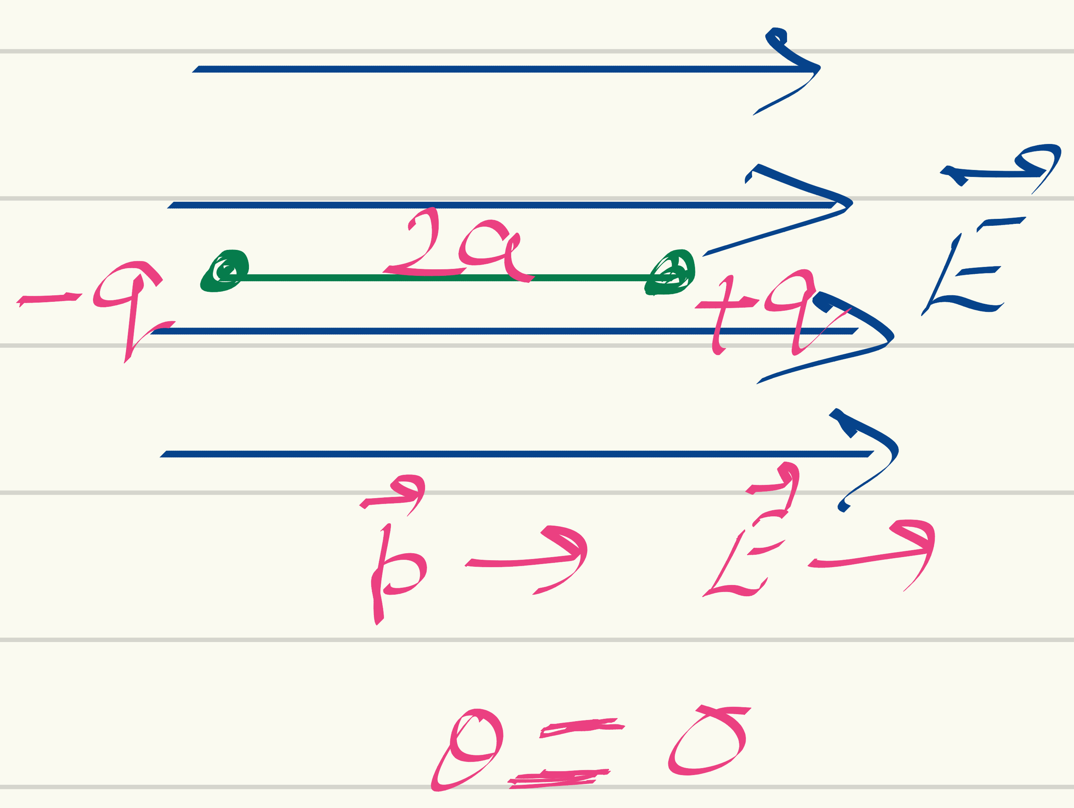
The direct of torque τ can be given by right hand thumb rule.

Special cases

(I) If $\theta = 0°$

by $\tau = pE\sin\theta$

$\boxed{\tau = 0}$



No torque is acting when $\vec{p}$ and $\vec{E}$ are parallel.

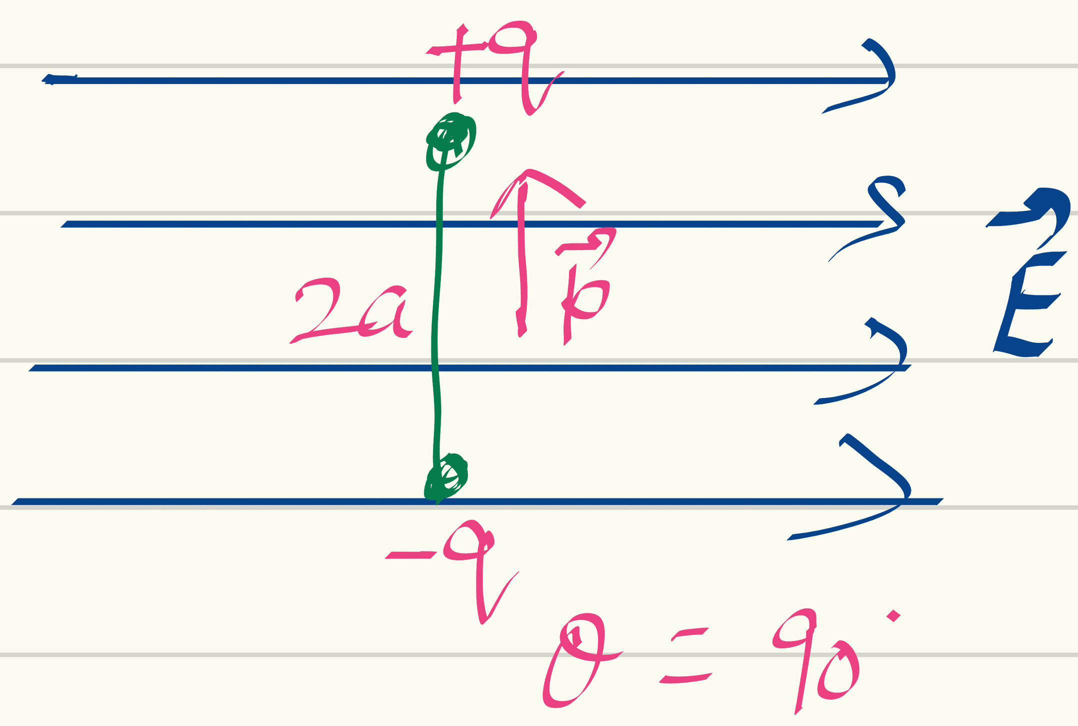
(* Stable equilibrium)

(II) If $\theta = 90°$

$\tau = pE\sin 90°$

or $\boxed{\tau = pE}$ [Max. torque]

