Chapter - 3

Date ___/___/___

Current Electricity

Electricity: The phenomenon associated with stationary or moving charges is known as electricity.

Electricity
→ Static Electricity (Generated by rubbing of charges)
→ Current Electricity (Generated by the flow of electrical charges in a conductor across an electric field)

Current Electricity: The study of electric charges in motion is called current electricity

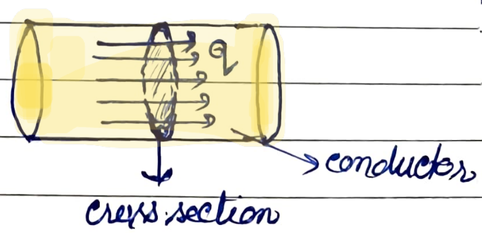
Electric current: Electric current is obtained ~~as~~ as the amount of electric charges flowing through any cross section of a conductor per unit time.

OR

The rate of flow of electric charges through a cross-section of a conductor is called electric current.

Let q charge crosses through a cross-section of a conductor in time t, then electric current

$$I = \frac{q}{t}$$



If n electrons cross through a cross-section in time t, then

$$I = \frac{ne}{t}$$

$e = 1.6 \times 10^{-19} C$ (charge of one electron)

Instantaneous Current: currents are not alway steady. Hence more generally if net charge Δq crosses cross-section area in a time t, then

Instantaneous current $I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$

$$I = \frac{dq}{dt}$$

or $\quad q = \int dq = \int I dt$

SI unit - Ampere (A)

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

$$1A = 1 C s^{-1}$$

One ampere: If one coulomb of charge crosses an area in one second, the the current through that area is one ampere.

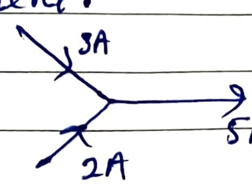
Smaller units -

$1 \, mA = 10^{-3} A$

$1 \, \mu A = 10^{-6} A$

* Current in a domestic appliance = 1A to 15 A
* current carried by a lightning = 10^4 A
* current in our nerves = 10^{-6} A

Electric current is a scalar quantity - Electric current has magnitude and direction both but it is a scalar quantity because laws of vector addition are not applicable to the addition of electric current.



Date ___/___/___

Electric currents in conductors

Carriers of current -

1. In solids:

In metallic conductors, electrons are the charge carriers.

In semiconductors, electrons (for n-type) and holes (for p-type) are the charge carriers

2. In liquids:

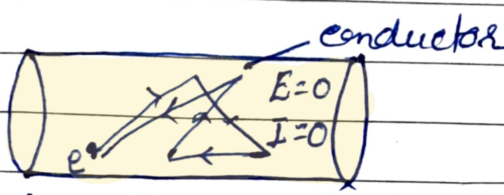
In electrolytic liquids the charge carriers are positively and negatively charged ions.

3. In gases:

In ionised gases positive and negative ions and electrons are the charge carriers.

In metals, atoms are closely packed. The valence electrons of one atom are close to the neighbouring atoms and experience electrical forces due to them. So they do not remain attached to a particular atom and are free to move throughout the lattice. These 'free electrons' are responsible for conduction in metals.

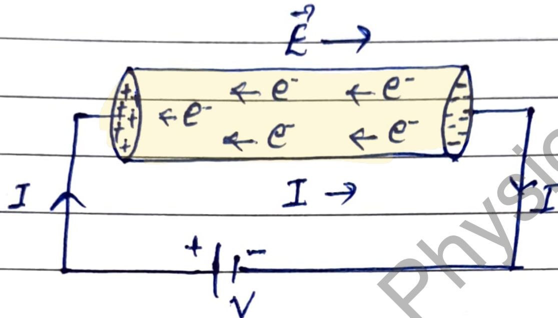
In absence of electric field, free electrons move randomly in the conductors and the net diplacement by them is zero. i.e overall flow of charge is not there. Hence no current flows through the conductor.



* When no electric field is applied, no current flows.

When an electric field is applied, electric charges experience a force. The electrons which are free to move are accelerated towards the positive charge. If electric field is steady there will be a continuous current.

Mechanism, which maintain a steady electric field is called cell or batteries.

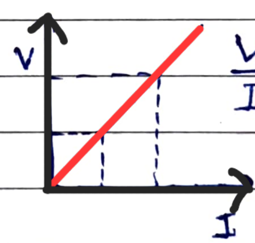


Ohm's Law: According to this law at constant temperature the current through a conductor between two points is directly proportional to the voltage across the two points -

$$V \propto I$$

$$V = RI$$

or $$\frac{V}{I} = R$$



$\frac{V}{I} = R = \text{Constant}$

Where R is the proportionality constant and known as resistance of the conductor.

SI unit - ohm (Ω)

Value of R depends upon -

(I) Nature of the material of the conductor

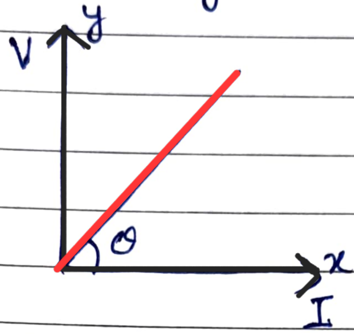
(II) Dimensions of the conductor (length, area)

(III) Temperature of the conductor

Date __/__/__

* It does not depend upon the value of V and I

<u>V-I characteristics</u>: It is represented by a straight line having a constant slope.



