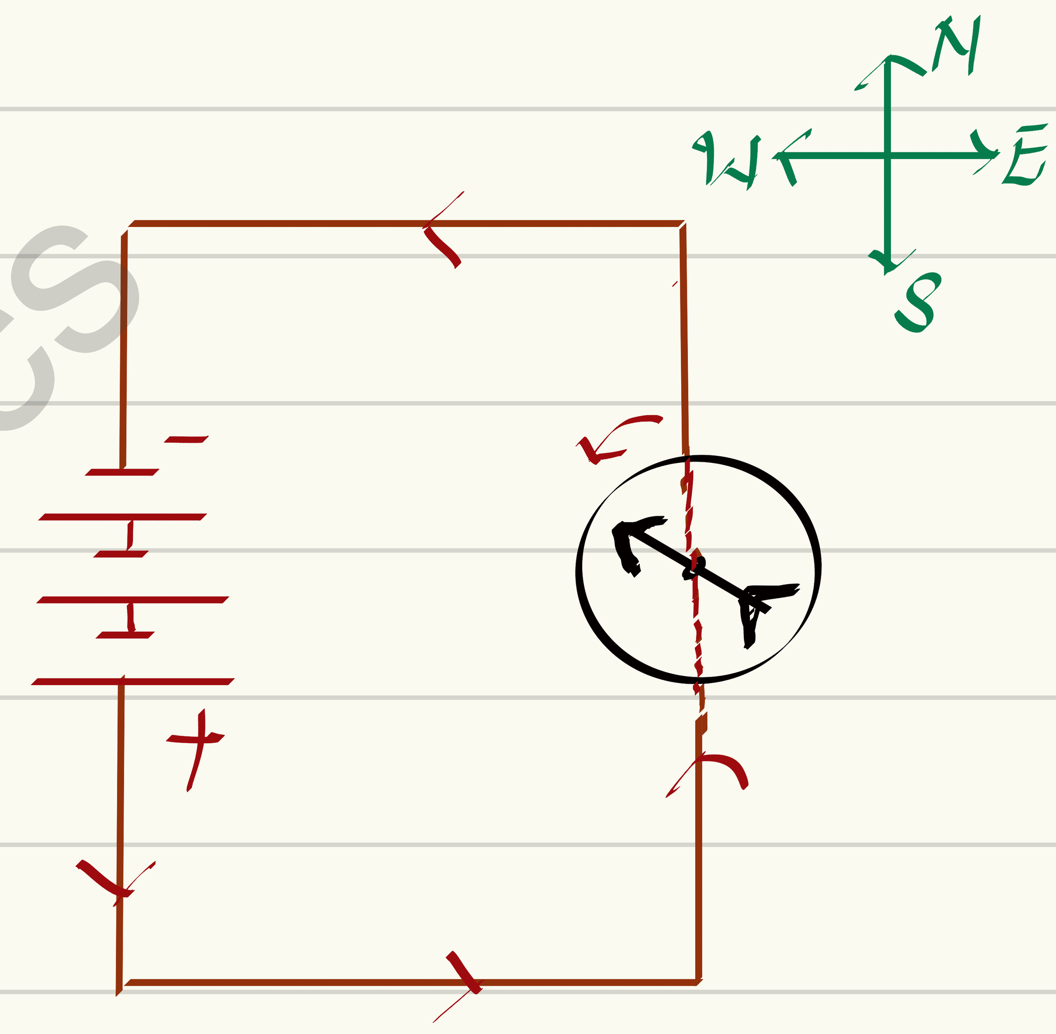
Moving Charges And Magnetism

Electricity and magnetism were known for more than 2000 years, but their relation was discovered only in 1820 by Hans Christian Oersted.

Oersted Experiment:

Oersted performed a simple experiment and observed that a current carrying wire deflects a magnetic compass, proving that electric current produces magnetic field.



Observations:

* Magnetic needle placed near a a current carrying wire shows deflection.
* On reversing the direction of current the direction of deflection also gets reversed
* The deflection angle increases with increase in current.

So he concluded that electric current in a conductor produces magnetic effect in the space around the conductor. i.e.

"Flow of electric charge is the source of magnetic field".

* Further research led to James Maxwell's equations (1864), unifying electricity and magnetism, leading to the discovery of radio waves.

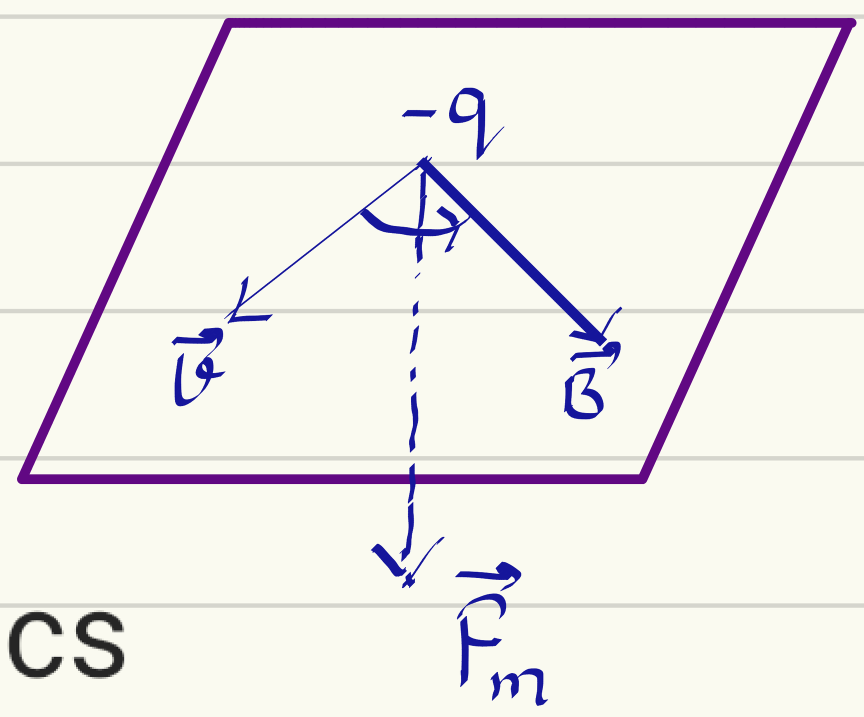
Magnetic Force: When a charge q moving with velocity v, enters in a magnetic field B at angle θ, then charge experiences a force. This force is called magnetic force.

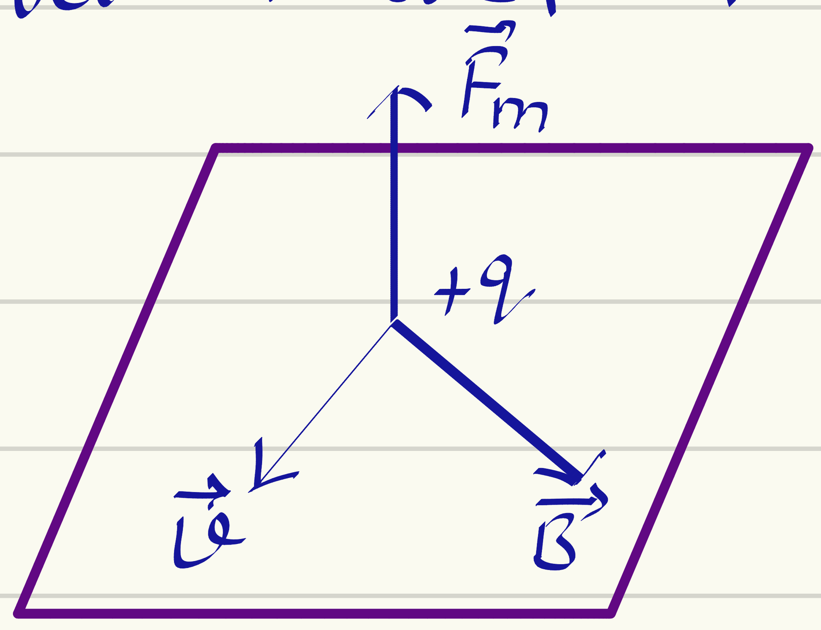
$$\boxed{\vec{F} = q(\vec{v} \times \vec{B})}$$

Magnitude of this force

$$\boxed{F = qvB\sin\theta}$$

Direction of F is ⊥ to the plane of $\vec{v}$ and $\vec{B}$ and can be found by right hand thumb rule.





<u>Special cases:</u>

(I) If charge q moves parallel to the magnetic field i.e $\theta = 0^\circ$ by $F = qvB\sin\theta$

$F = qvB\sin 0^\circ$

$\boxed{F = 0}$ No force is experienced by the charge.

(II) If charge q moves antiparall to the magnetic field. i.e $\theta = 180^\circ$

$F = qvB\sin 180^\circ$

$\boxed{F = 0}$ No force is experienced by the charge.

(III) If charge enters perpendicularly to the magnetic field, $\theta = 90^\circ$

$F = qvB\sin 90^\circ$

$\boxed{F = qvB}$ Maximum force is experience.

(IV) If charge particle is at rest i.e $v = 0$, then

$\boxed{F = 0}$

Charge at rest does not experience the mag. force.