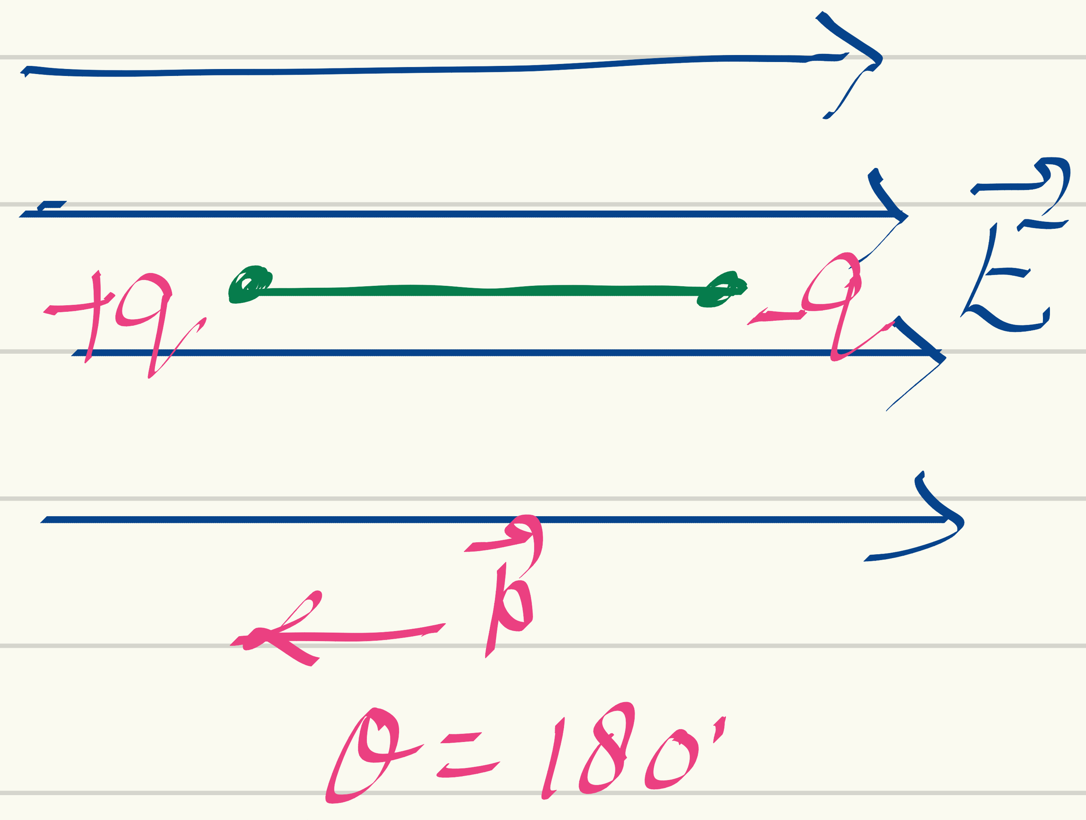
τ_{max}



(iii) If $\theta = 180°$

$\tau = pE \sin 180$

or $\boxed{\tau = 0}$



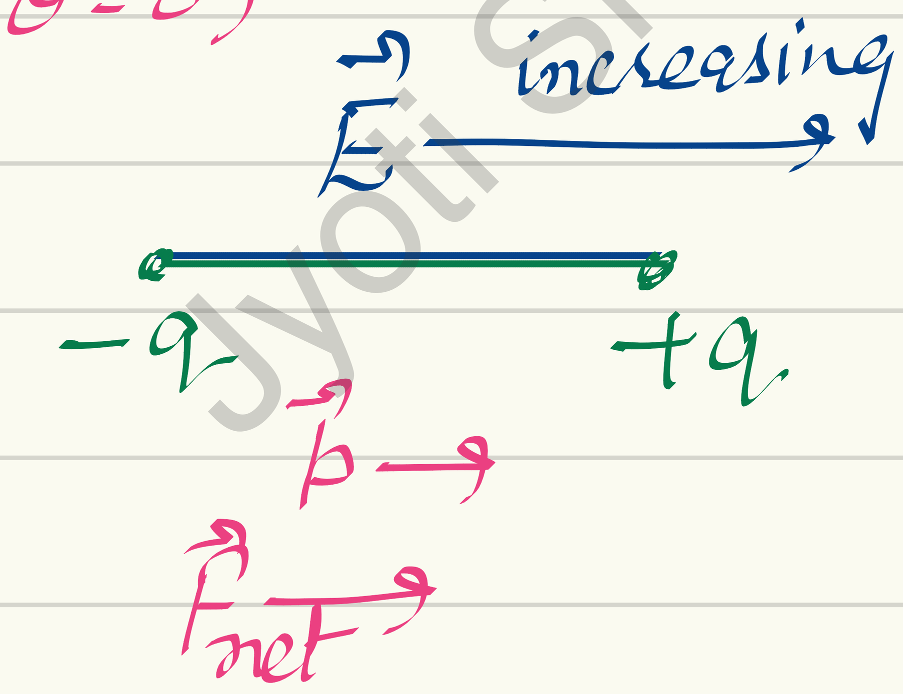
No torque is acting when $\vec{p}$ and $\vec{E}$ are antiparallel.
(Unstable equilibrium)

Dipole in Non-Uniform Electric field:

When field is not uniform, the net force will not be zero.

Let when $\vec{p}$ is parallel or antiparallel to $\vec{E}$
* Net force is not zero
* Net torque is zero.

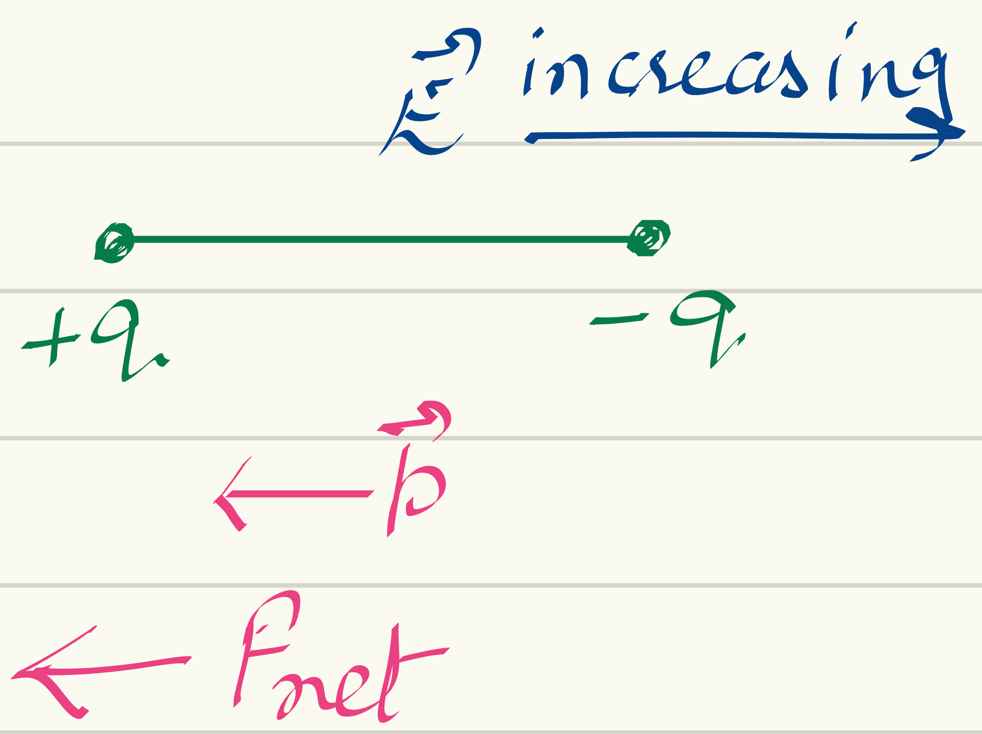
(i) • $\vec{p} \parallel \vec{E}$ $(\theta = 0°)$



Net force $F_{net} \neq 0$
(In the dirⁿ of increasing field)

Net torque is zero
(Dipole will not rotate)

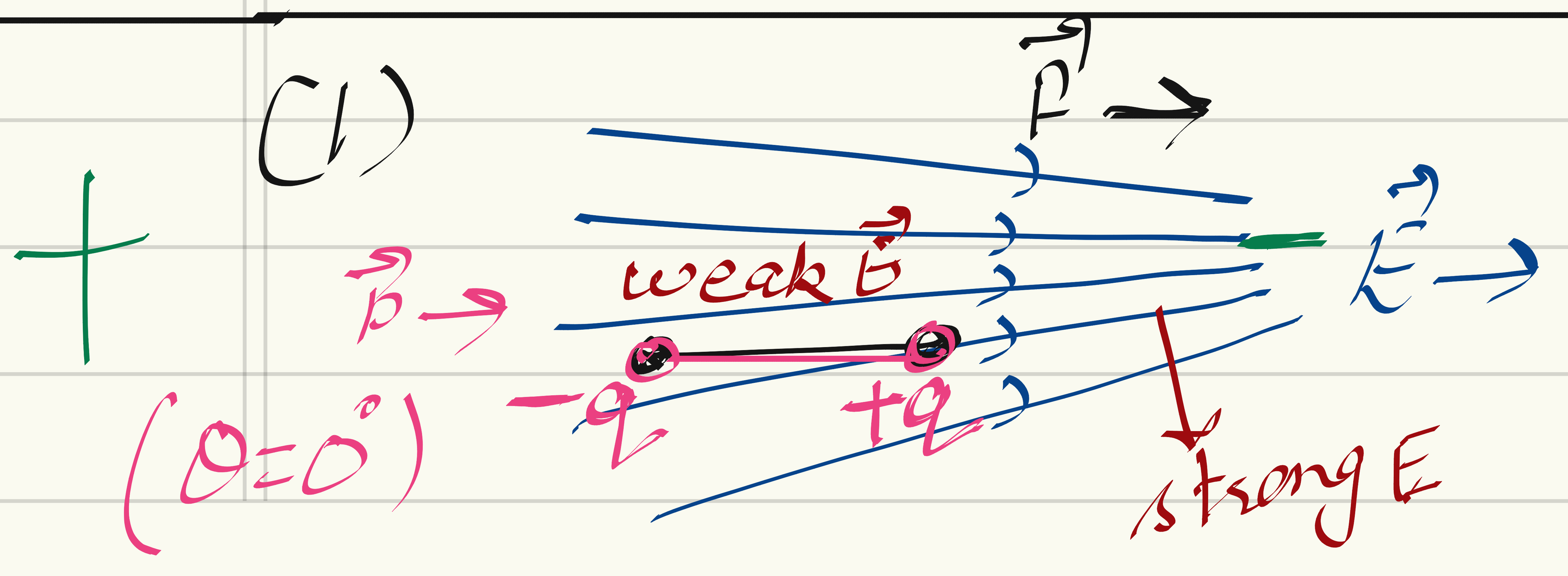
(ii) • $\vec{p}$ anti ∥ $\vec{E}$ $(\theta = 180°)$