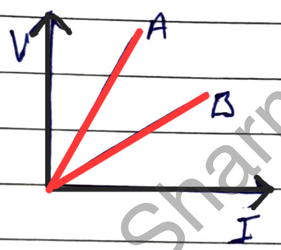
$V = IR$

$y = mx$

here $m = \frac{y}{x} = \frac{V}{I} = \tan\theta$
$= R$

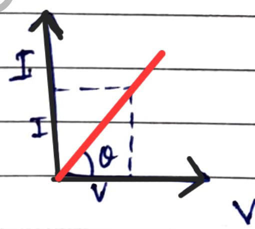
* Higher the slope more the resistance.

e.g



For two conductors A & B as shown in given V-I graph A has more resistance

* For I-V graph. $R = \frac{V}{I} = \cot\theta \Rightarrow \frac{I}{V} = \frac{1}{R}$



For smaller slope, resitance will be higher.

<u>Electrical Resistance</u>: Resistance of a conductor is the opposition offered to the flow of electric charge in the conductor.
It is defined as the ratio of V and I.

$$R = \frac{V}{I}$$

<u>Cause of Resistance</u>: When free electrons flow one end to another end of the conductor, they collide with the ions. Their collision offer resistance to the flow of electron.

Resistance $R \propto l$ (length of the conductor)

and $R \propto \frac{1}{A}$ $A \rightarrow$ Area of cross-section

i.e. $R \propto \frac{l}{A}$

$$\boxed{\begin{array}{l} \text{If } l\uparrow, R\uparrow \\ \text{If } A\uparrow \ R\downarrow \end{array}}$$

or $R = \frac{\rho l}{A}$

Where ρ is the resistivity.

* ρ depends on the material of the conductor but not on its dimensions.

$$\rho = \frac{RA}{l}$$

Resistivity depends on temperature

SI unit - Ω-m

Definition of ρ:

if $A = 1$, $l = 1$, then

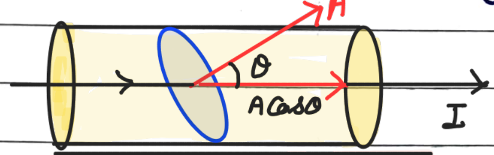
$\rho = R$

i.e The specific resistance of a material is equal to the resistance of a wire with unit cross-section area and unit length.

Current density (j): current per unit area (taken normal to the current) is called current density.

$$j = \frac{I}{A}$$

or $\vec{j} = \frac{I}{\vec{A}}$



$$\boxed{I = \vec{j}\cdot\vec{A} = jA\cos\theta}$$

SI unit - A/m^2. It is a vector quantity.

Date ___/___/___

Vector form of Ohm's law:

We know,

Resistance, $R = \frac{\rho l}{A}$

$\rho \rightarrow$ Resistivity
$l \rightarrow$ length of wire
$A \rightarrow$ Area of cross section

or $IR = \frac{I \rho l}{A}$

or $\frac{V}{l} = j\rho$ $\quad [\because \frac{I}{A} = j]$

or $E = j\rho$ $\quad [\because E = \frac{V}{l}]$

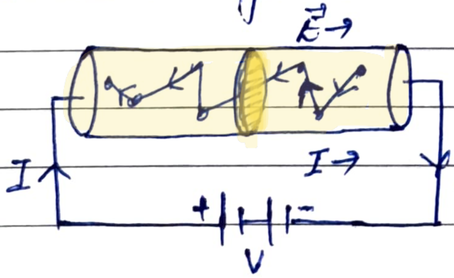
or $\vec{E} = \vec{j}\rho$

or $\vec{j} = \sigma \vec{E}$ $\quad [\because \frac{1}{\rho} = \sigma]$

where $\sigma = \frac{1}{\rho}$ is called conductivity (Reciprocal of resistivity)

This relation $\vec{j} = \sigma \vec{E}$ is known as vector form of ohm's law.

Drift velocity: It is defined as the average velocity with which free electrons get drifted towards the +ve terminal under the effect of applied electric field.



In absence of electric field the average thermal velocity

$$u_{av} = \frac{u_1 + u_2 + \cdots u_n}{n} = 0$$

$$\boxed{u_{av} = 0}$$

While electric field is applied free electrons are accelerated in the opposite direction of electric field.

acceleration $a = \frac{F}{m}$

$$a = \frac{eE}{m} \qquad [\because F = qE = eE]$$

· The electrons accelerate for an average time interval τ (relaxation time)
(average time b/w successive collisions)

Therefore the drift velocity

$$v_d = u_{av} + a\tau$$

$$v_d = 0 + \left(\frac{eE}{m}\right)\tau$$

or $$v_d = \frac{eE}{m}\tau$$

vector form $$\vec{v}_d = \left(-\frac{e\vec{E}}{m}\right)\tau$$

−ve sign shows that electrons are drifted in the opposite direction of electric field.

* The equation of drift velocity tells us that electrons move with an average velocity which is independent of time although electrons are accelerated.

* <u>Relaxation time (τ)</u> - The small interval of time b/w two successive collisions of electron and ion.

* The motion of electrons is uniformly accelerated motion b/w two successive collisions.

* The drift speed of electrons is much smaller about 10^{-5} times of the thermal speed at ordinary temperature.

Mobility (μ): Mobility of a current carrier is defined as the magnitude of the drift velocity per unit electric field.

$$\text{Mobility } \mu = \frac{v_d}{E}$$

[* Mobility is how quickly electrons pass through a conductor]

$$= \left(\frac{eE\tau}{m}\right) \cdot \frac{1}{E}$$

$$\text{or} \quad \mu = \left(\frac{e}{m}\right)\tau$$

For an electron $\mu_e = \frac{e\tau_e}{m_e}$

For an hole $\mu_h = \frac{e\tau_h}{m_h}$

For both, electron and hole μ is positive.