<u>Magnetic Field:</u> It is the space around a magnetic material or a moving charge where magnetic forces can be detected.

By $F = qvB\sin\theta$

$$B = \frac{F}{qv\sin\theta}$$

Meaning of $\otimes$ & $\odot$
$\otimes \rightarrow$ dir^n is into the page
$\odot \rightarrow$ dir^n is out of the page

Mathematically, the magnetic field B is defined as the force per unit charge per unit velocity experienced by a moving a charge in the field.

It is a vector quantity.

SI unit - Tesla = $N m^{-1} A^{-1}$

CGS unit - Gauss

$1 \text{ Gauss} = 10^{-4} \text{ Tesla (T)}$

* Earth magnetic field is about 3.6×10^{-5} T
* Force experienced by the charge particle in electric field is independent of its speed.
* A charge at rest is a source of electric field only but a moving charge is the source of electric as well as magnetic field.
* Electric force accelerates or retards a charge.

__Lorentz Force:__ Lorentz force is the total force experienced by a charged particle moving in an electric field and a magnetic field.

It is given by

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$F_e \rightarrow$ Electric force
$F_m \rightarrow$ Magnetic force

here $\vec{F}_e = q\vec{E}$, $\vec{F}_m = q(\vec{v} \times \vec{B})$

then,

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

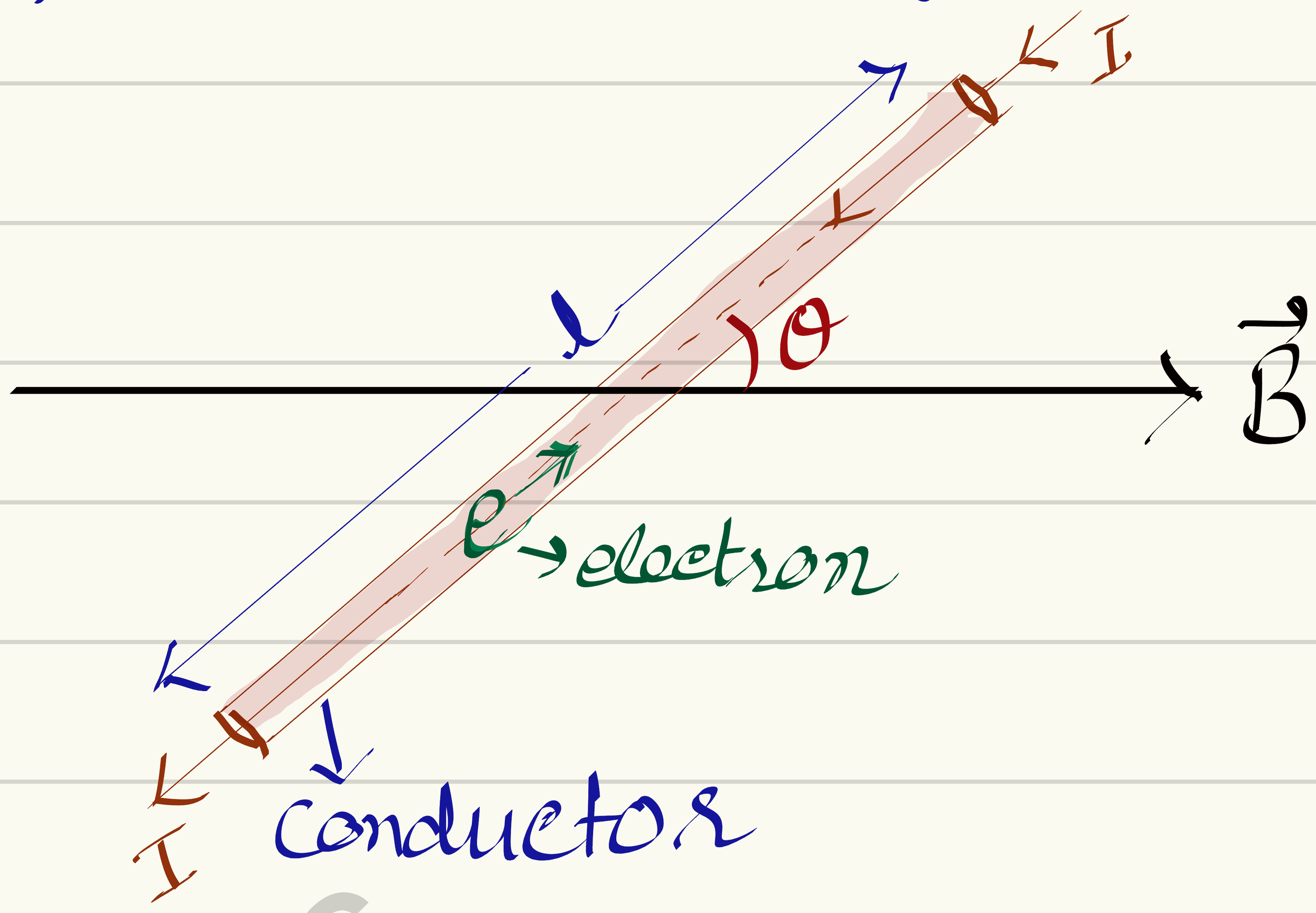
$$\boxed{\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]}$$

* Magnetic force $\vec{F}_m$ does no work on a charged particle when the magnetic field is steady or uniform because $F_m \perp v$, means no component of force in the direction of motion, so it does no work.

Magnetic force on a current carrying conductor Place in uniform magnetic field:

Consider a conductor of length 'l' carrying a current I placed in a uniform magnetic field $\vec{B}$.

Let us consider a conductor of length 'l' and area of cross section A placed at an angle θ to the direction of magnetic field as shown in fig.



The total number of free electrons in the conductor

$N = n \times$ volume of conductor.

$n \rightarrow$ electron density (free electrons per unit volume)

so $N = nAl$ [volume $= Al$] —— (1)

Force on each electron

$$\vec{f} = e(\vec{v}_d \times \vec{B}) \quad [\because \vec{F} = q(\vec{v} \times \vec{B})]$$

here v_d is drift velocity of free electrons.

$$f = e v_d B \sin\theta \quad \text{—— (2)}$$

Now total force experienced by all drifted electrons

$$F = N \times f$$

$$= nAl \times e v_d B \sin\theta \quad [\text{from (1) and (2)}]$$

$$= (neAv_d)\, l B \sin\theta$$

$$\boxed{F = I l B \sin\theta} \quad [\because I = neAv_d]$$

or $\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$ θ is angle b/w $\vec{l}$ and $\vec{B}$

Dir^n of $\vec{l}$ is in the dir^n of current I

* The direction of force is always ⊥ to the plane containing $(\vec{l} \times \vec{B})$. It can be found by using Fleming's left hand rule.

* Special cases!

(1) If $\theta = 0°$ or $180°$, $\sin\theta = 0$, $F = 0$

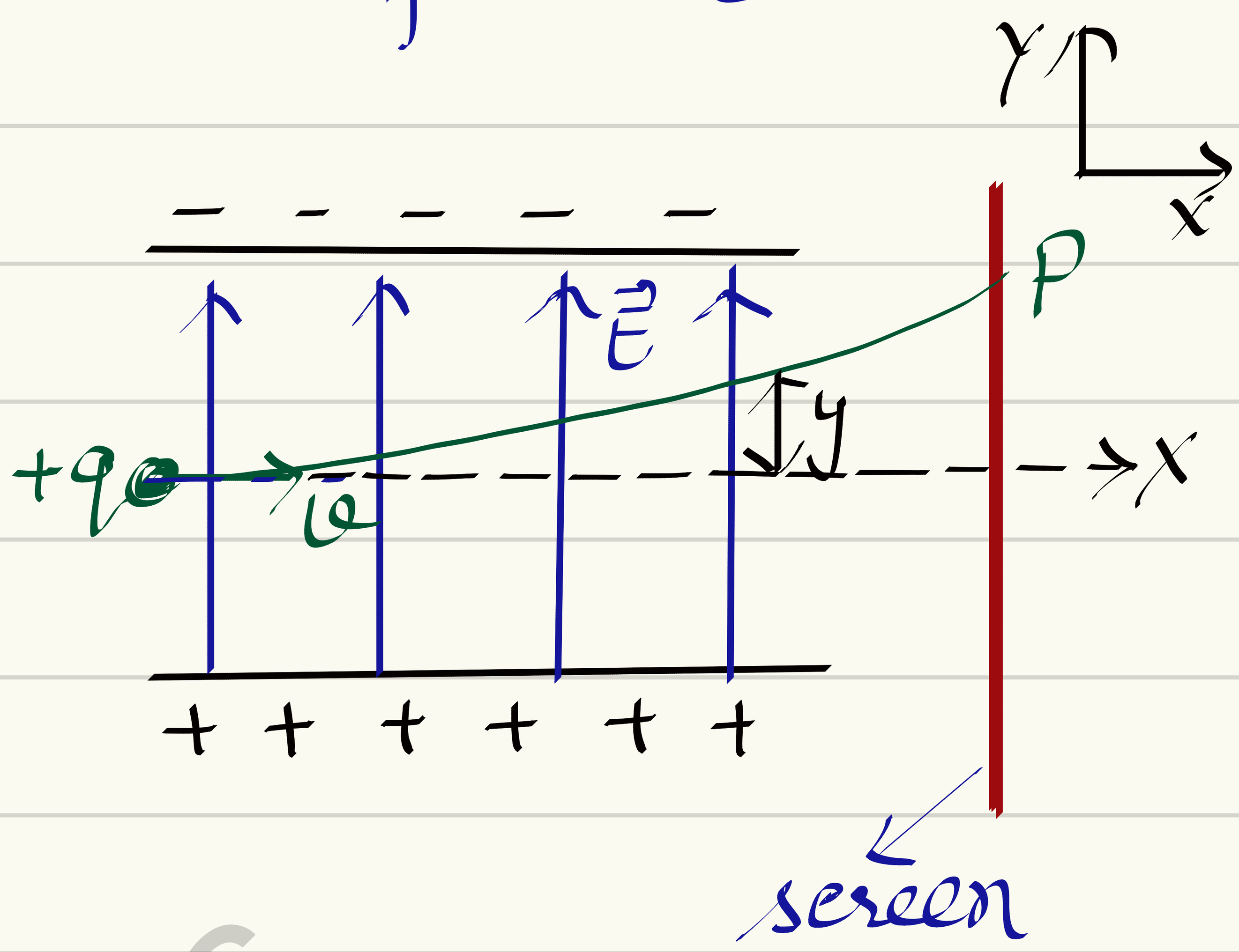
Thus current carrying conductor placed parallel and antiparallel to the direction of magnetic field does not experience any force.