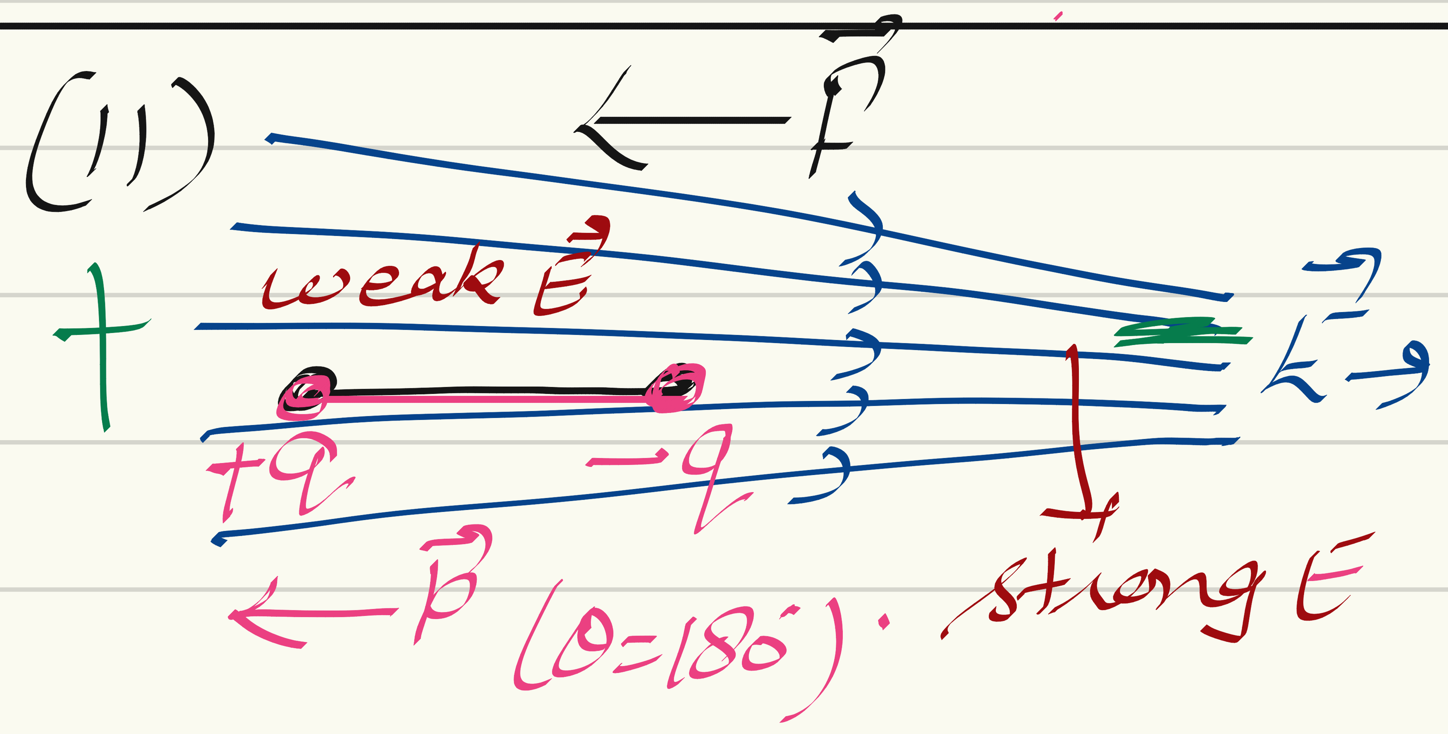
Net force $F_{net} \neq 0$
(In the dirⁿ of decreasing field)

Net torque is zero
(Dipole will not rotate)

(i)



(ii)



Continuous Charge Distribution

At macroscopic level charge is considered as continuous.

Types of continuous charge distribution are

1. <u>Linear charge Distribution</u>

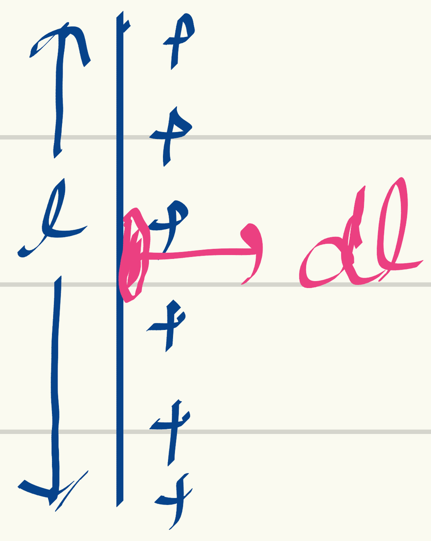
The linear charge density λ is defined as

$$\lambda = \frac{\Delta Q}{\Delta l} \quad [\text{charge per unit length}]$$

for small line element

$$\lambda = \frac{dq}{dl}$$

SI unit - $C\text{-}m^{-1}$

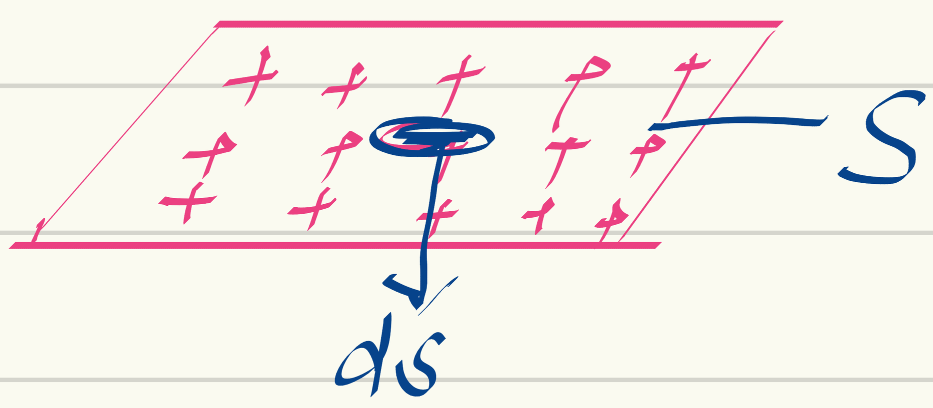


2. <u>Surface charge Distribution</u>

The surface charge density σ is defined as

$$\sigma = \frac{\Delta Q}{\Delta S} = \frac{dq}{dS} \quad [\text{charge per unit area}]$$

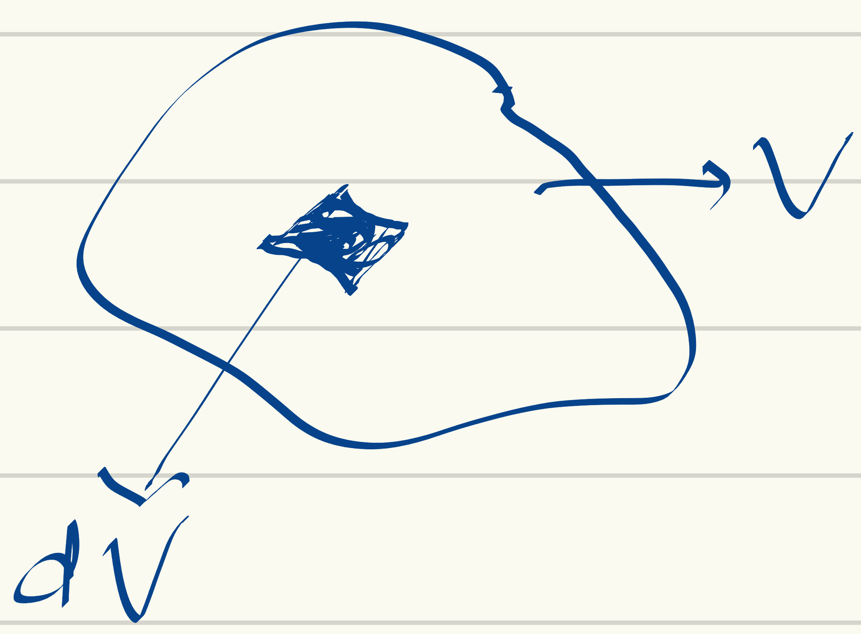
SI unit - $C\text{-}m^{-2}$



3. <u>Volume charge Distribution</u>

Volume charge density ρ is defined as

$$\rho = \frac{\Delta q}{\Delta V} = \frac{dq}{dV} \quad [\text{charge per unit volume}]$$



✱ The notion of continuous charge distribution is similar to continuous mass distribution in mechanics. We ignore discrete molecular distribution of mass.

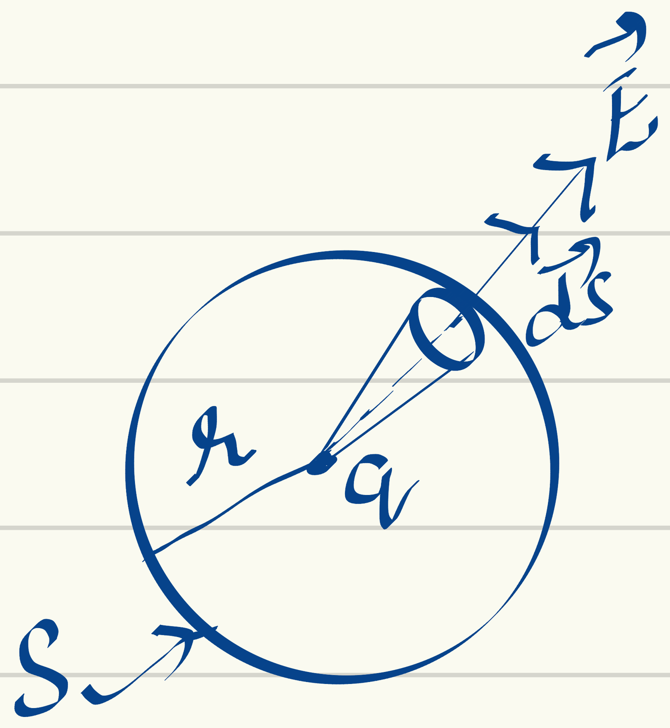
Gauss Law: Electric flux through a closed surface S is equal to $\frac{1}{\epsilon_0}$ times of charge enclosed by S.

$$\phi = \frac{q}{\epsilon_0}$$

where q = total charge enclosed

* not in syllabus

Proof:



Consider the total flux through a sphere of radius r which encloses a point charge q at its centre.