SI unit - m^2/Vs. It is scalar quantity

Practical unit - $cm^2V^{-1}s^{-1}$

* $\mu \propto \tau$

and $\mu \propto \frac{1}{m}$

Relation b/w I and μ

We have

$$I = neAv_d$$

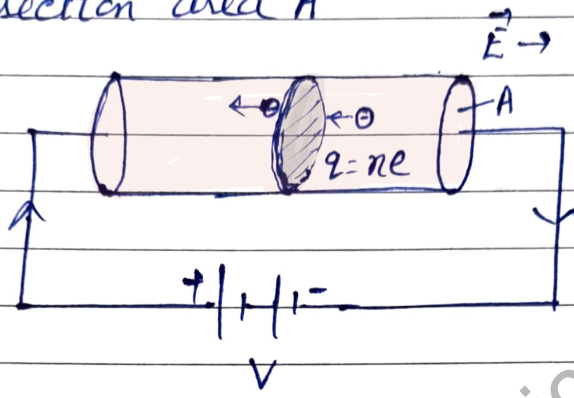
$$\text{or} \quad \boxed{I = neA\mu E} \qquad \left[\because \mu = \frac{v_d}{E}\right]$$

* conductivity arises due to mobile charge carriers

Date ___/___/_____

Relation between drift velocity and electric current:

Consider a conductor of length l and uniform cross section area A



Let V be the applied potential difference the electric field $E = \frac{V}{l}$

Now the total charge in the conductor,

$q = (nAl)e$ [Al = Volume of conductor]

Time taken by charge q to cross the conductor

$t = \frac{l}{v_d}$ [$\because$ time = $\frac{\text{distance}}{\text{speed}}$]

By $I = \frac{q}{t} = \frac{(nAl)e}{l/v_d}$

or $\boxed{I = neAv_d}$

$I \propto v_d$ (as n, e and A are constant)

This is the relation b/w I and v_d.

* $v_d \propto \frac{I}{A} \Rightarrow v_d \propto j$ (current density), $v_d \propto \frac{1}{\text{Temp}}$
* v_d is not dependent on cross-sectional area or length of the wire.
* For a constant electric field, v_d is dependent on μ. (mobility)

Deduction of Ohm's law

By $I = neAv_d$

$$\frac{I}{A} = nev_d$$

$$j = nev_d \qquad \left[\because \frac{I}{A} = j \text{ (current density)}\right]$$

or $$j = ne\left(\frac{eE}{m}\tau\right) \qquad \left[\because v_d = \frac{eE}{m}\tau\right]$$

or $$j = \left(\frac{ne^2}{m}\tau\right)E \quad \text{--- (1)}$$

On comparing this relation with $j = \sigma E$ we see it is exactly the Ohm's law, where

$$\text{conductivity } \sigma = \frac{ne^2}{m}\tau$$

Hence eq^n (1) represent the Ohm's law.

* $\sigma \propto \tau$
* $\rho \propto \frac{1}{\tau}$ $\quad \left[\because \text{Resistivity} = \frac{1}{\text{conductivity}}\right]$

i.e if no. of collisions increases, relaxation time τ decreases and hence conductivit σ also decreases, or resistivity ρ increases.

Effect of temperature on the resistance and resistivity-

In a conductor resistance is offered due to the collisions b/w the electrons and ions. When temperature $\uparrow$, collisions become more frequent and resistance also increases.

Date ___/___/___

i.e $R \uparrow$ with increase in temperature T.
and $R \downarrow$ with decrease in temp.

For a conductor, change in resistance

$(R_T - R_0) \propto R_0$ $\quad$ $R_0 \rightarrow$ Resistance at 0°C
and $(R_T - R_0) \propto \Delta T$ $\quad$ $R_T \rightarrow$ Resistance at T°C

i.e $(R_T - R_0) \propto R_0 \Delta T$

or $(R_T - R_0) = \alpha R_0 \Delta T$

Where α is the temperature coefficient of resistance.

also $\boxed{R_T = R_0(1 + \alpha \Delta T)}$ —(1)

or $R_T = R_0[1 + \alpha(T - T_0)]$

and $\boxed{\alpha = \frac{R_T - R_0}{R_0 \Delta T}}$ —(2)

Temperature coefficient: It is defined as the change in resistance per unit original resistance per degree rise in temperature.

SI unit - K^{-1} or $°C^{-1}$

Dimension - $[T^{-1}]$

We know that $R = \frac{\rho l}{A}$

if we ignore the change in $\frac{l}{A}$ on changing temp.
then, $R = \rho$

and eqn (1) and (2) becomes

$\rho = \rho_0(1 + \alpha \Delta T)$

$$\alpha = \frac{\rho_T - \rho_0}{\rho_0 \Delta T} = \frac{\Delta \rho}{\rho_0 \Delta T}$$

For unit change in temp. ($\Delta T = 1$)

$$\boxed{\alpha = \frac{\Delta \rho}{\rho_0}}$$

* Over a limited range of temp. the resistivity is given by

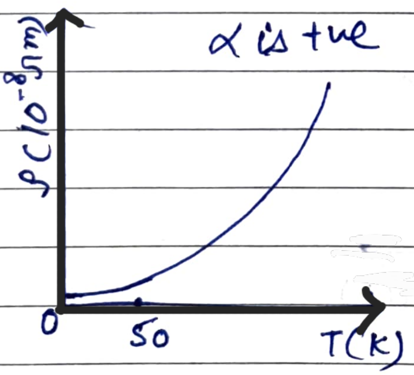
$$\rho = \rho_0 (1 + \alpha \Delta T)$$

and the graph b/w ρ and T is approximately a straight line for metals

* A temp. much lower than 0°C the graph deviates from a straight line.

