

Topic Motion In A straight Line Date .....

2.1  
 Example 32 The position of . . . . . and  $t = 4 \text{ s}$  ?

Solution: Position of the object is given by

$$x = a + bt^2$$

where  $a = 8.5 \text{ m}$  and  $b = 2.5 \text{ m/s}^2$   
 We know

$$v = \frac{dx}{dt}$$

$$= \frac{d(a + bt^2)}{dt}$$

$$\text{or } v = 0 + 2bt \quad \left[ \because \frac{d t^n}{dt} = n t^{n-1} \right]$$

$$\text{or } v = 2bt$$

Now at  $t = 0$ ,  $v = 0$  Ans

and at  $t = 2.0 \text{ s}$

$$v = 2b \times 2$$

$$= 4b$$

$$= 4 \times 2.5$$

$$v = 10 \text{ m/s} \quad \text{Ans} \quad [\because b = 2.5 \text{ m/s}^2]$$

for average velocity

$$\text{by } x = a + bt^2$$

displacement at  $t = 2.0 \text{ sec}$

$$x_1 = a + b(2)^2 = a + 4b$$

displacement at  $t = 4.0 \text{ sec}$

$$x_2 = a + b(4)^2 = a + 16b$$

We know

$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{a + 16b - (a + 4b)}{4 - 2}$$

$$\text{or } v_{av} = \frac{12b}{2} = 6b$$

$$\text{or } v_{av} = 6 \times 2.5 = 15 \text{ m/s} \quad \text{Ans}$$

## Example 2.2

Calculus Method:  $u \rightarrow$  velocity at  $t=0$ ,  $v \rightarrow$  velocity at time  $t$

### 1) First Equation

We have,  $a = \frac{dv}{dt}$

$$dv = a dt$$

$$\int_u^v dv = \int_0^t a dt$$

$$[v]_u^v = a[t]_0^t$$

$$v - u = a(t - 0)$$

$$\boxed{v = u + at}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$v = \frac{dx}{dt}, a = \frac{dv}{dt}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int dx = x, \int dv = v$$

$$\int dt = t, \int t dt = \frac{t^2}{2}$$

$$\int_u^v v dv = \left[ \frac{v^2}{2} \right]_u^v = \frac{v^2 - u^2}{2}$$

### 2) Second Equation

We know,  $v = \frac{dx}{dt}$

$$dx = v dt$$

$$dx = (u + at) dt$$

or  $dx = u dt + a t dt$

On integrating within the limits,

$$\int_{x_0}^x dx = \int_0^t u dt + \int_0^t a t dt$$

$$\text{or } [x]_{x_0}^x = u[t]_0^t + a \left[ \frac{t^2}{2} \right]_0^t$$

$$\text{or } x - x_0 = u(t - 0) + a \left( \frac{t^2}{2} - 0 \right)$$

put  $x - x_0 = s$ ,

$$\boxed{s = ut + \frac{1}{2} at^2}$$

### (iii) Third Equation

$$\text{We know, } a = \frac{dv}{dt}$$

$$= \frac{dv}{dt} \cdot \frac{dx}{dx}$$

$$= \frac{dv}{dx} \cdot \frac{dx}{dt}$$

$$a = \frac{dv}{dx} \cdot v$$

$$\text{or } a = v \frac{dv}{dx}$$

$$\text{or } v dv = a dx$$

On integrating within the limit,

$$\int_u^v v dv = \int_{x_0}^x a dx$$

$$\left[ \frac{v^2}{2} \right]_u^v = a \left[ x \right]_{x_0}^x$$

$$\text{or } \frac{v^2}{2} - \frac{u^2}{2} = a(x - x_0)$$

$$\text{put } x - x_0 = s$$

$$\frac{v^2 - u^2}{2} = as$$

$$\text{or } \boxed{v^2 - u^2 = 2as}$$

Example 3.4 <sup>2.3</sup> A ball is thrown - - - - hits the ground?  
Take  $g = 10 \text{ m/s}^2$

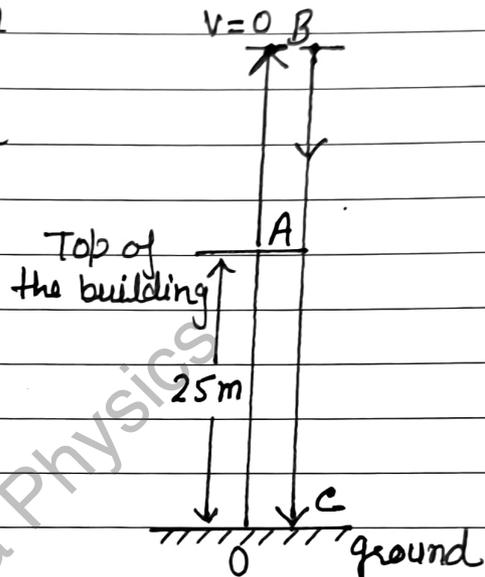
Solution

The ball is thrown from the top of the building with initial velocity

$$u = 20 \text{ m/s}$$

height of the building  $h = 25 \text{ m}$

$$\text{ang } g = 10 \text{ m/s}^2$$



(a) by III eq<sup>n</sup> of motion

$$v^2 = u^2 + 2gh$$

at maximum height (At B)