The flux through a small area element $\vec{ds}$

$$\phi = \int d\phi = \int \vec{E} \cdot \vec{ds}$$

or $$\phi = \int E \, ds \cos 0^\circ \qquad [\theta = 0^\circ]$$

$$= E \int ds$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \int ds \qquad \left[E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\right]$$

$$= \frac{1}{\cancel{4\pi}\epsilon_0} \cdot \frac{q}{\cancel{r^2}} \times \cancel{4\pi r^2} \qquad \left[\because \int ds = s = 4\pi r^2\right]$$

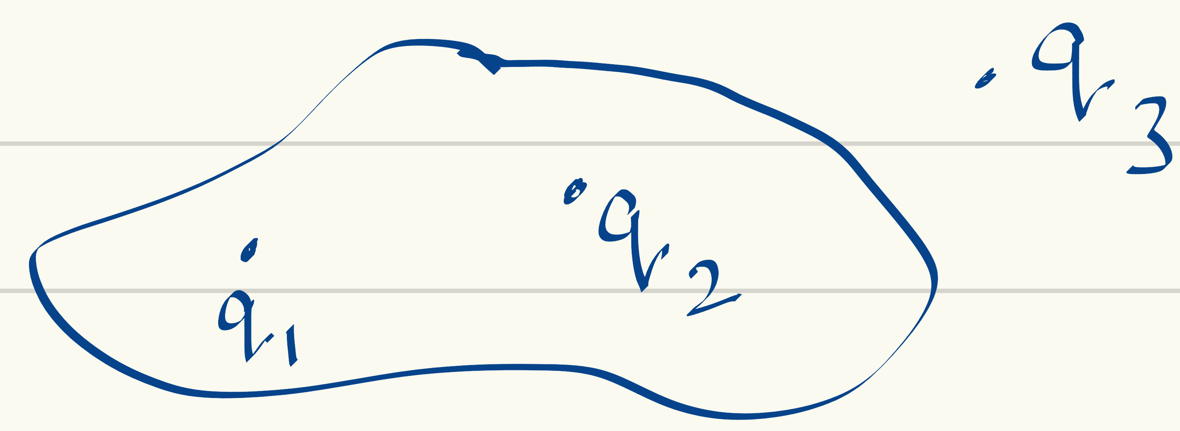
or $$\boxed{\phi = \frac{q}{\epsilon_0}}$$

Proved.

* If no charge encloses the total flux is zero.
* Gauss law is true for any closed surface, no matter what is it shape or size.
* In Gauss law q represents sum of all charges enclosed by the surface S.

* If some charges are inside and outside the surface then the flux ϕ will be due to all the charges but q however represents only the total charge inside S.

e.g.



In Gauss law, $\phi = \frac{q}{\epsilon_0}$

the term $\phi \left(= \int \vec{E} \cdot \vec{ds}\right)$ is due to q_1, q_2 and q_3 but the term $q = q_1 + q_2$ only.

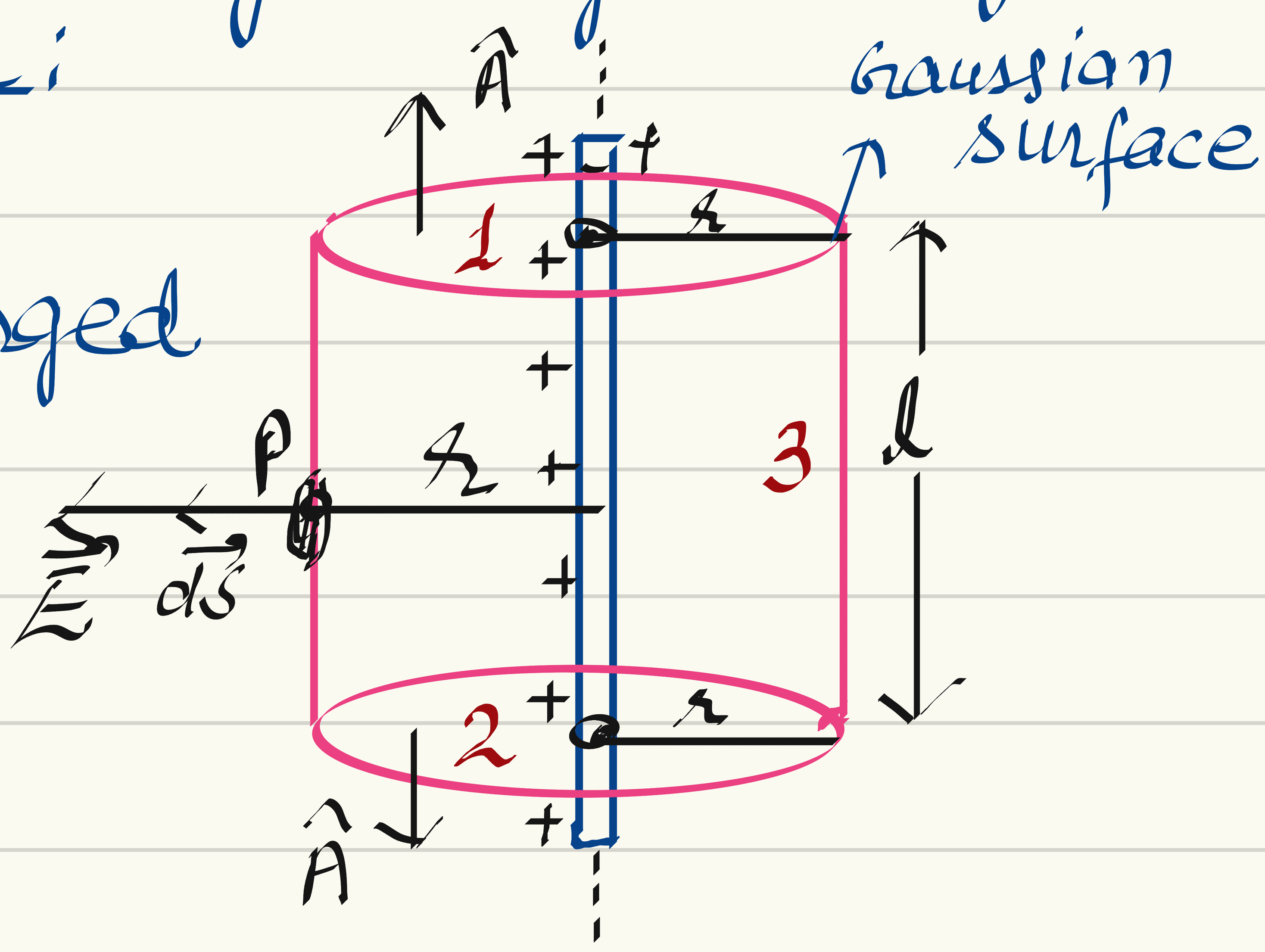
* Gaussian Surface - Gaussian surface is an imaginary surface around a symmetric charge distribution and electric field intensity is same at all the points of surface.

* Gauss law is based on the inverse square dependence on distance contained in coulomb's law. violation of Gauss's law will indicate departure from inverse square law.

Applications of Gauss's law

For some symmetric charge configuration it is possible to obtain the electric field in a simple way using the Gauss law.

1. **Field due to an infinitely long straight uniformly charged wire:**



Consider a uniformly charged thin straight wire of charge density λ.

Let $\lambda > 0$, so dir^n of $\vec{E}$ is radially outward.

Let 'P' is the point where electric field is to be determined.
Draw the cylindrical gaussian surface passing through 'P' as show in fig.