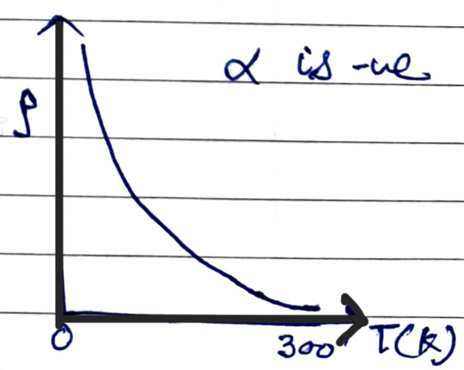
* α is +ve for metallic conductors. (R↑ if T↑)

ρ-T graph for copper (Metals)



The variation of resistivity of copper with temp. is parabolic in nature.

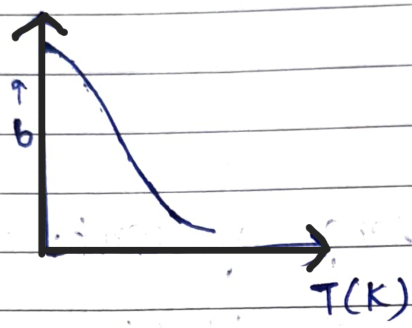
ρ-T graph for semiconductor



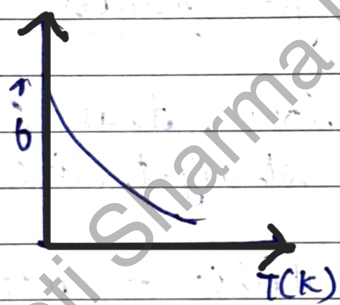
α is -ve means if temp increases, resistance decreases.

Conductivity and temperature graph
(σ - t graph)

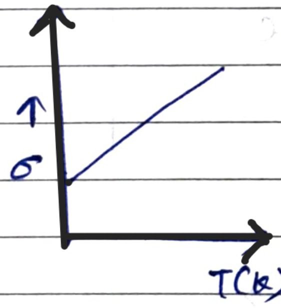
(i) For metals



(ii) For alloys

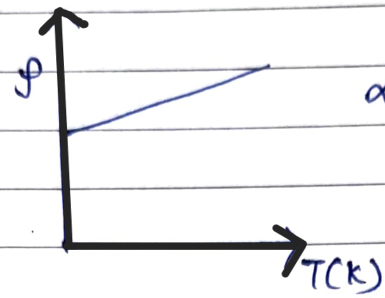


(iii) For semiconductor



Date ___/___/___

ρ - T graph for Alloy (nichrome)



α is +ve and very small

For alloys α is very small +ve value.
i.e Resistance does not change appreciably with change in temp.
Therefore alloys are used for making standard resistance.

* Dependence of ρ upon T

We know

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

where $n \rightarrow$ free electrons per unit volume
$\tau \rightarrow$ Relaxation time

here $\rho \propto \frac{1}{n}$ and $\rho \propto \frac{1}{\tau}$

In metals n is independent of temperature.
Thus $\uparrow$ in temp, $\downarrow$ in τ and hence $\uparrow$ in ρ

In insulators and semiconductors
n increases with temp which is much more than decrease in τ. ($T\uparrow$, $\rho\downarrow$ or $\sigma\uparrow$)

* **Superconductivity:** The complete loss of resistivity of certain metal or alloy below a certain temp. is called super conductivity. Mercury behaves like a super conductor ~~below 4.2K~~ below 4.2 K.

Date __/__/__

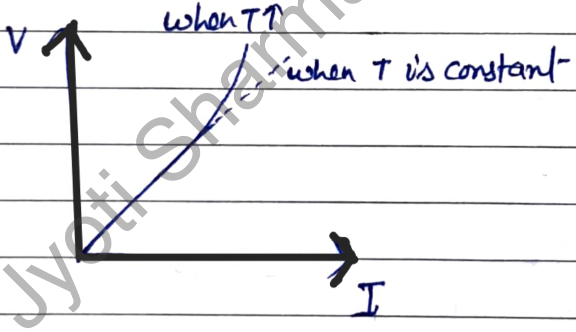
Limitations of Ohm's law

Ohm's law is not considered to be a fundametal law. It is possible that some materials may not strictly follow the Ohm's law. i.e $V \propto I$ does not hold.

Examples -

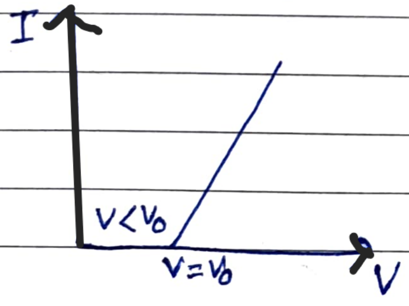
(I) Change in the nature of material occurs with passage of current (non linear behaviour between V and I)

When current ↑, temperature ↑ and temp. does not remain same as per Ohm's law condition. In this situation conductors show non linear behaviour.



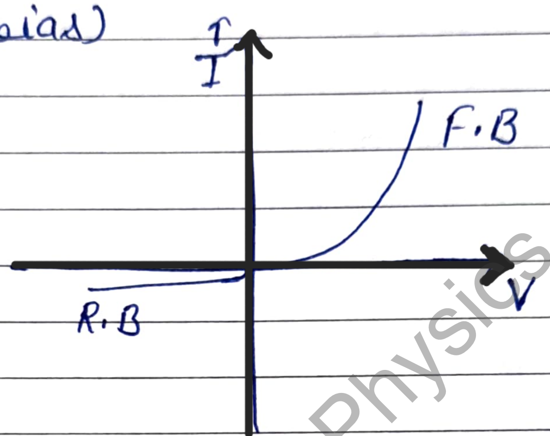
(II) Measurable current start flowing only at a perticular potential difference.

In water valtmeter current begins to flow when $V = V_0$. When $V < V_0$ current does not flow even potential difference is present, which is not according to the Ohm's law.