(ii) If $\theta = 90^\circ$, $F = IlB$, Maximum force is experienced by the conductor.

Charged particle moving in uniform Electric field:

Force on a charge '+q' due to electric field E

$F = qE$



Now acceleration produced

$$a = \frac{F}{m} = \frac{qE}{m}$$

charge particle will accelerate in the direction of electric field $\vec{E}$.

By $s = ut + \frac{1}{2}at^2$

Along x axis, $x = vt$

Along y axis, $y = 0 + \frac{1}{2}\frac{qE}{m}\left(\frac{x}{v}\right)^2 \quad \left[\because t = \frac{x}{v}\right]$

$$y = \left(\frac{qE}{2mv^2}\right)x^2$$

or $y = kx^2$ where $k = \frac{qE}{2mv^2}$ = Constant

This equation represent a parabola ($y \propto x^2$) i.e path of the charged particle is parabolic.

Charged particle moving in uniform magnetic field:

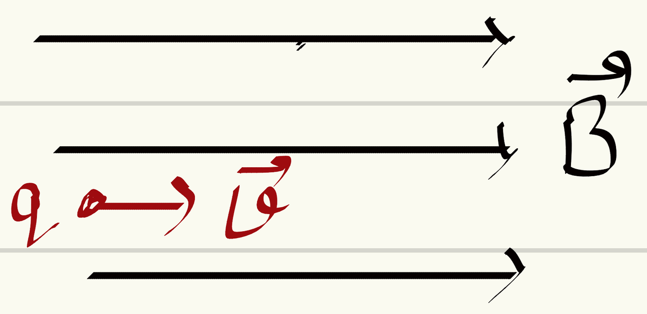
Case I When motion of a charged particle is parallel or antiparallel to the magnetic field -

$\theta = 0^\circ$ or 180°

$F = qvB\sin\theta$

$F = 0 \quad [\sin 0^\circ = 0, \sin 180^\circ = 0]$

No matter charge is +ve or -ve $F = 0$, If $\theta = 0^\circ$ or 180°



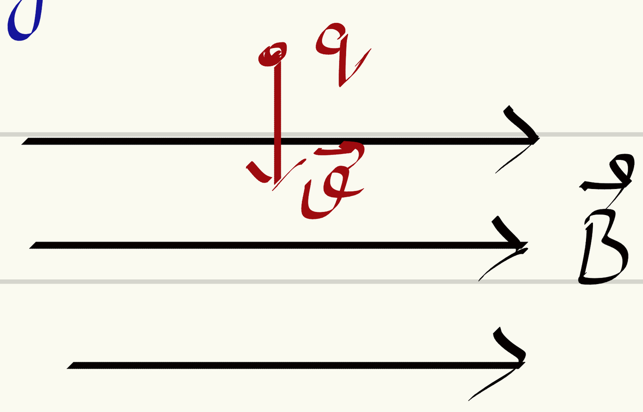
$\theta = 0^\circ$

$\theta = 180^\circ$

<u>Case II:</u> When charged particle moved at right angle to the magnetic field

Consider a uniform magnetic field $\vec{B}$, perpendicular to the plane of paper, directed into page.

Here force on the charge particle

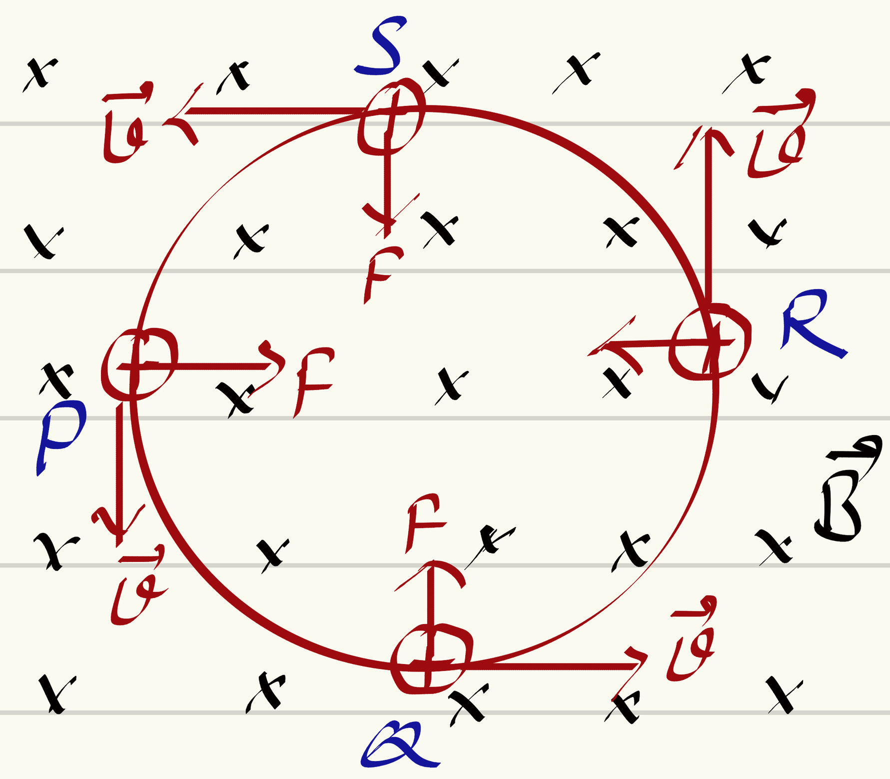


$$F = qvB\sin 90^\circ$$
$$F = qvB$$

Direction of this force is always perpendicular to the direction of motion as well as direction of magnetic field. $[\vec{F} \perp \vec{v}$ and $\vec{F} \perp \vec{B}]$

Direction of this force can be determined by right hand thumb rule.

Charge particle follows a <u>circular path</u> when it enters perpendicularly to the magnetic field ($\theta = 90^\circ$), as shown in figure.



<u>Radius of the circular path:</u>

Here centripetal force is provided by magnetic force.

$$F_m = F_c$$
$$qvB = \frac{mv^2}{r}$$

or $\boxed{r = \frac{mv}{qB}}$ or $\boxed{r = \frac{v}{\left(\frac{q}{m}\right)B}}$

Clearly, $r \propto v$. i.e. smaller the radius slower the speed and larger the radius faster the speed.

<u>Time Period (T):</u> (Time taken to complete one revolution)

$$T = \frac{\text{Distance}}{\text{Speed}} = \frac{2\pi r}{v}$$

put $r = \frac{mv}{qB}$

$$T = \frac{2\pi m \cancel{v}}{qB\cancel{v}}$$

$$\boxed{T = \frac{2\pi m}{qB}}$$

Frequency (ν): (number of revolutions per second)

$$\nu = \frac{1}{T}$$

or $$\boxed{\nu = \frac{qB}{2\pi m}}$$

* It is clear that time period or frequency is independent of speed and radius of the charged particle.
* It depends on magnetic fied and nature of $\frac{q}{m}$. Means all particles of same $\frac{q}{m}$ will revolve at the same frequency.

Angular Frequency (ω)

$$\omega = 2\pi\nu$$

$$\omega = 2\pi \times \frac{qB}{2\pi m}$$

$$\boxed{\omega = \frac{qB}{m}}$$

This angular frequency is called gyro-frequency.

* When two different charged particles having same momentum enter perpendicularly into the uniform magnetic field their paths are equally curved.

Case III: When the charge particle moves at an angle to the magnetic field (other than 0°, 90° and 180°)

From fig it is clear that due to the component $v_x = v\cos\theta$ charge particle moves with constant velocity.