Flux through the gaussian surface

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$= 0 + 0 + \int E\,dS \cos 0°$$

[∵ for ϕ_1 and ϕ_2 $\theta = 90°$]

So,

$$\phi = E \int dS$$

$$\phi = E \times 2\pi r l$$

[∵ $\int dS$ = surface area of curved surface]

by Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

hence $$\frac{q}{\epsilon_0} = E \times 2\pi r l$$

or $$E = \frac{1}{2\pi\epsilon_0} \cdot \frac{q}{rl}$$

$$\boxed{E = \frac{q}{2\pi\epsilon_0 r l}}$$

put $\frac{q}{l} = \lambda$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

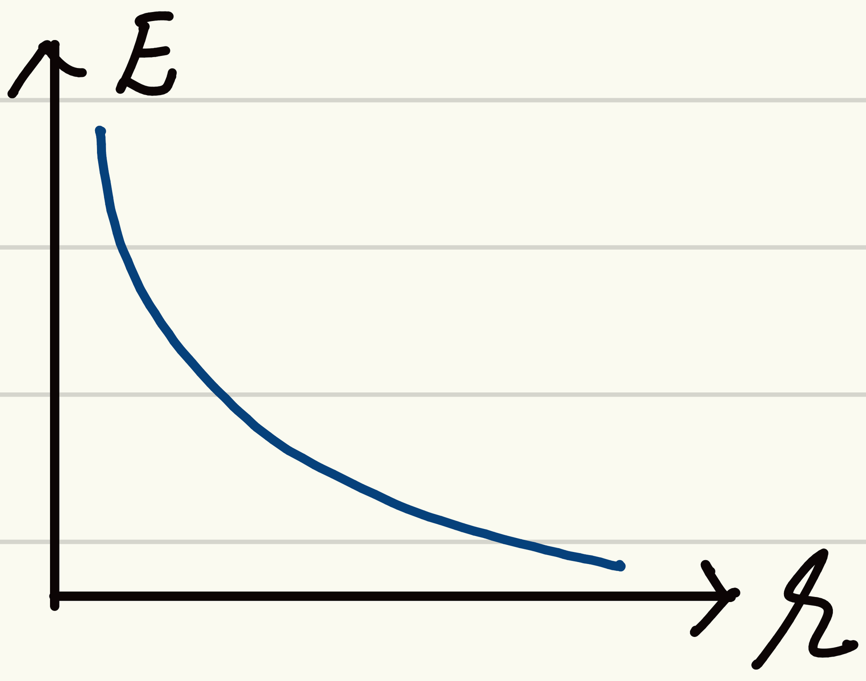
Vector form - $$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r} \cdot \hat{n}}$$

$\hat{n}$ is the unit vector in the dirn of electric field.

* E is outward if λ is +ve and inward if λ is -ve.

* For line charge $\boxed{E \propto \frac{1}{r}}$

* Graph-



* K.E of an electron revolving a line charge

$F_e = F_c$

$eE = \frac{mv^2}{r}$

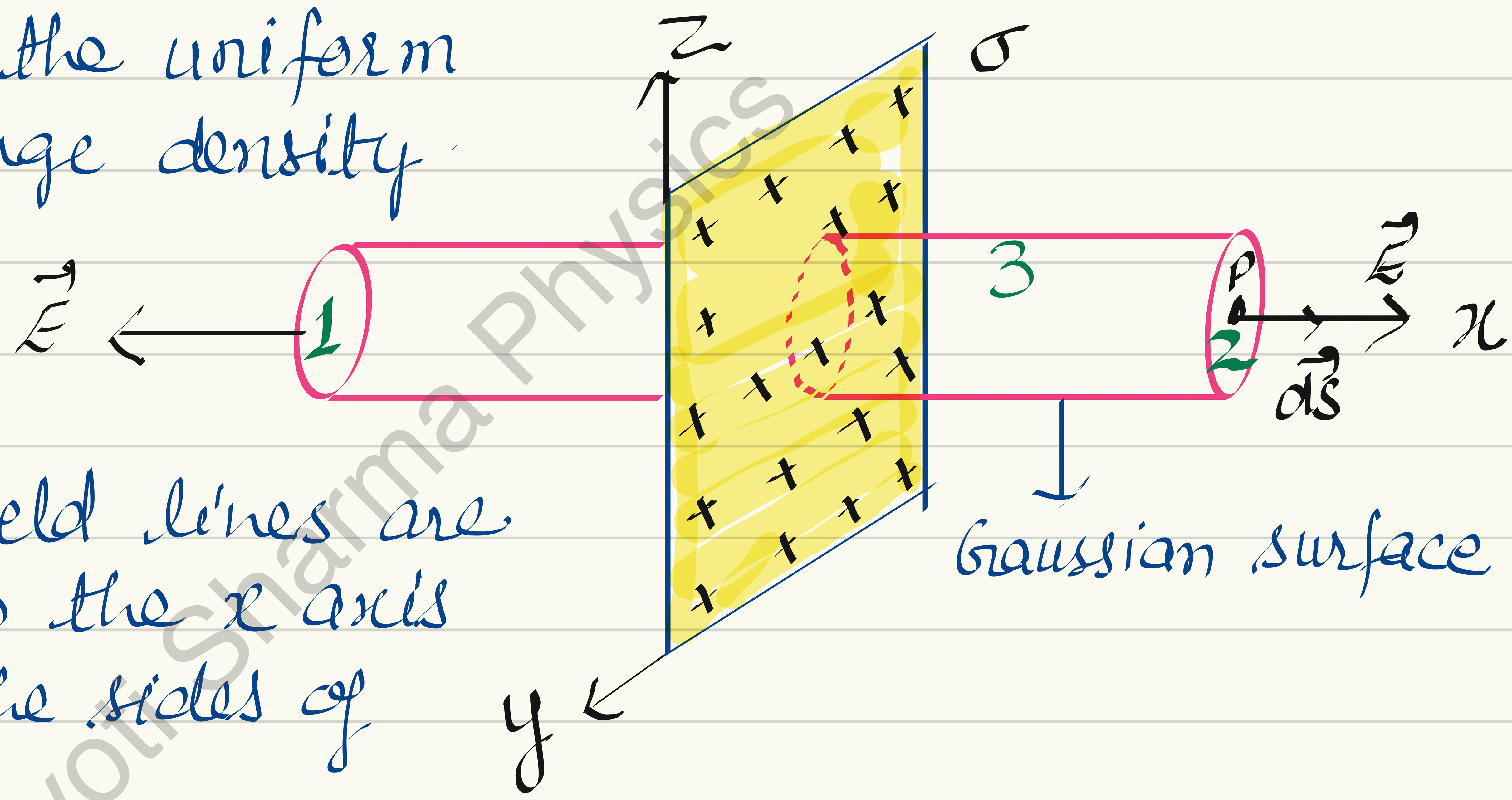
$\frac{e\lambda}{2\pi\epsilon_0 \cancel{r}} = \frac{mv^2}{\cancel{r}}$

$mv^2 = \frac{e\lambda}{2\pi\epsilon_0}$

$\boxed{K.E = \frac{1}{2}mv^2 = \frac{e\lambda}{4\pi\epsilon_0}}$

2. Field due to a uniformly charged infinite plane

Let σ be the uniform surface charge density.



Electric field lines are parallel to the x axis on both the sides of sheet.

Now net flux

$$\phi = \phi_1 + \phi_2 + \phi_3$$

Only the faces 1 and 2 contribute to flux as $\theta = 0°$, and $\phi_3 = 0$ as $\theta = 90°$

so

$$\phi = \int E\,ds + \int E\,ds$$

$$= 2E\int ds$$

$$\phi = 2EA \qquad \left[\because \int ds = A\right]$$

by Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

So $$\frac{q}{\epsilon_0} = 2EA$$

but $q = \sigma A$

$$\frac{\sigma \cancel{A}}{\epsilon_0} = 2E\cancel{A}$$

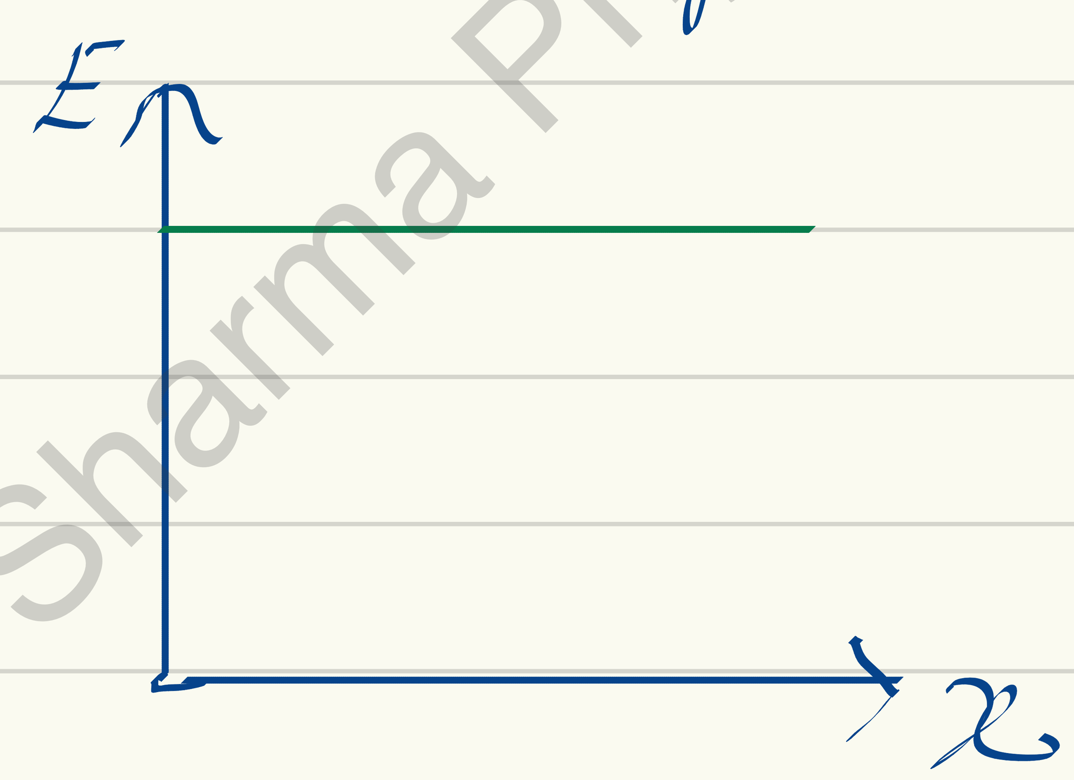
or $$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