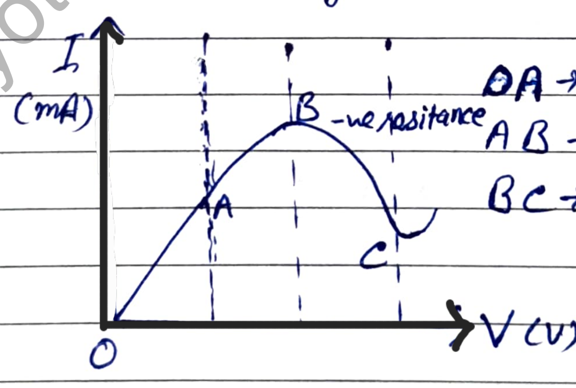


Date __/__/__

(III) V-I relationship varies with change of polarity of applied voltage.
A diode offers less resistance to a perticular polarity of battery (Forward bias) but offers very high resistance when polarity is reversed (Reverse bias)



(IV) Decrease in the current is possible with increase in voltage.
Graph shows the variation of current with voltage for GaAs. The curve BC shows that current I ↓ when voltage V increases.



OA → linear
AB → non-linear region
BC → -ve resistance region

In the graph there is more than one value of V for the same I

Date __/__/__

Resistivity of various materials:

Materials are classified as conductors, semiconductors and insulators depending on their resistivities.
Metals have low resistivities in the range of $10^{-8}\Omega m$ to $10^{-6}\Omega m$.
Insulators like ceramic, rubber and plastics have resistivities 10^{18} times greater than metals i.e in the range of $10^{10}\Omega m$ or more.
Resistivity of semiconductor lies between conductors and insulators but can be decreased by adding small amount of suitable impurities.
* (Table 3.1 in NCERT shows resistivities of some materials)

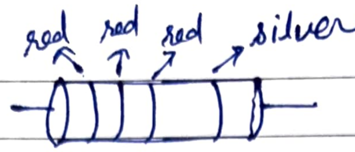
Not in new syllabus * **Resistors:** Commercially produced resistors are of two major types -
(i) Wire bound resistors (made of alloy)
(ii) carbon resistors (made of carbon)

Resistors in higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus are used widely in electronic circuits.

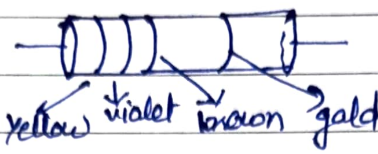
Resistors colour codes

Colour	Number	Multiplier	Tolerence (%)
Black	0	1	
Brown	1	10	5% for gold
Red	2	10^2	10% for silver
Orange	3	10^3	20% for no colour
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	

Date ___/___/___

e.g (i) 

$= (22 \times 10^2 \Omega) \pm 10\%$

(ii) 

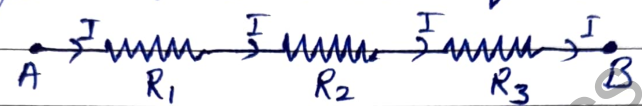
$= (47 \times 10 \Omega) \pm 5\%$

Not in new syllabus *

Combination of Resistors

1. In series

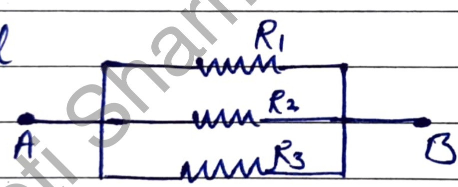
(i) Equivalent resistance $R_{eq} = R_1 + R_2 + R_3$



(ii) Current through each resistor is same

(iii) Sum of potential difference across individual resistors is equal to the potential difference applied by the source.

2. In parallel



(i) Equivalent resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

(ii) Potential difference across each resistor is same.

(iii) Sum of electric currents flowing through the individual resistors is equal to the electrical current drawn from the source.

* In series combination, net resistance increases therefore current in the circuit decreases.
* In parallel combination net resistance decreases, therefore current in the circuit increases.

* For n identical resistors –
  (I) Connected in series

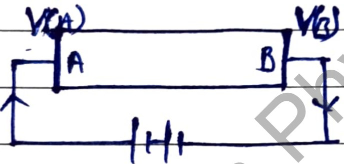
$$R_{eq} = nR$$

  (II) connected in parallel

$$R_{eq} = \frac{R}{n}$$

Electrical Energy and Power

Consider a conductor with end points A and B as shown in fig



$V_A > V_B$

Let in time interval Δt an amount of charge $\Delta Q = I\Delta t$ travels from A to B.
The change in P.E b/w A and B

$$\Delta U = \Delta Q\,[V(B) - V(A)]$$
$$= -\Delta Q\,V$$
$$= -(I\Delta t)V$$

or $\Delta U = -IV\Delta t < 0$

If charges move without collisions through the conductor, the by the conservation of energy

$$\Delta K = -\Delta U$$

or $\Delta K = IV\Delta t > 0$ $\quad [\because \Delta U = -IV\Delta t]$

* Thus in case charges were moving without collision through the conductor under the action of electric field, their K.E would increase as they move.
* But on the average, charge carriers do ~~not~~ move with acceleration, they move with steady drift velocity.

* During collisions, the energy gained by the charges thus is shared with the atoms. The atoms vibrate more vigorously, i.e the conductor heats up.
* Therefore in an actual conductor, the amount of energy dissipated as heat in the conductor..

$$\Delta W = IV\Delta t$$

The energy dissipated per unit time is the power dissipated

$$P = \frac{\Delta W}{\Delta t}$$

or $$P = \frac{IV\Delta t}{\Delta t}$$

or $$\boxed{P = IV = VI}$$

This is the power dissipated.