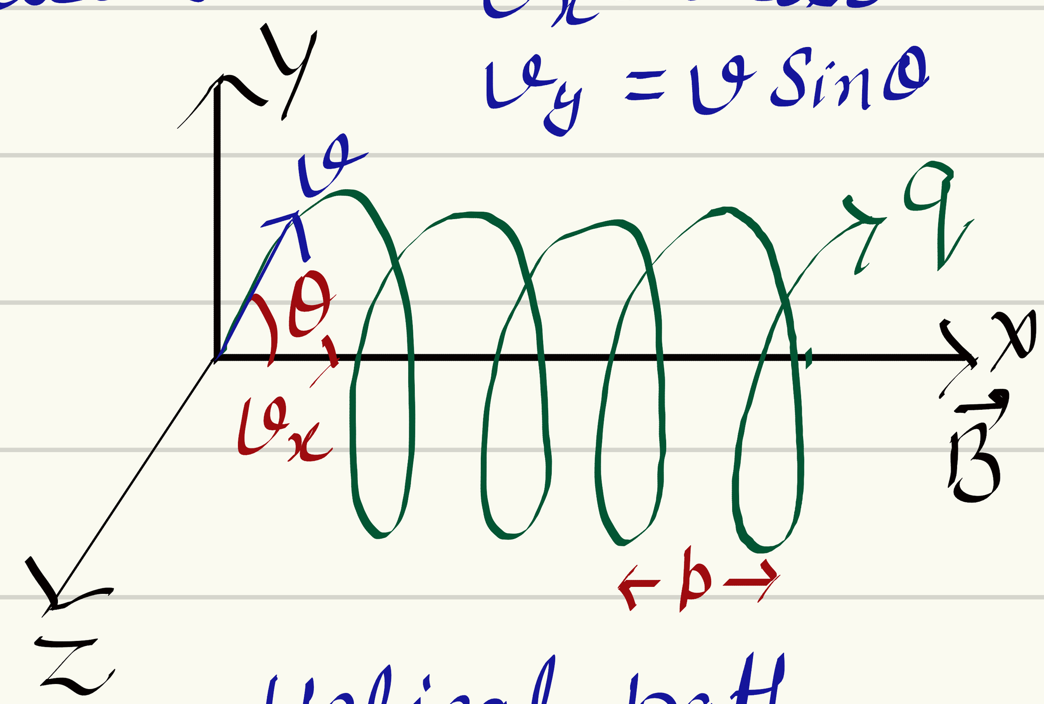
Due to $v_y = v\sin\theta$ particle moves on a circular path.

Therefore the path becomes helical.

$v_x = v\cos\theta$
$v_y = v\sin\theta$

Here centripetal force is provided by Lorentz magnetic force.

$$\frac{m(v\sin\theta)^2}{r} = q(v\sin\theta)B\sin 90°$$



Helical path

$$\boxed{r = \frac{mv\sin\theta}{qB}}$$ $r \rightarrow$ Radius of circular path

Time period $T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi}{v\sin\theta} \times \frac{mv\sin\theta}{qB}$

or $$\boxed{T = \frac{2\pi m}{qB}}$$

* Under the combined effect of two components of velocity, the resultant path is helix with is axis parallel to $\vec{B}$
* T is independent of v.

Pitch of Helix: The linear distance travelled by the charged particle in one rotation is called pitch.

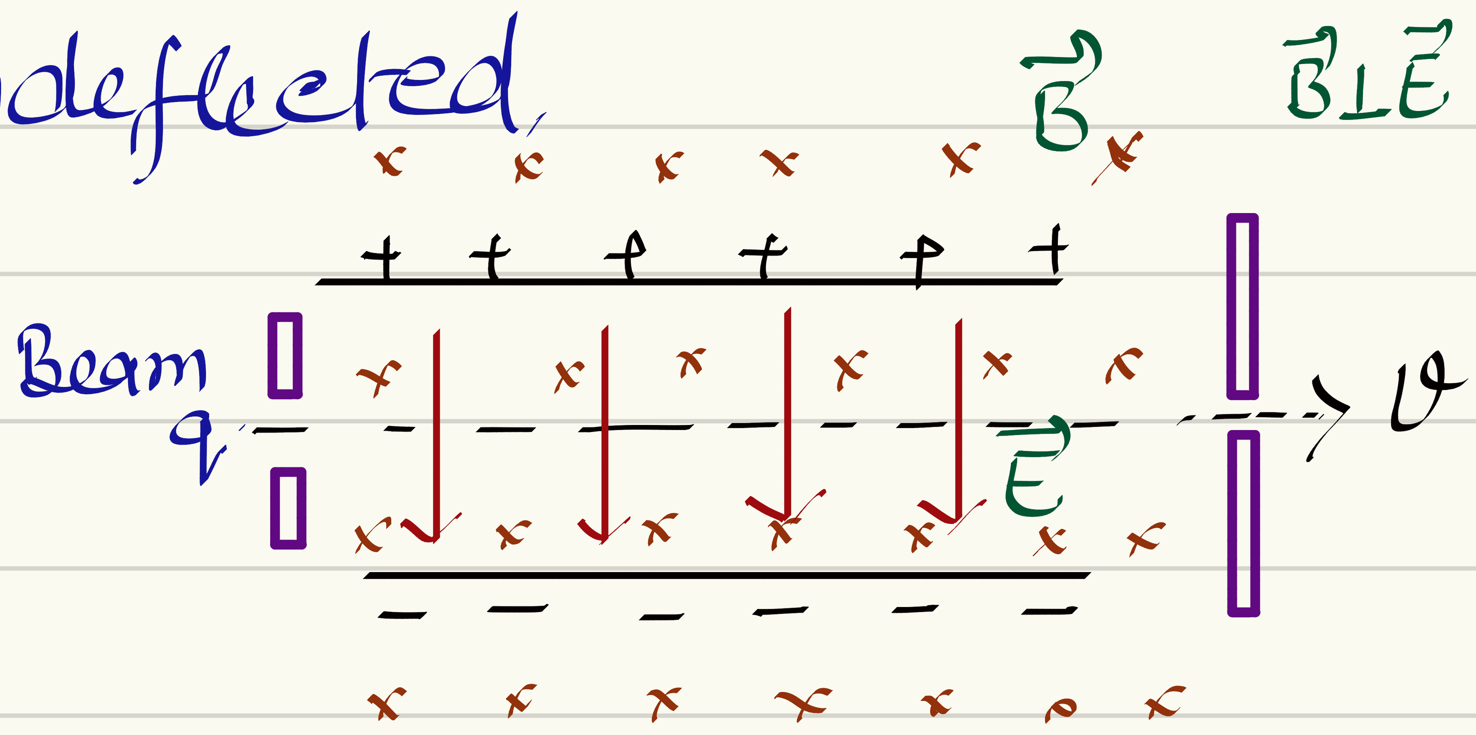
$$p = (v\cos\theta) \times T$$
$$= v\cos\theta \times \frac{2\pi m}{qB}$$

or $$\boxed{p = \frac{2\pi m v\cos\theta}{qB}}$$

* Velocity Selector (Velocity filter): It is a device that allows only charged particles with specific velocity to pass through it while deflecting others. It works by using mutually perpendicular electric field (E) ang magnetic field (B)

For a particle to pass undeflected,

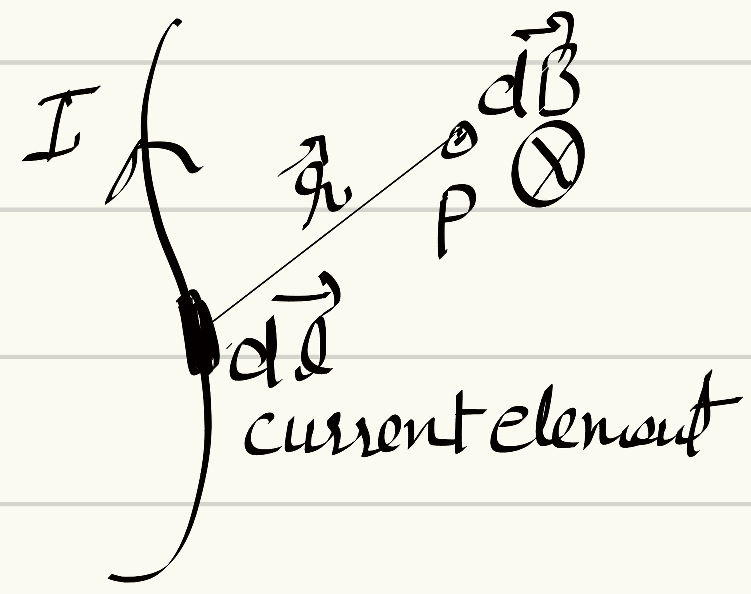
$$F_E = F_B$$
$$qE = qvB$$
$$\boxed{v = \frac{E}{B}}$$



Only particles with velocity $v = \frac{E}{B}$, pass through undeflected.