$$v = 0$$

then

$$0 = 20^2 + 2(-10)h \quad [ \because g = -10 \text{ m/s}^2 ]$$

$$\text{or } h = \frac{20 \times 20}{20}$$

$$\text{or } h = 20 \text{ m Ans}$$

(b) Net displacement  $AC = s = -25 \text{ m}$

-ve sign is taken because displacement is in the opposite direction of initial velocity.

$$\text{by } s = ut + \frac{1}{2}gt^2$$

$$-25 = 20t + \frac{1}{2}(-10)t^2$$

$$\text{or } 5t^2 - 20t - 25 = 0$$

$$\text{or } t^2 - 4t - 5 = 0$$

$$\text{or } (t-5)(t+1) = 0$$

$$\text{or } t = 5 \text{ sec as } t \neq -1$$

Therefore time taken to hit the ground = 5 Sec

Ans

2.4

Example 3.5 Discuss the motion - - - - - air resistance.

Solution: We know that for verticle motion, Equations of motion become

$$(i) v = u + gt$$

$$(ii) h = ut + \frac{1}{2}gt^2$$

$$(iii) v^2 = u^2 + 2gh$$

Here we assume that motion is in y-direction.

The object is released from rest at  $y = 0$

Therefore we take -y direction and  $u = 0$   
also we take  $a = -g (= -9.8 \text{ m/s}^2)$

Hence the eq<sup>n</sup>s become

$$(i) v = 0 - gt$$

$$v = -gt \Rightarrow \boxed{v = -9.8t \text{ m/s}}$$

$$(ii) y = 0 - \frac{1}{2}gt^2$$

$$\text{or } y = -\frac{1}{2}gt^2 \Rightarrow \boxed{y = -4.9t^2 \text{ m}}$$

$$(iii) v^2 = 0 - 2gy$$

$$\text{or } v^2 = -2gy \Rightarrow \boxed{v^2 = -19.6y \text{ m}^2/\text{s}^2}$$

2.5

Example 3.6

Galileo's law of odd numbers: The distances traversed - - - - Prove it.

Solution:

Let us divide the time interval of motion of an object under free fall into many equal intervals.

We know for free fall

$$y = -\frac{1}{2}gt^2 \quad [\because u=0]$$

$$\text{or } y = -4.9t^2 \quad [\because g = 9.8 \text{ m/s}^2]$$

by using this relation we find the distances after 1s, 2s, 3s - - - in terms of  $y_0$  which is distance travelled in 1st second.

We write these values in a table -

$t$ (sec)	$y (= -4.9t^2)$	distance traversed in successive interval	Ratio
$t=0$	$y=0$	-	-
$t=1$	$y = -4.9 = y_0$ (say)	$y_0 - 0 = y_0$	1
$t=2$	$y = -4.9 \times 2^2 = -4 \times 4.9 = 4y_0$	$4y_0 - y_0 = 3y_0$	3
$t=3$	$y = -4.9 \times 3^2 = -9 \times 4.9 = 9y_0$	$9y_0 - 4y_0 = 5y_0$	5
$t=4$	$y = -4.9 \times 4^2 = -16 \times 4.9 = 16y_0$	$16y_0 - 9y_0 = 7y_0$	7
$t=5$	$y = -4.9 \times 5^2 = -25 \times 4.9 = 25y_0$	$25y_0 - 16y_0 = 9y_0$	9

From the table it is clear that the distances are in the simple ratio 1:3:5:7:9:11 - - -