By using Ohm's law $V = IR$, we get

$$P = I^2R = \frac{V^2}{R}$$

* (Power dissipated → heat, light, sound etc.)
* Power dissipated depends on the current and voltage.

Electrical Energy: The energy generated by the movement of electrons from one point to another.

Formula $\boxed{W = VI\Delta t}$ $\boxed{W = VQ}$

Units - SI unit is joule or watt second. Board of Trade

Commercial unit - kilowatt hour (KWh) or (B.O.T)

$1\ KWh = 3.6 \times 10^6$ joule or WS or one unit

Date ___/___/___

Examples of electrical energy- fan, bulb etc.

Electric Power: It is the rate at which work is done or energy is transformed in an electric circuit.

OR

The rate of transfer of electrical energy by an electric circuit per unit time is called electric power.

* Power and energy are scalar units.

* The electrical power comes from an external source like a cell in the circuit.

Power Transmission: Electric power is transmitted from power station to homes and factories via transmission cables.

There is a power loss during the transmission. Let the connecting wires from the power station to the device has a finite resistance R_c.

The power dissipated in the connecting wires

$$P_c = I^2 R_c$$

$$= \left(\frac{P}{V}\right)^2 R_c \qquad [\because P = VI]$$

$$= \frac{P^2 R_c}{V^2}$$

i.e the power wasted $P_c \propto \frac{1}{V^2}$

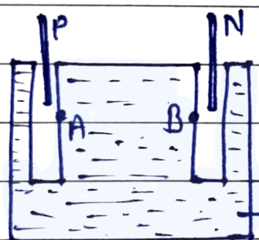
Therefore to reduce P_c the cable wires carry current at enormous value of V. Due to this, high voltage danger sign is used on transmission

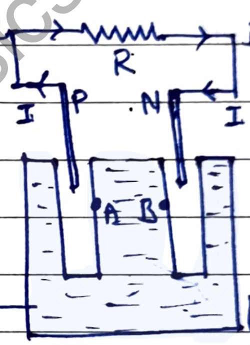
Date __/__/__

Cells, EMF, Internal Resistance

cells, emf and internal resistance are the components which complete the circuit and help the flow of electricity within the circuit. These are inter-related to one another. Batteries i.e cells are posses internal resistance, and potential difference.

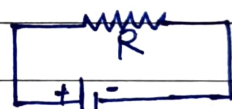
Cell: Cell is a simple device to maintain a ~~steady~~ steady current in an electrical circuit. Basically a cell has two electrodes - called positive(P) and negative(N)





(open circuit) electrolyte (closed circuit)

Electrolytic cell



symbol

Positive electrode has p.d V_+ and negative electrode has p.d V_-

$$\mathcal{E} = V_+ - (-V_-) = V_+ + V_-$$

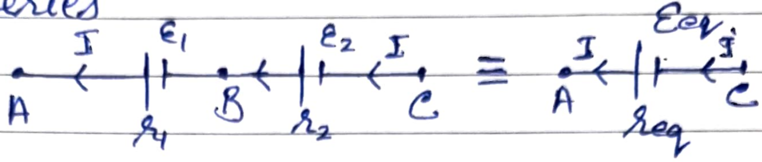
This difference is called the electromotive force (emf) of the cell ($\mathcal{E}$).

EMF: is the potential difference between the positive and negative electrodes in an open circuit i.e. when no current is flowing.

Internal Resistance(r) Internal resistance is the resistance offered by the electrolyte and electrodes when current flows.

Cells in series and parallel

(1) Cells in series



Consider two cells in series as shown in fig.

Now

$$V_{AB} = \varepsilon_1 - I r_1$$

and $$V_{BC} = \varepsilon_2 - I r_2$$

hence,

$$V_{AC} = V_{AB} + V_{BC}$$
$$= \varepsilon_1 - I r_1 + \varepsilon_2 - I r_2$$
$$V_{AC} = \varepsilon_1 + \varepsilon_2 - I(r_1 + r_2)$$

On comparing with

$$V_{AC} = \varepsilon_{eq} - I r_{eq}$$

we get

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$

and $$r_{eq} = r_1 + r_2$$

Similarly for n cells

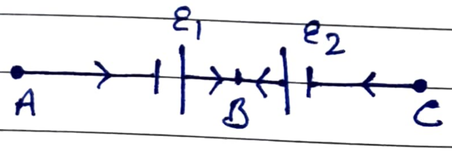
$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \cdots \varepsilon_n}$$

and $$\boxed{r_{eq} = r_1 + r_2 + r_3 + \cdots r_n}$$

* For n identical cells of emf ε each

$$\left[\begin{array}{l} \varepsilon_{eq} = n\varepsilon \\ r_{eq} = nr \end{array}\right]$$

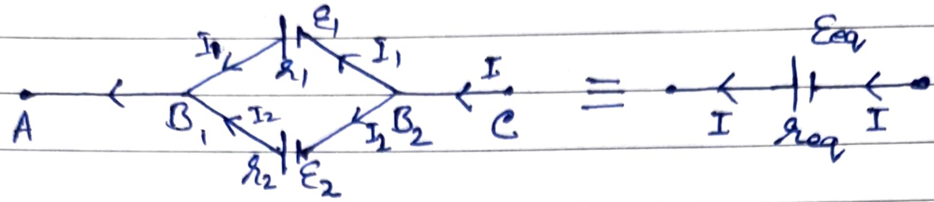
* If two negative electrodes are connected together then $\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2$ [for $\varepsilon_1 > \varepsilon_2$]



$\varepsilon_1 > \varepsilon_2$

Date ___/___/___

(ii) Cell in parallel



In parallel combination I_1 and I_2 are the currents leaving the positive electrodes of the cell as shown in fig.