For electromagnetic waves in free space
$c = \frac{E_0}{B_0}$ $c \rightarrow$ speed of light
E_0 and B_0 are peak value of E & B

Biot Savart's Law: According to this law magnetic field at any point due to current element $d\vec{l}$



$$dB \propto \frac{I\,dl\,\sin\theta}{r^2}$$

or $$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}$$

Here μ_0 is permeability of free space

Its SI unit is $Tm^{-1}A^{-1}$

(Permeability of free space represents the ability of vacuum to support the formation of magnetic field)

Value, $\mu_0 = 4\pi \times 10^{-7}\ NA^{-2}$

Vector form of Biot Savart's law

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I(d\vec{l} \times \vec{r})}{r^3}}$$

Direction of dB is given by right hand thumb rule and in the dir^n of $(d\vec{l} \times \vec{r})$.

* Biot Savart's law for magnetic field is similar to coulomb's law for electric field.

- Both follow inverse square law.
- Both obey principle of superposition.
- Both are long range force.
- Magnetic field produces by vector source $Id\vec{l}$ and electric field is produce by scalar quantity charge.
- Magnetic force depends on angle ($\sin\theta$) but coulomb force does not depend on angle.

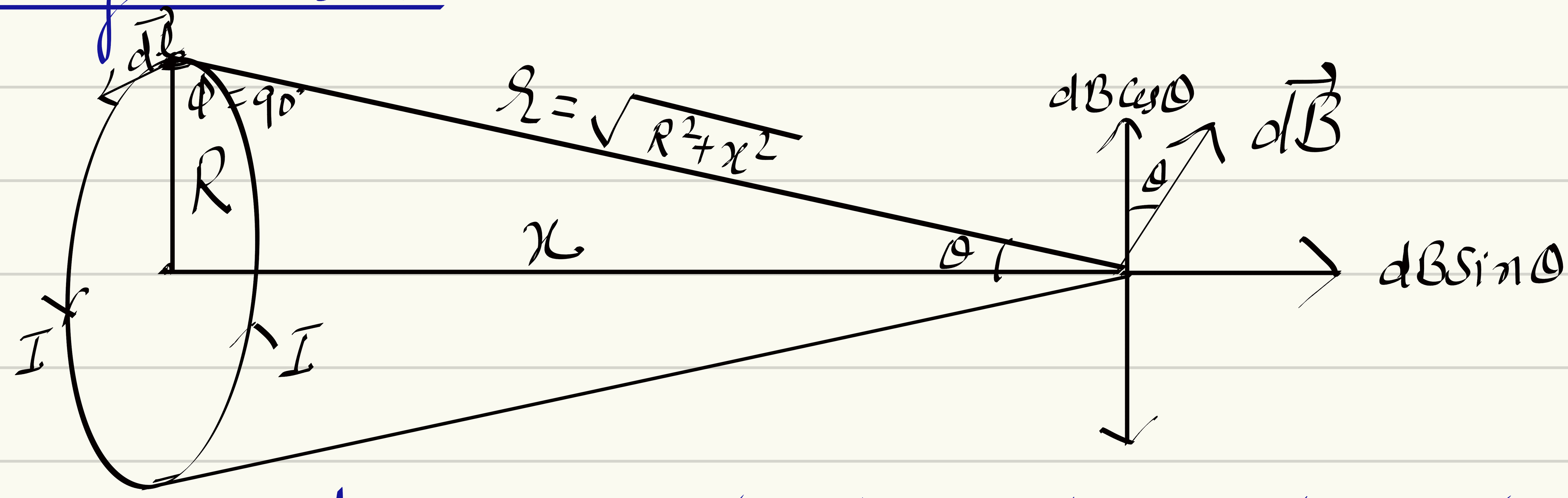
* Relation b/w ϵ_0, μ_0 and speed of light c

$$\epsilon_0\mu_0 = (4\pi\epsilon_0)\left(\frac{\mu_0}{4\pi}\right) = \frac{1}{9\times10^9}\times10^{-7} = \frac{1}{(3\times10^8)^2} = \frac{1}{c^2}$$

or $$\boxed{c = \frac{1}{\sqrt{\epsilon_0\mu_0}}}$$

Applications of Biot Savart's law:

1. **Magnetic field on the axis of a circular loop carrying current**



Consider a current carrying circular loop as shown in fig.
Let 'P' is the point on its axis where magnetic field is to be determined.

Consider a current element $\vec{dl}$ at a distance 'r' from 'P'.
According to Biot Savart's law magnitude of 'dB' at point 'P' due to $\vec{dl}$ is given by

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin 90°}{r^2} \qquad [\phi = 90°]$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

On resolving $\vec{dB}$ into its components, we see $dB\cos\theta$ cancel out and the total magnetic field is given by

$$B = \int dB\sin\theta$$

$$= \int \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}\cdot\frac{R}{r} \qquad \left[\because \sin\theta = \frac{R}{r}\right]$$

$$= \frac{\mu_0 I R}{4\pi r^3}\int dl$$

$$= \frac{\mu_0 I R}{4\pi r^3} \times 2\pi R \qquad \left[\because \int dl = 2\pi R\right]$$

$$B = \frac{\mu_0}{4\pi}\frac{IR^2}{r^3} \times 2\pi$$

or $$B = \frac{\mu_0}{4\pi}\frac{IR^2}{\left(\sqrt{R^2+x^2}\right)^3} \times 2\pi \qquad \left[\because r = \sqrt{R^2+x^2}\right]$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{IR^2}{(R^2+x^2)^{3/2}} \times 2\pi} \qquad \text{—— (1)}$$

or $$\boxed{B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}} \quad \text{——— (2)}$$

* If coil has N turns, then

$$\boxed{B = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}}$$

Special cases:

(I) If $x >>> R$ then R^2 can be neglected and

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0}{4\pi}\,\frac{IR^2}{x^3} \times 2\pi$$

mag. moment $M = IR^2$

$$B = \frac{\mu_0}{2\pi}\,\frac{M}{x^3}$$

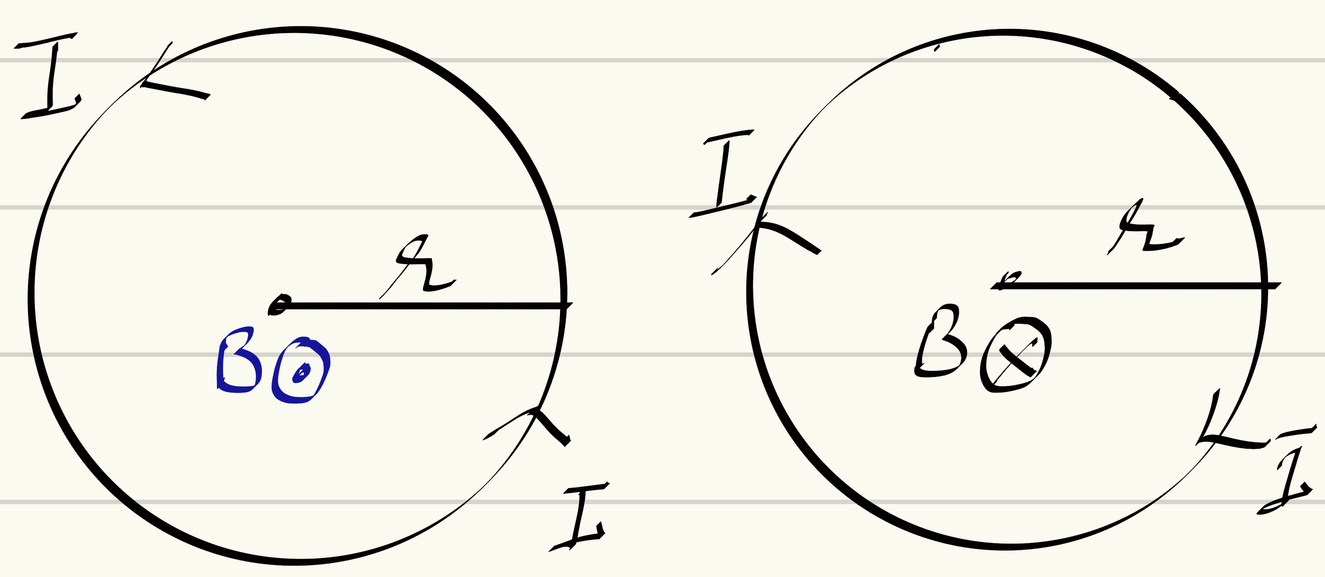
For $x >> R$ coil behaves as bar magnet or mag. dipole