vector form

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

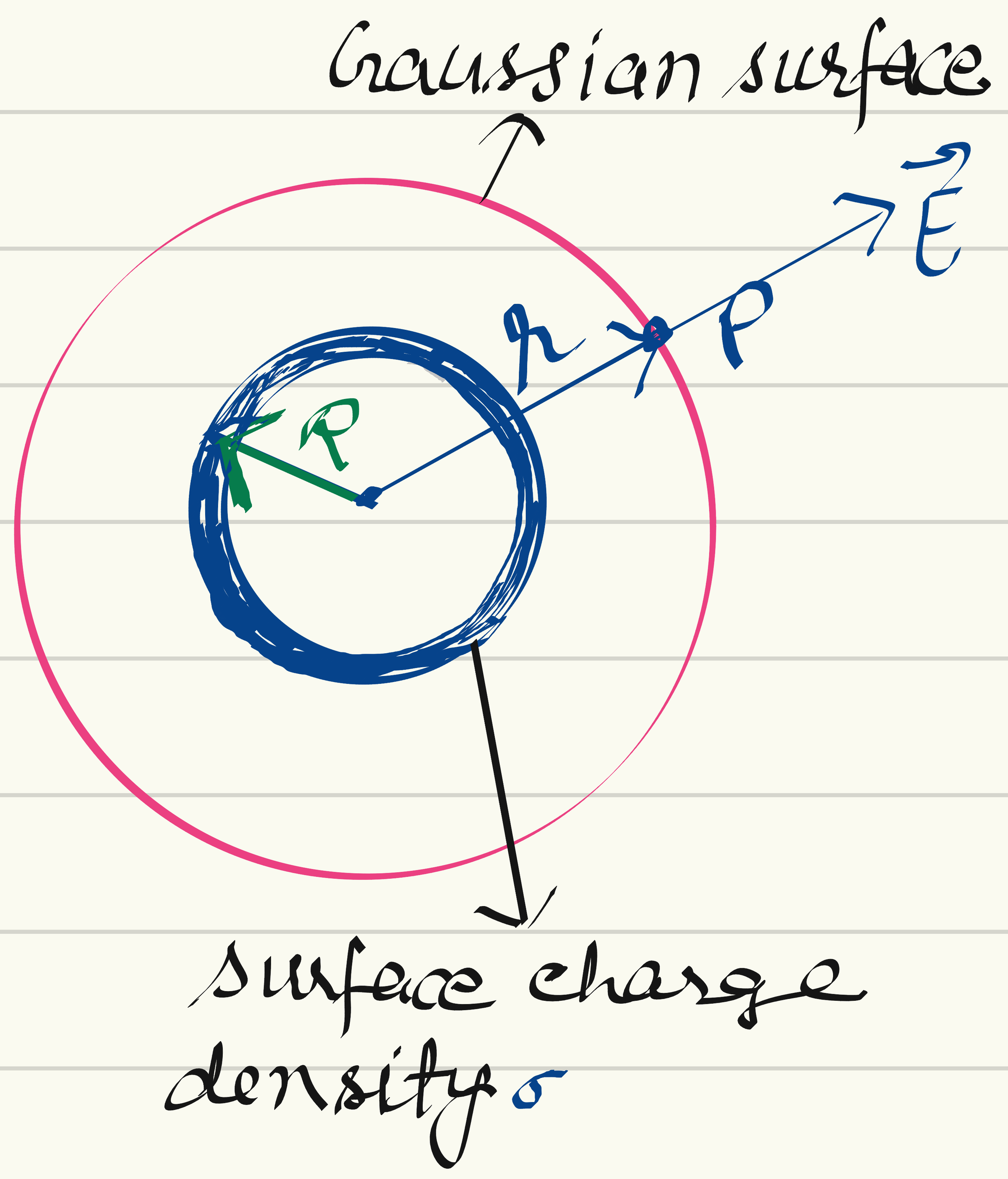
Where $\hat{n}$ is the unit vector normal to the plane and going away from it.

* E is away from the plate if $\sigma > 0$ [+ve]
* E is towards the plate if $\sigma < 0$ [-ve]
* E is independent of x
* Graph -



3. Field due to a uniformly charged thin spherical shell: (For conducting or non-conducting)

Let σ be the uniform surface charge density of a thin spherical shell of radius R.



(1) Field outside the shell: $(r > R)$

Consider a point P outside the shell with radius vector $\vec{r}$.

At every point of Gaussian surface $\vec{E}$ and $\vec{ds}$ are parallel $(\theta = 0^\circ)$

Now net flux

$$\phi = \int E\, ds \cos 0^\circ$$

or $\phi = E \int ds$

$$= E \times 4\pi r^2 \qquad [\because \int ds = 4\pi r^2]$$

by Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

So $\frac{q}{\epsilon_0} = E \times 4\pi r^2$

or $$\boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}}$$

put $q = \sigma A = \sigma \times 4\pi R^2$ [$q \to$ total charge on shell]

we get

$$E = \frac{1}{\cancel{4\pi}\epsilon_0} \cdot \frac{\sigma \times \cancel{4\pi} R^2}{r^2}$$

or $$\boxed{E = \frac{\sigma R^2}{\epsilon_0 r^2}}$$

Vector form $$\boxed{\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$$

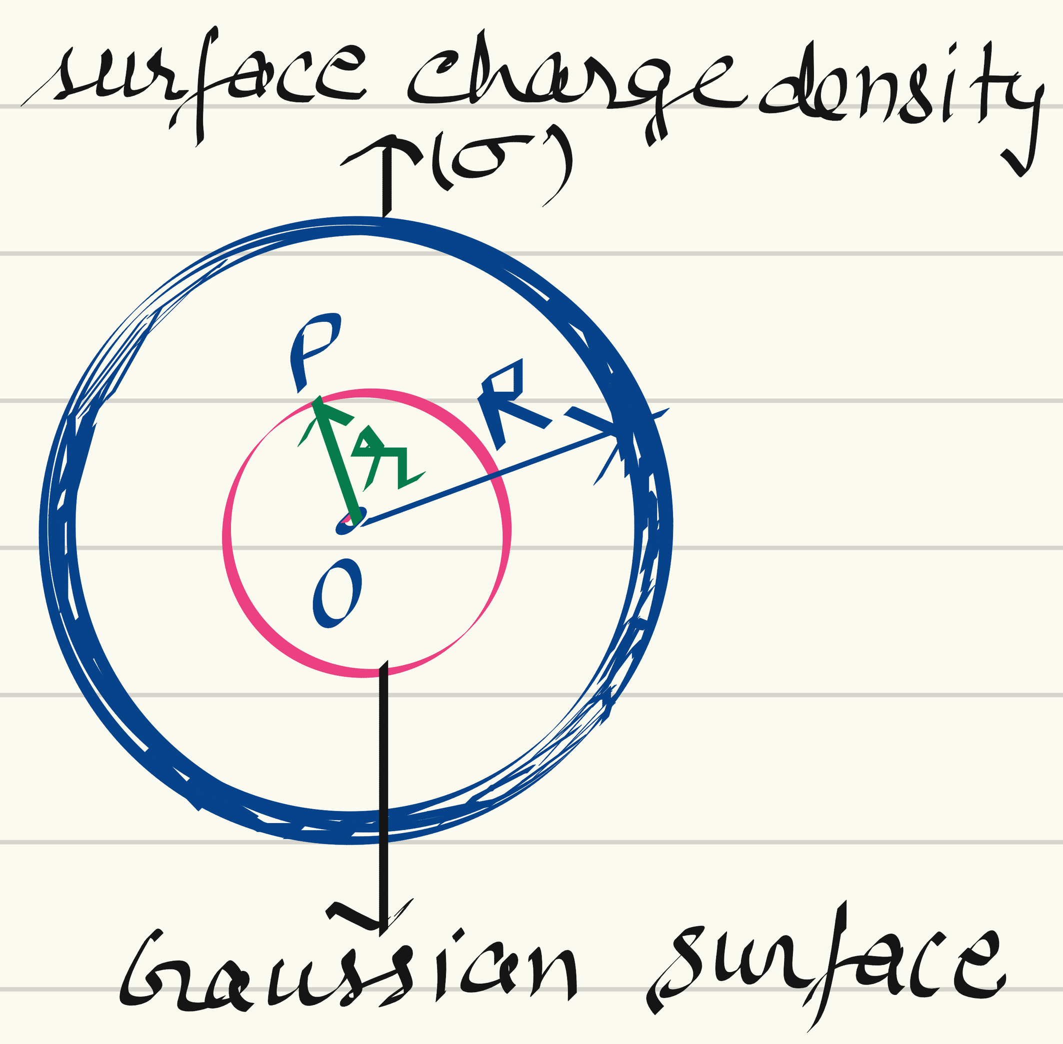
* E is directed outward if $q > 0$ and inward if $q < 0$.
* Field outside the shell is as if entire charge of the shell is situated at its centre.