We have $I = I_1 + I_2$

$$= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

or

$$I = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

or

$$V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$

or

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

On comparing with $V = \varepsilon_{eq} - I r_{eq}$, we get

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \Rightarrow \frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

and

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \Rightarrow \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

For n cells

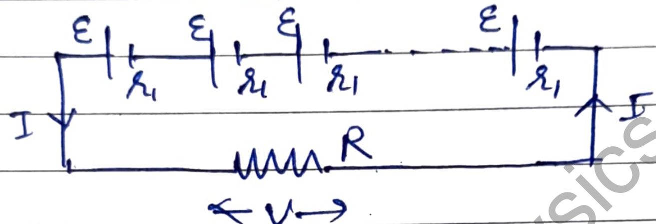
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \cdots \frac{\varepsilon_n}{r_n}$$

Date __/__/__

and $\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots \frac{1}{r_n}$

* $\frac{\varepsilon_{eq}}{r_{eq}} = \frac{n\varepsilon}{r}$ and $\frac{1}{r_{eq}} = \frac{n}{r}$ [for n identical cells]

* for n identical cells in series with external R



- total emf $= n\varepsilon$
- total internal resistance $= nr$

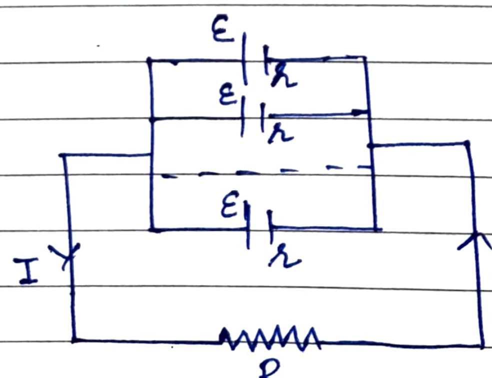
and by $I = \frac{\varepsilon}{R+r}$

$$I = \frac{n\varepsilon}{R+nr}$$

(i) If $R << nr$, $I = \frac{n\varepsilon}{nr} \Rightarrow \boxed{I = \frac{\varepsilon}{r}}$

(ii) If $R >> nr$, $\boxed{I = \frac{n\varepsilon}{R}}$

* for n identical cells in ~~series~~ parallel



- $r_{eq} = \frac{r}{n}$
- emf of the battery is same as of a single cell
- current divided equally

Date __/__/__

Total emf $= \varepsilon$

Total resistance $= \frac{r}{n} + R$

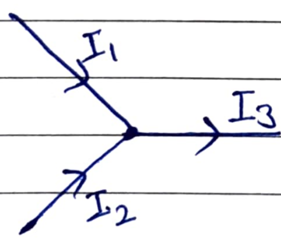
(i) If $R \ll \frac{r}{n}$ then $I = \frac{n\varepsilon}{r}$ $\quad \left[\because I = \frac{\varepsilon}{R + \frac{r}{n}}\right]$

(ii) If $\frac{r}{n} \ll R$ then $I = \frac{\varepsilon}{R}$

Kirchhoff's Rules: Kirchhoff rules are used to solve complex electric circuit. Formulae of series, parallel combination are not always sufficient to determine currents and potential difference in the circuit. The two Kirchhoff's rules are used in such cases.

1. Junction Rule / Current Rule

At any junction, the sum of currents entering the junction is equal to the sum of current leaving the junction. ie $\Sigma I = 0$



for given fig. $I_1 + I_2 = I_3$

2. Loop Rule / Voltage Rule

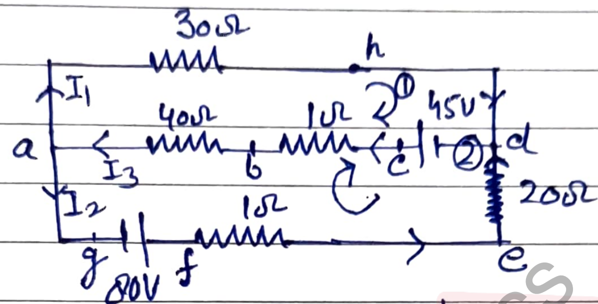
In any closed loop of electric circuit the algebraic sum of emfs of cells and product of current and resistances is always equal to zero.

ie $\Sigma \varepsilon + \Sigma IR = 0$

OR

The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. i.e $\Sigma \Delta V = 0$

example -



Steps to apply the rules

(i) Choose the loop

(ii) Assume any direction (CW/ACW)

(iii) IR is -ve in the dirⁿ of current and vice-versa.

(iv) →|⊢→ -ve Emf ε

←|⊢← +ve emf ε

* IR is -ve in your assumed dirⁿ of loop (CW/ACW) and +ve in the opposite dir

In loop ① ahdcba

$$-30I_1 + 45 - (1+40)I_3 = 0$$

or $-30I_1 + 45 - 41I_3 = 0$

or $30I_1 + 41I_3 = 45$ —(1)

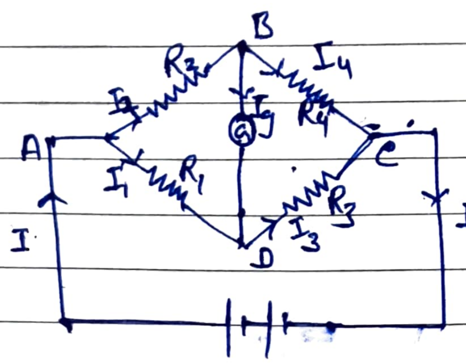
In loop ② ahdefga

$$-30I_1 + 21I_2 - 80 = 0$$

or $-30I_1 + 21I_2 = 80$ —(2)

Wheatstone Bridge: (Application of Kirchhoff's Rule)

The circuit shown in fig is called 'Wheatstone bridge'