(II) Magnetic field at the centre of the loop (At $x=0$)

$$B = \frac{\mu_0 I R^2}{2(R^2)^{3/2}} \quad [\text{put } x=0 \text{ in eq}^n (2)]$$

$$B = \frac{\mu_0 I R^2}{2R^3}$$

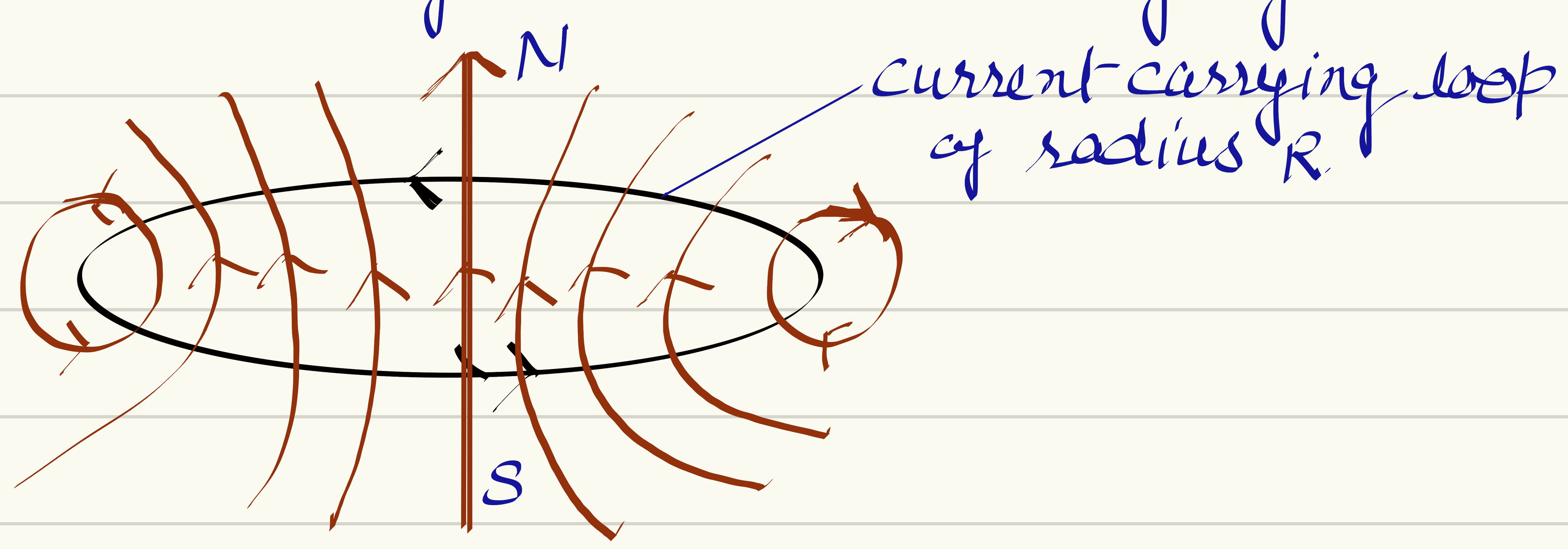
or $$\boxed{B = \frac{\mu_0 I}{2R}}$$



Also $$\boxed{B = \frac{\mu_0}{4\pi}\cdot\frac{I}{R} \times 2\pi}$$

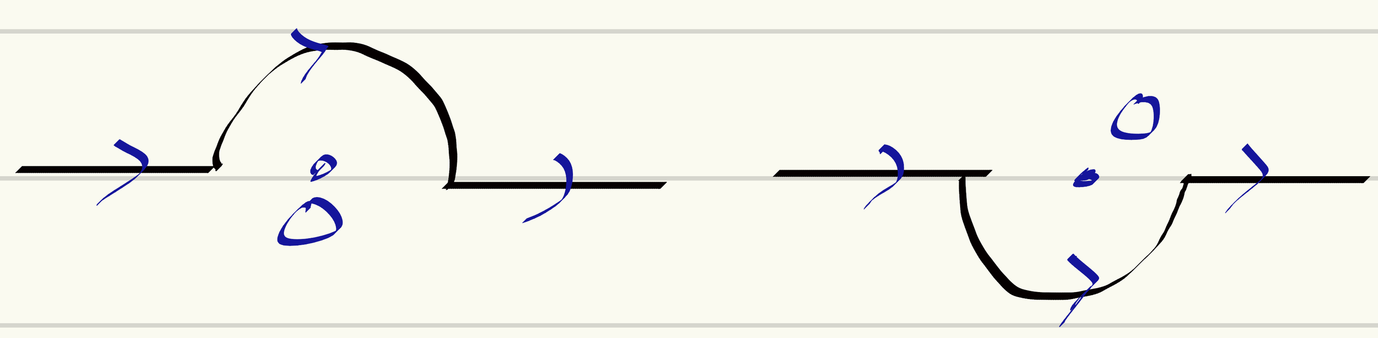
Magnetic field lines of a current carrying circular coil:

$$\vec{B}_0 = \frac{\mu_0 I}{2R}\,\hat{i}$$



Direction of magnetic field is given by right hand thumb rule.

* For semi-circular loop



$$B_{semi} = \frac{\mu_0 I}{4R}$$

[Due to straight wire B at 'O' = zero.

* Amperian loop is an imaginary closed loop around the conductor.

Ampere's Circuital Law:

This law states the relationship between the current and the magnetic field created by it.

"According to this law the integral of magnetic field density (B) along an imaginary closed path is equal to the μ_0 times of the current enclosed by the path."

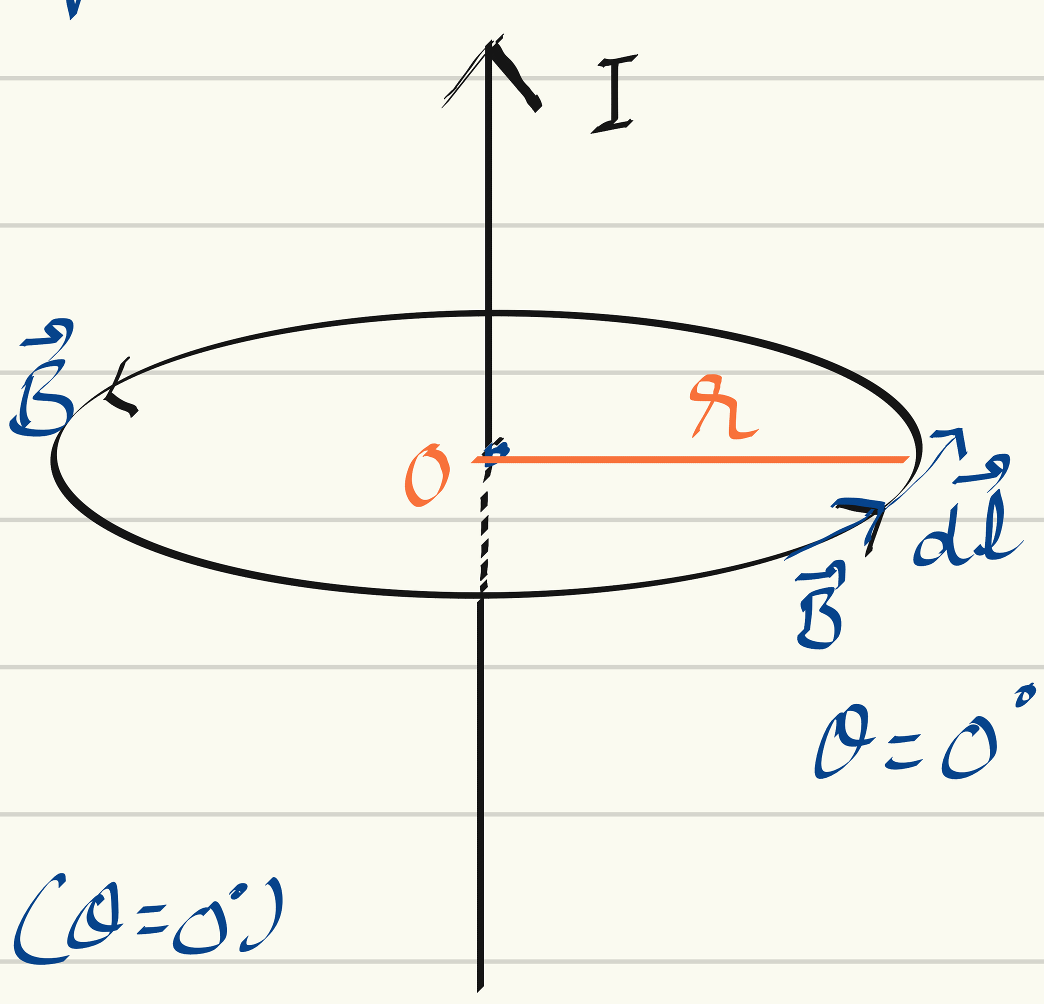
OR

"The line integral of the magnetic field ($\vec{B}$) around any closed path is equal to μ_0 times the total current (i) passing through the closed path."

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

where μ_0 is the permeability of free space.

<u>Proof</u>: From Biot-Savart's law, Magnetic field due to long straight wire



$$B = \frac{\mu_0 I}{2\pi r}$$

here $\vec{B}$ and $\vec{dl}$ are in same direction ($\theta = 0°$)

So

$$\oint \vec{B} \cdot \vec{dl} = \oint B\,dl \cos 0° = \oint B\,dl \qquad [\cos 0° = 1]$$

and

$$= B \oint dl$$

$$= \frac{\mu_0 I}{2\pi r} \oint dl \qquad \left[\because B = \frac{\mu_0 I}{2\pi r}\right]$$

$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r \qquad \left[\because \oint dl = 2\pi r\right]$$

or

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I}$$

* This relation involves a sign convention given by right hand thumb rule.