(ii) Field Inside the shell : $(r < R)$

Let the point 'P' is inside the shell at a distance r from O.

Gaussian surface is passing through P.

Net flux through the gaussian surface $\phi = E \int ds$



or $\phi = E \times 4\pi r^2$

but inside the shell $q = 0$

so by Gauss's law

$$\phi = \frac{q}{\epsilon_0} = 0$$

or $E \times 4\pi r^2 = 0$

or $\boxed{E = 0}$

(iii) **Field at the surface ($r = R$)**

By Gauss' law

$$\phi = \frac{q}{\epsilon_0} = E \times 4\pi R^2$$

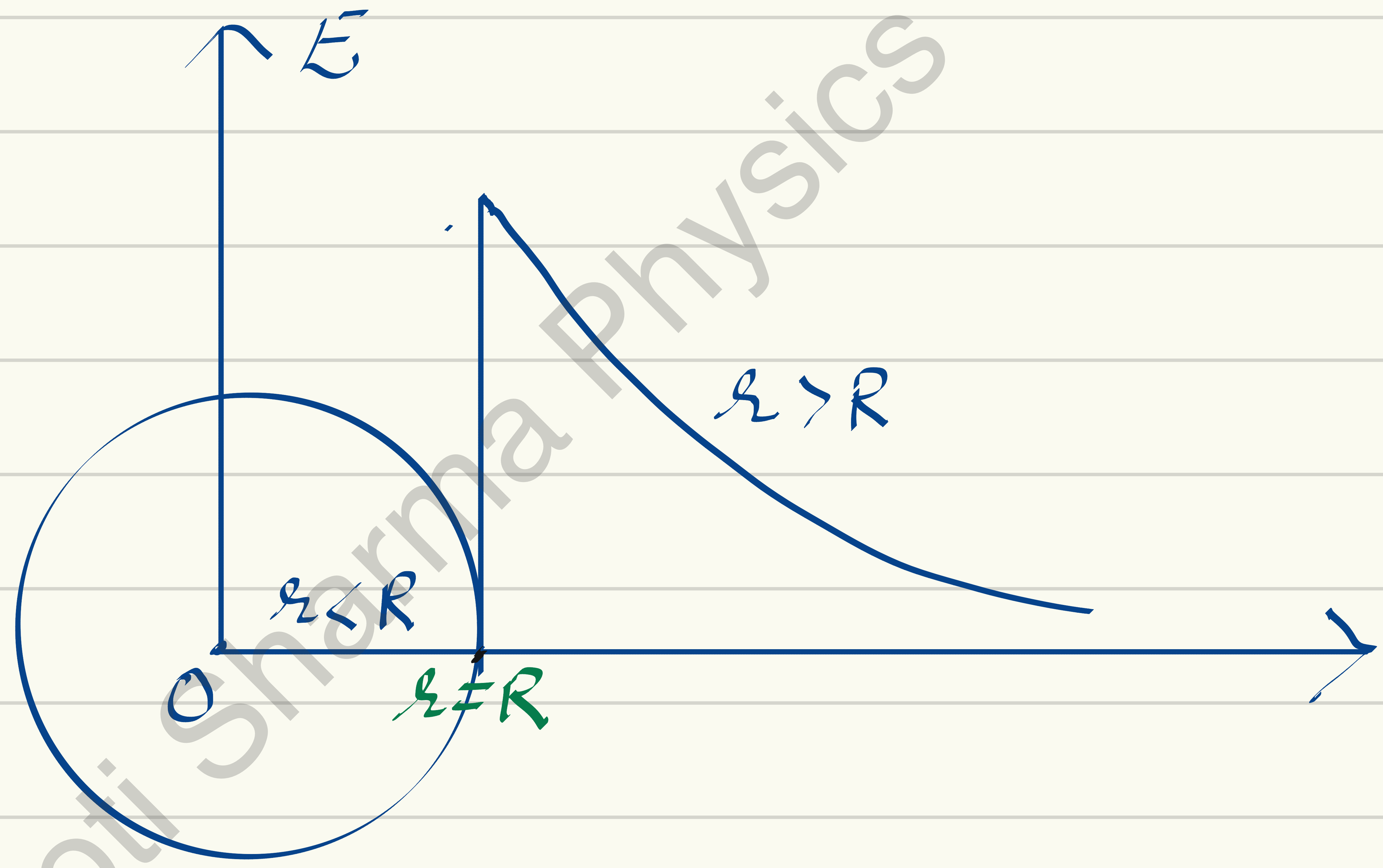
or $\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}}$

Also by $E = \frac{\sigma R^2}{\epsilon_0 r^2}$

$$E = \frac{\sigma}{\epsilon_0} \quad [\because r = R]$$

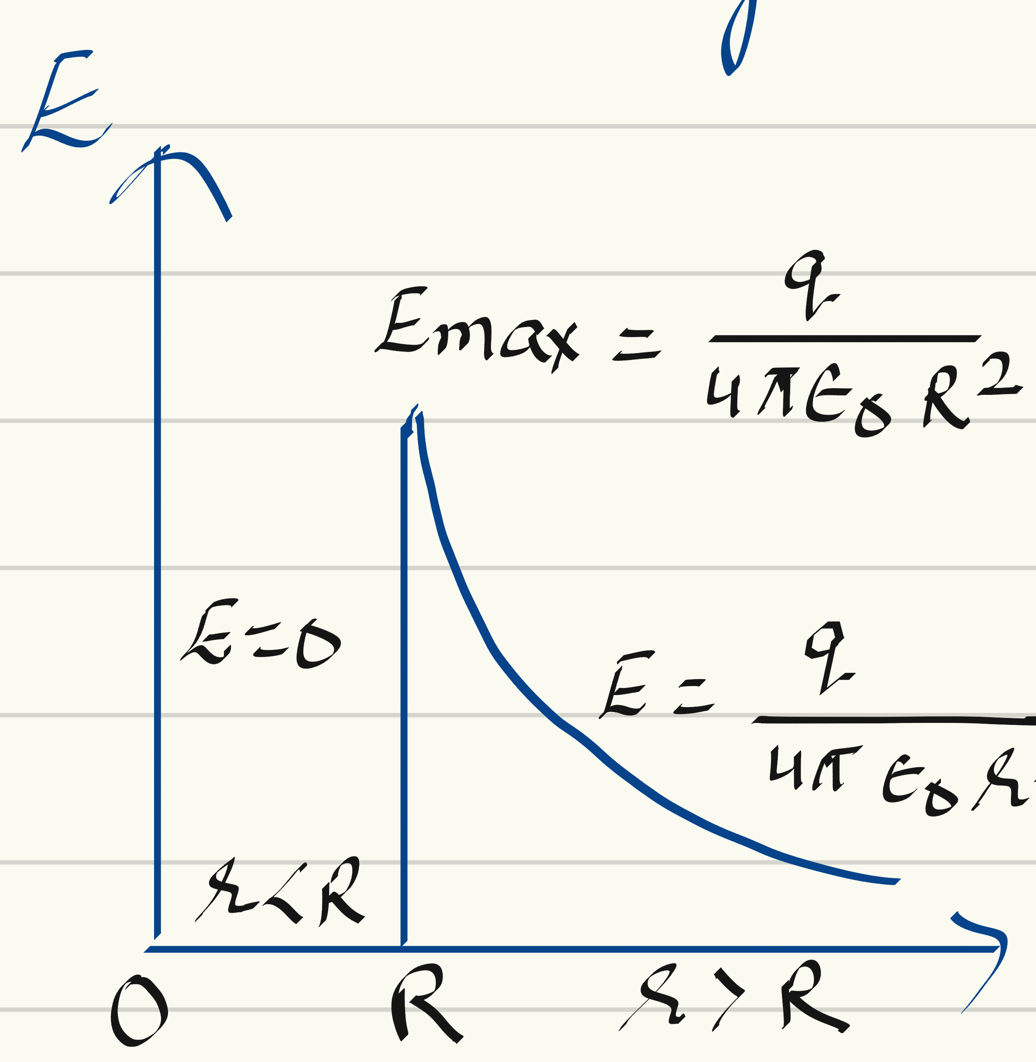
* Inside the spherical shell (for conducting and non conducting both) electric field is zero.