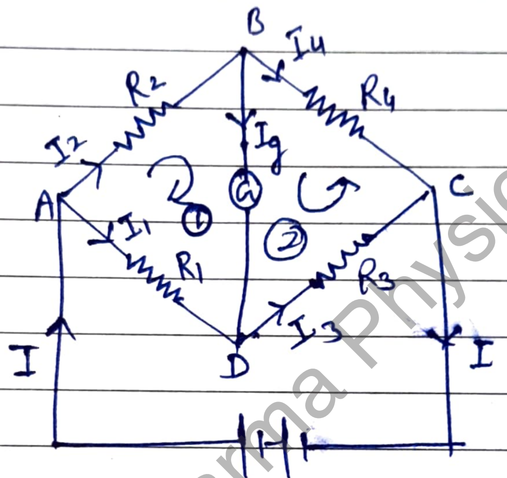


* R_1, R_2, R_3 and R_4 are the four resistors
* Across AC battery is connected
* Between BD, galvanometer is connected

Neglect internal resistance of the battery.

Balanced Bridge: In case of balanced bridge $V_B = V_D$.
i.e $I_g = 0$ (Galvanometer shows zero reading)

Condition for 'Balanced Bridge'-



When bridge is balanced no current flows through arm B.D. i.e $I_g = 0$, which gives

$$I_2 = I_4 \quad \text{and} \quad I_1 = I_3$$

Now in loop ① ABDA, apply Kirchhoff's rule

$$-I_2R_2 - I_g + I_1R_1 = 0$$

or $\quad I_1R_1 = I_2R_2 \qquad [\because I_g = 0]$

or $\quad \dfrac{I_1}{I_2} = \dfrac{R_2}{R_1} \qquad -(1)$

Now in loop ② BDCB, by Kirchhoff's rule we get

$$-I_g - I_3R_3 + I_4R_4 = 0$$

or $\quad I_3R_3 = I_4R_4 \qquad [\because I_g = 0]$

or $\quad \dfrac{I_3}{I_4} = \dfrac{R_4}{R_3}$

but $I_3 = I_1$ and $I_4 = I_2$, then

$$\frac{I_1}{I_2} = \frac{R_4}{R_3} \quad \text{--- (2)}$$

from eq^n (1) and (2)

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

or $$\boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

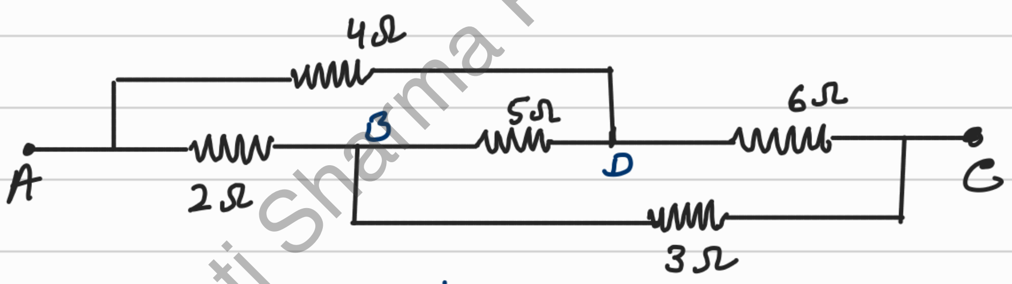
or $$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

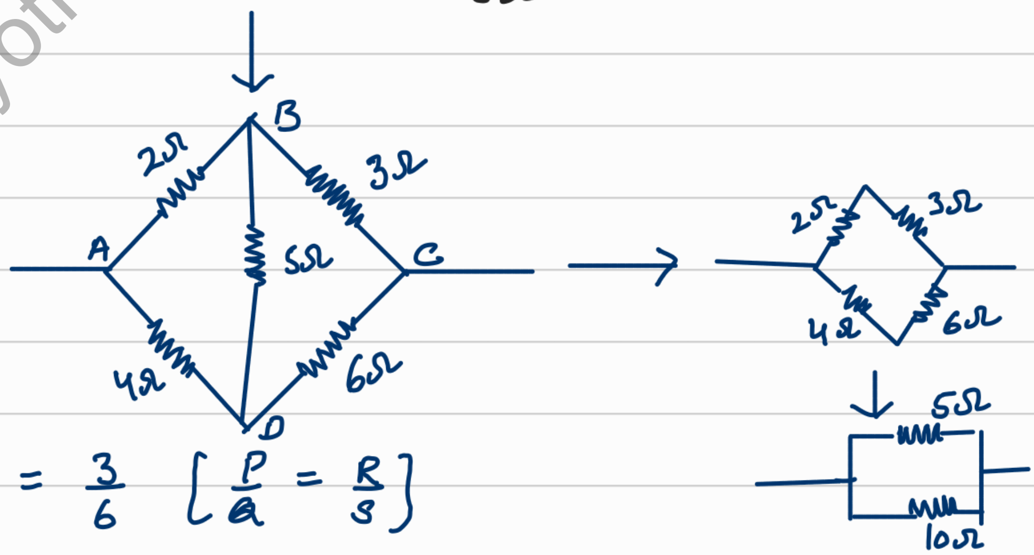
[If $R_1 \to P$, $R_2 \to Q$, $R_3 \to R$ and $R_4 \to S$

This is the condition for balanced bridge.

e.g.

To find the equivalent resistance of given circuit





Since $\frac{2}{4} = \frac{3}{6}$ $\left[\frac{P}{Q} = \frac{R}{S}\right]$

The circuit is balanced Wheatstone bridge,

ie $V_B = V_D$, so no current in 5Ω resistance. Then 2Ω and 3Ω are in series, also 4Ω and 6Ω in series

so $R_{eq} = \frac{5 \times 10}{5+10} = \frac{50^{10}}{15_3} = 3.33\Omega$ Ans