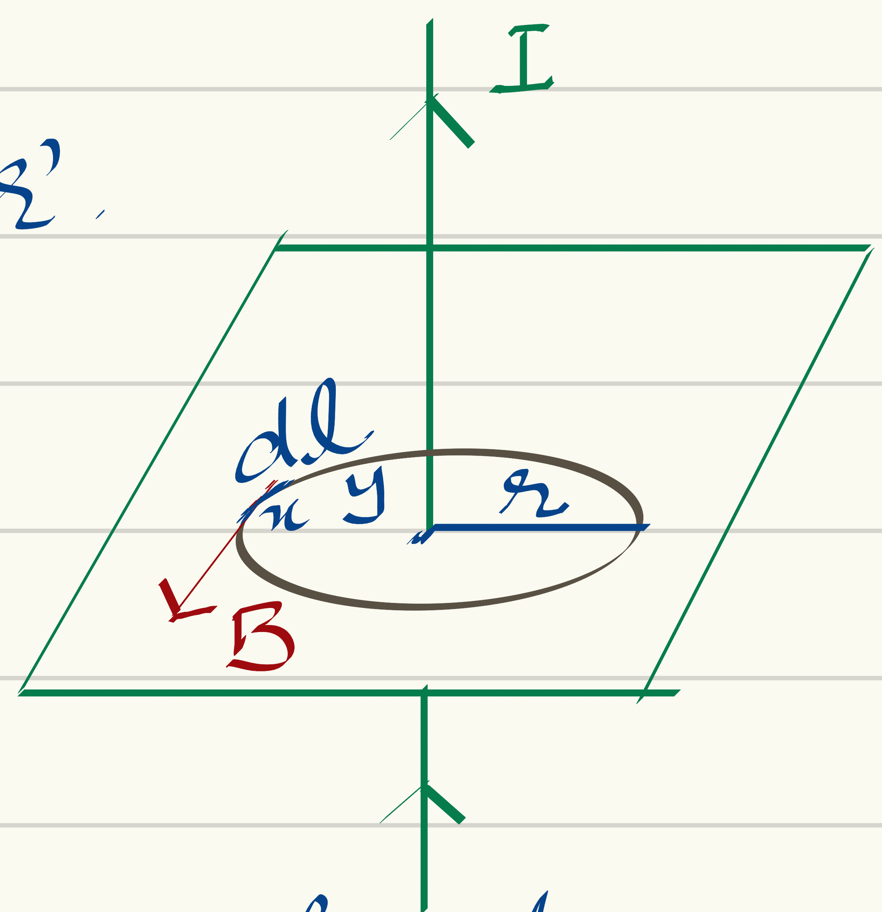
* Ampere's law is to Biot-Savert law what Gauss's law is to coulomb's law.

* It is possible to choose the amperian loop such that at each point of the loop, either
(1) B is tangential to the loop (B is constant and non zero)
(11) B is normal to the loop, or
(111) B vanishes.

* Bio Savart law based on the experimental results whereas Ampere's law based on mathematical

Applications of Ampere's Circuital law

1. Magnetic field due to thin current carrying straight conductor



Consider a radius of radius 'r'. Let xy be the small element of length dl. $\vec{B}$ and $\vec{dl}$ are in the same dirⁿ ($\theta = 0°$)

By A.C.L.

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \qquad I \rightarrow \text{enclosed}$$

$$\oint B\,dl\cos 0° = \mu_0 I$$

$$B\oint dl = \mu_0 I$$

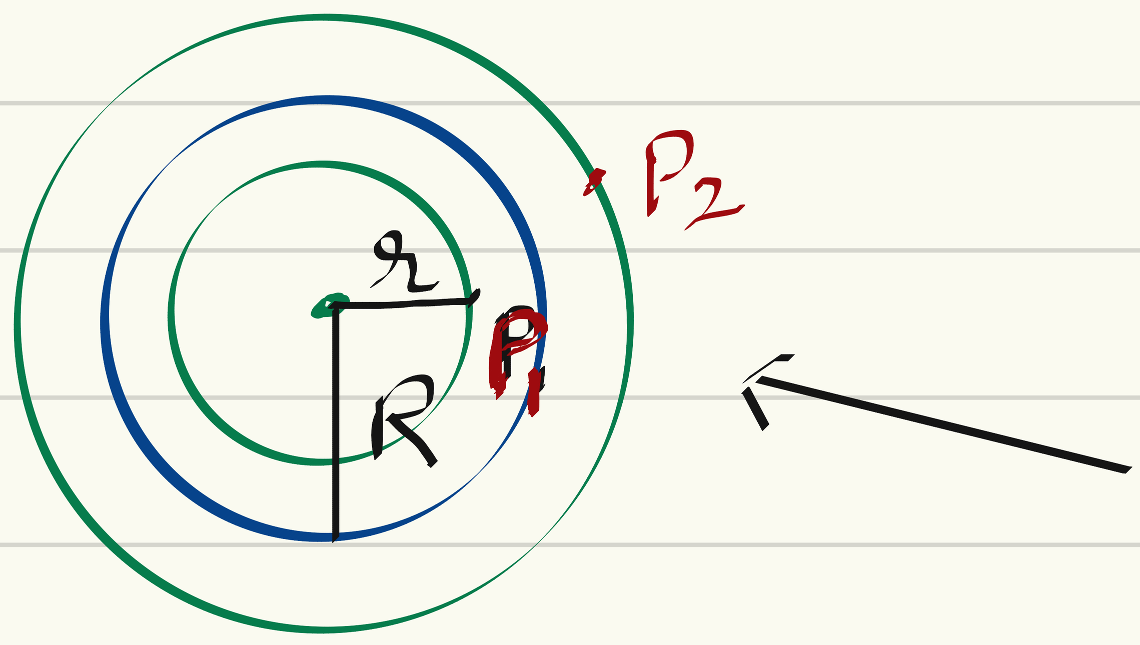
$$B \times 2\pi r = \mu_0 I \qquad \left[\because \oint dl = 2\pi r\right]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

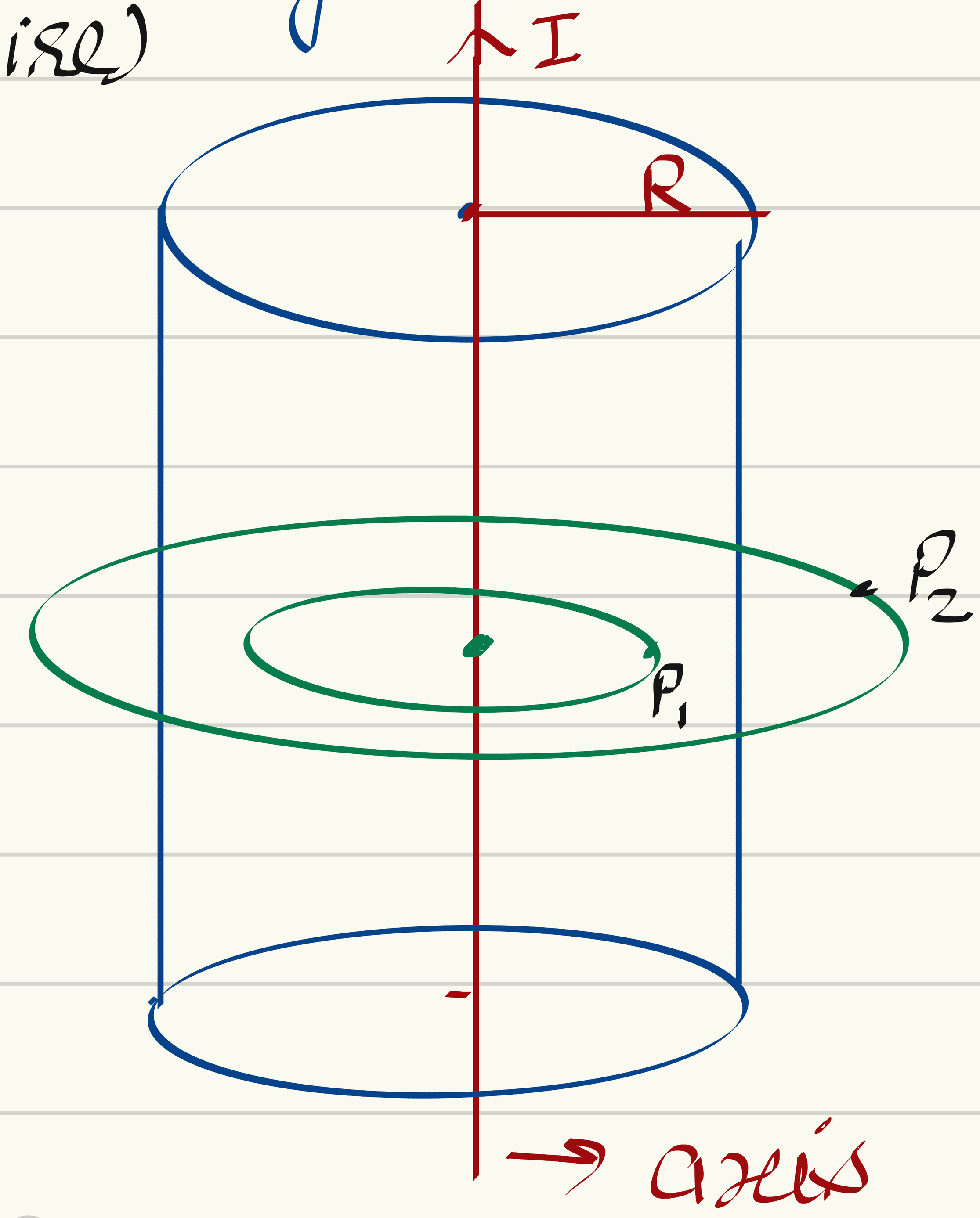
Dirⁿ of B is determined by right hand thumb rule

* When a straight conductor carries current it produces a magnetic field around it, which follows Amper's circuital law and Biot Savart's law.