* Graph-

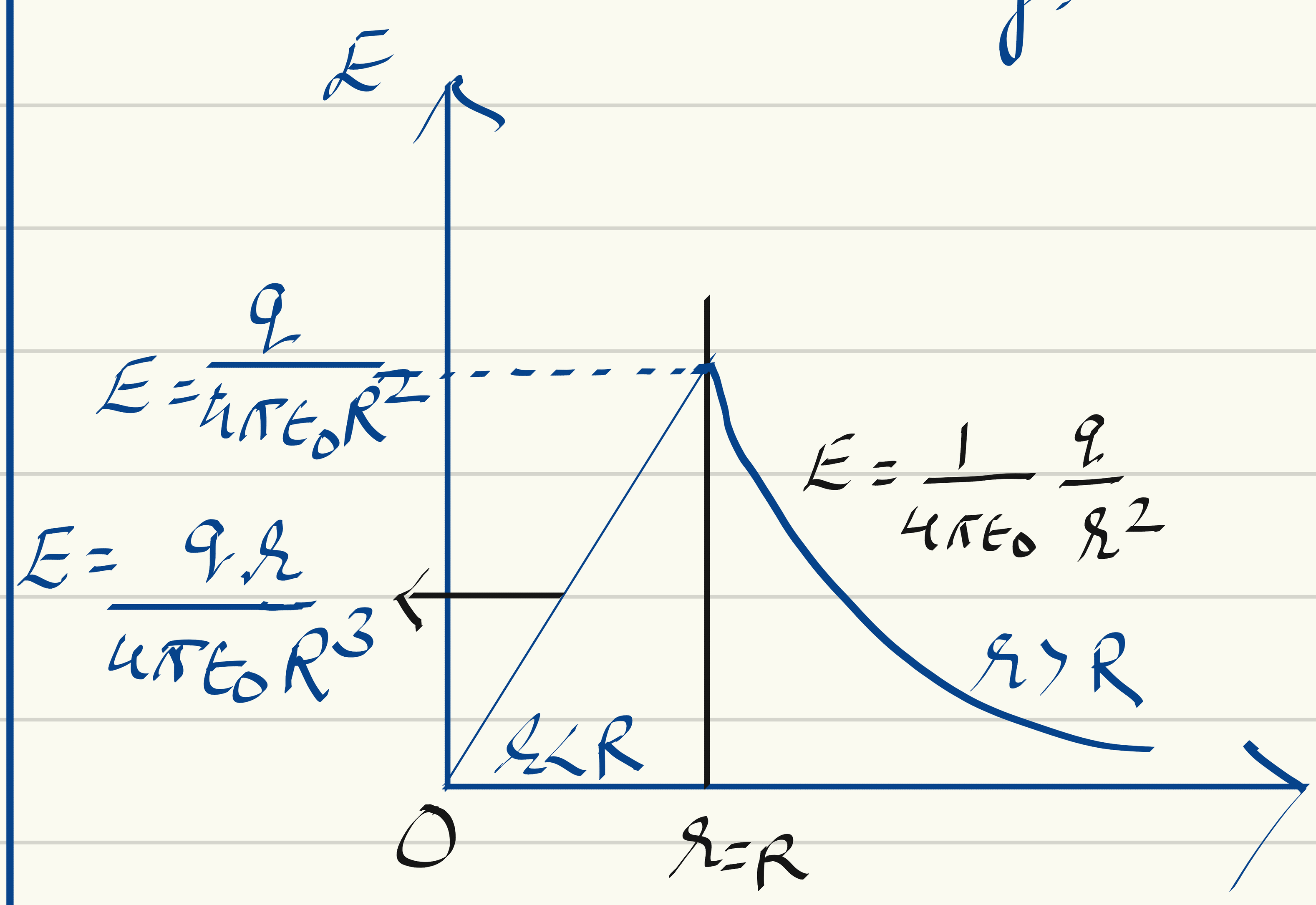


For solid sphere

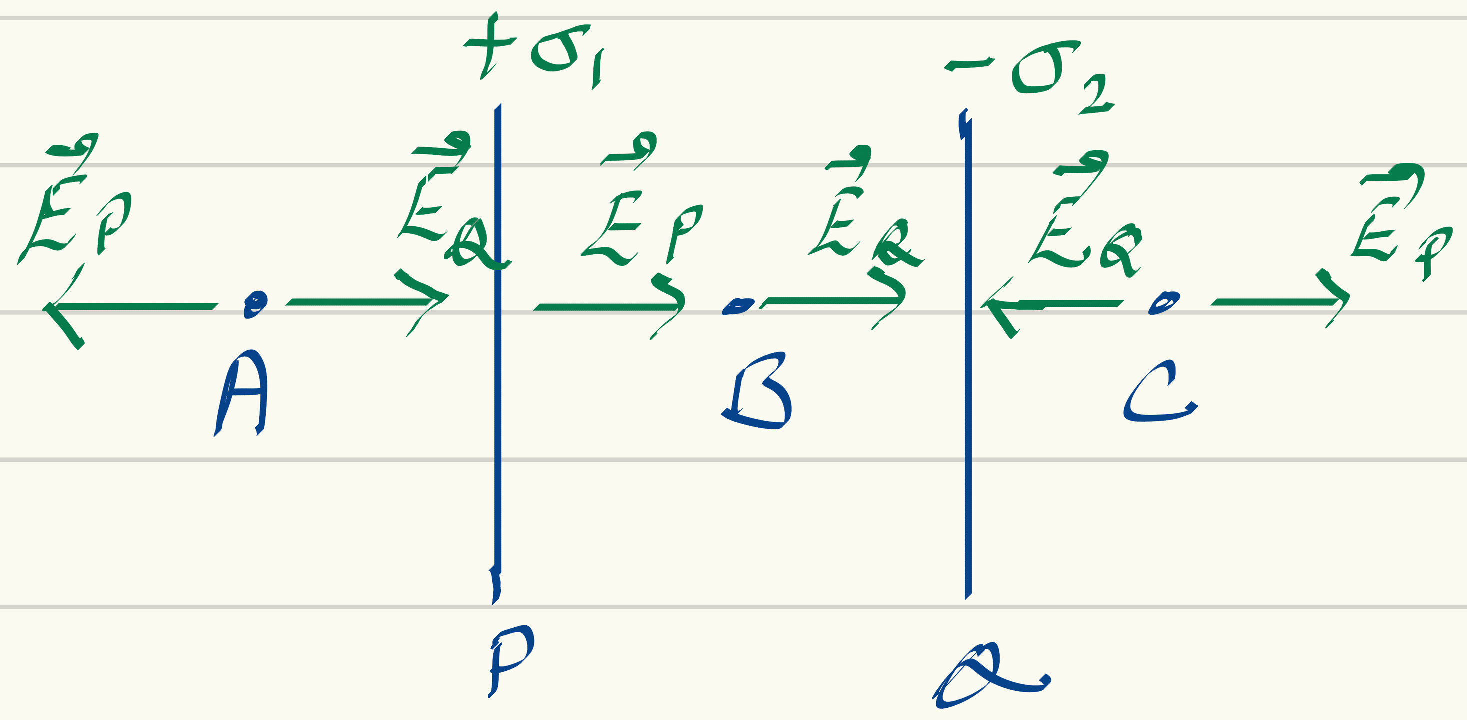
conducting



Non-conducting



Two infinite parallel sheets of charge



P & Q are uniformly charged sheets of charge densities $+\sigma_1$ and $-\sigma_2$ respectively.

Net field at 'A'

$$E_A = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2) \quad [\because \sigma_1 > \sigma_2]$$

Net field at 'B'

$$E_B = \frac{1}{2\epsilon_0}(\sigma_1 + \sigma_2) \quad [\text{towards right}]$$

Net field at 'C'

$$E_C = \frac{1}{2\epsilon_0}(\sigma_1 - \sigma_2) \quad [\because \sigma_1 > \sigma_2]$$