Proved

## II Method

The distance traversed in  $n$ th second by a freely falling body is given by

$$S_{nth} = u + \frac{a}{2} (2n-1)$$

For free fall

$$u = 0 \text{ and } a = g = 9.8 \text{ m/s}^2$$

so

$$S_{nth} = \frac{9.8}{2} (2n-1)$$

$$\text{or } S_{nth} = 4.9(2n-1)$$

Now for  $n=1$

$$S_1 = 4.9(2 \times 1 - 1)$$

$$\text{or } S_1 = 4.9 = 1y_0 \text{ (say)}$$

for  $n=2$

$$S_2 = 4.9(2 \times 2 - 1)$$

$$\text{or } S_2 = 3 \times 4.9 = 3y_0$$

for  $n=3$

$$S_3 = 4.9(2 \times 3 - 1)$$

$$\text{or } S_3 = 5 \times 4.9 = 5y_0$$

similarly

$$S_4 = 7 \times 4.9 = 7y_0$$

$$S_5 = 9 \times 4.9 = 9y_0$$

$$S_6 = 11 \times 4.9 = 11y_0$$

It is clear that

$$S_1 : S_2 : S_3 : S_4 : S_5 : \dots = 1 : 3 : 5 : 7 : 9 : \dots$$

Proved

### Example 3.7<sup>2.6</sup>

Initial velocity of vehicle =  $u_0$

Deceleration =  $-a$

Final velocity  $v = 0$

Let the stopping distance is  $ds$

We have III eq<sup>n</sup> of motion

$$v^2 = u_0^2 + 2as$$

$$0 = u_0^2 + 2(-a)ds$$

$$\text{or } ds = \frac{u_0^2}{2a}$$

If  $u_0 \rightarrow 2u_0$

then  $ds \rightarrow 4ds$

### Example 3.8<sup>2.7</sup>

Given  $h = d = 21 \text{ cm} = 0.21 \text{ m}$

$u = ?$ ,  $u = 0$

by  $h = ut + \frac{1}{2}gt^2$

$$0.21 = \frac{1}{2} \times 9.8 t^2$$

$$\text{or } t^2 = \frac{0.21}{4.9}$$

$$t = \sqrt{\frac{0.21 \times 30}{4.9 \times 100}} = \sqrt{\frac{30}{7 \times 100}}$$

$$= \frac{\sqrt{4.3}}{10} \approx \frac{2}{10} = 0.2 \text{ Sec}$$

Teacher's Sign .....  
R.T.  $t = 0.2 \text{ S}$  A<sub>3</sub>

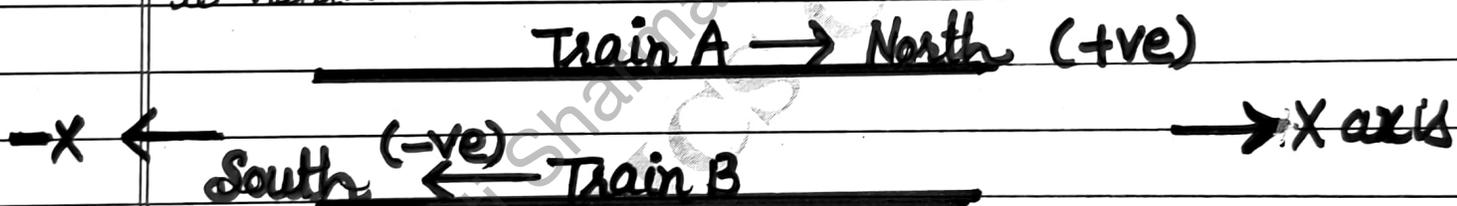
Example 3.9: Two parallel rail - - - - on the ground?  
*(Not in new syllabus)*

Solution: Given,

$$\begin{aligned} \text{Speed of train A } V_A &= 54 \text{ km/h} \\ &= \frac{54 \times 5}{18} = 15 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{Speed of train B } V_B &= 90 \text{ km/h} \\ &= \frac{90 \times 5}{18} = 25 \text{ m/s} \end{aligned}$$

Train A moves north and train B moves south. Let positive direction of x-axis is south to north.



(a) Velocity of B with respect to A

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$= -25 - 15 \quad [ \because \vec{V}_B = -25 \text{ m/s} ]$$

$$= -40 \text{ m/s} \quad \text{Ans.}$$

The train B appears to A to move with a speed of 40 m/s from the north to south.

(b) Relative velocity of ground with respect to B

$$\vec{V}_{GB} = \vec{V}_G - \vec{V}_B$$

$$\vec{V}_{AB} = 0 - (-25) \quad [ \because V_G = 0 ]$$

$$= 25 \text{ m/s} \quad \text{Ans}$$

i.e. the ground appears to B to move with a speed of 25 m/s from south to north.

(C) Relative velocity of monkey with respect to the train A

$$\vec{V}_{MA} = -18 \text{ km/h}$$

$$= -18 \times \frac{5}{18} = -5 \text{ m/s}$$

$$\vec{V}_A = 15 \text{ m/s} \quad \text{and} \quad \vec{V}_M = ?$$

$$\vec{V}_{MA} = \vec{V}_M - \vec{V}_A$$

$$-5 = \vec{V}_M - 15$$

$$\text{or } \vec{V}_M = 15 - 5$$

$$\text{or } \vec{V}_M = 10 \text{ m/s} \quad \text{Ans}$$

i.e. the velocity of monkey with respect to ground  $V_M = 10 \text{ m/s}$

Monkey will appear to move from south to north.