2 Magnetic field due to ifinite long solid cylindrical conductor (Thickwise)



cross-sectional view



(i) For a point 'P_1' inside the cylinder, $(r < R)$

current from area $\pi R^2 = I$

so current from area πr^2 (I_{en}) $= \dfrac{I}{\pi R^2} \times \pi r^2$

$$I_{en} = \frac{I r^2}{R^2}$$

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By ACL for 'P_1'

$B \times 2\pi r = \mu_0 I_{en}$ $\quad \left[\because \oint \vec{B}\cdot\vec{dl} = B \times 2\pi r\right.$

or $B \cdot 2\pi \cancel{r} = \mu_0 \dfrac{I r^{\cancel{2}}}{R^2}$

or $\boxed{B = \dfrac{\mu_0 I r}{2\pi R^2}} \Rightarrow \boxed{B \propto r}$

(ii) For a point on the axis of cylinder

as $r = 0$, $B = 0$

i.e $\boxed{B_{axis} = 0}$

(iii) For a point on the surface of cylinder
($r = R$)

$$B_s \times 2\pi R = \mu_0 I$$

or $$\boxed{B_s = \frac{\mu_0 I}{2\pi R}}$$ [B_{max}]

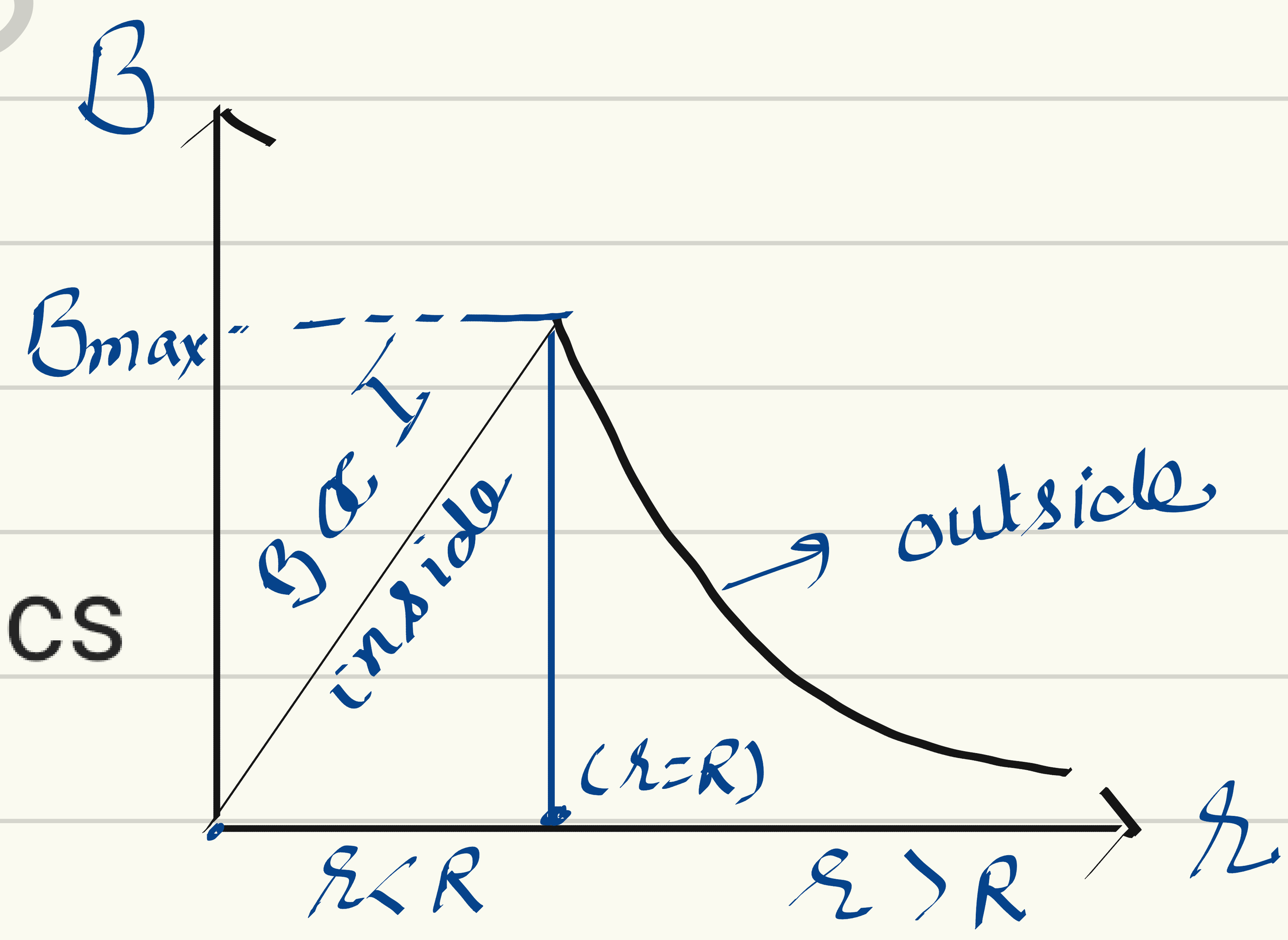
(iv) For a point outside the cylinder
($r > R$)

$$B_{out} \times 2\pi r = \mu_0 I$$

or $$\boxed{B_{out} = \frac{\mu_0 I}{2\pi r}} \Rightarrow \boxed{B \propto \frac{1}{r}}$$

* Magnetic field outside the cylinder conductor (wire) does not depend upon nature of conductor like radius, area, solid, hollow etc.

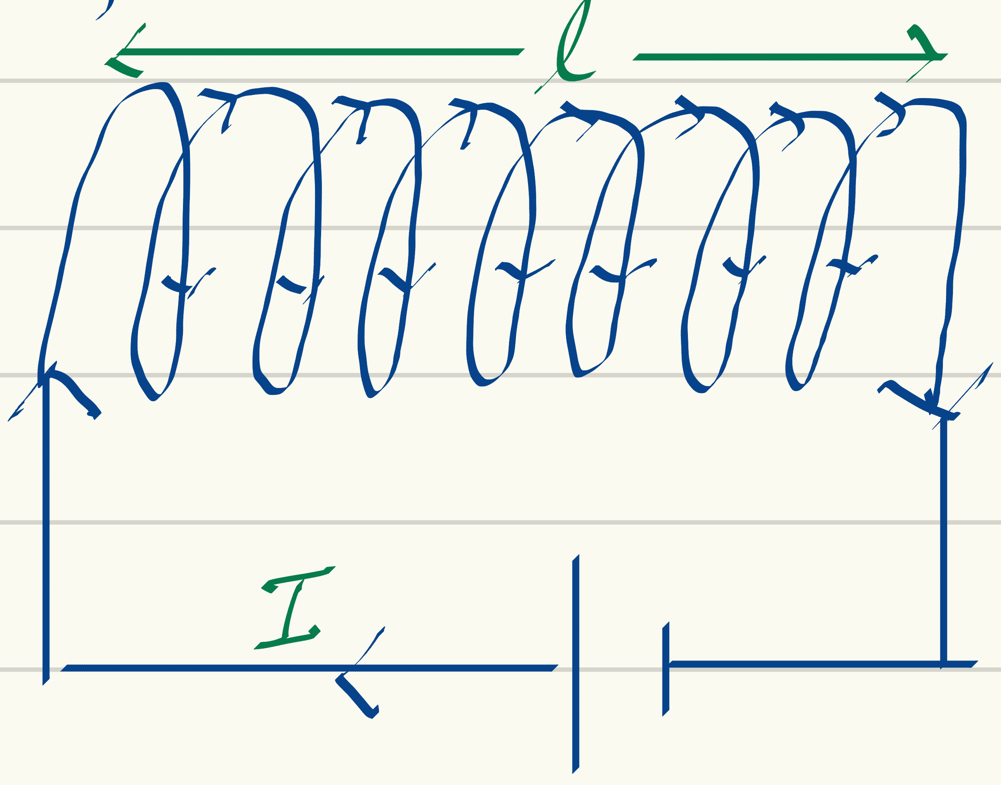
* Graph b/w B and r

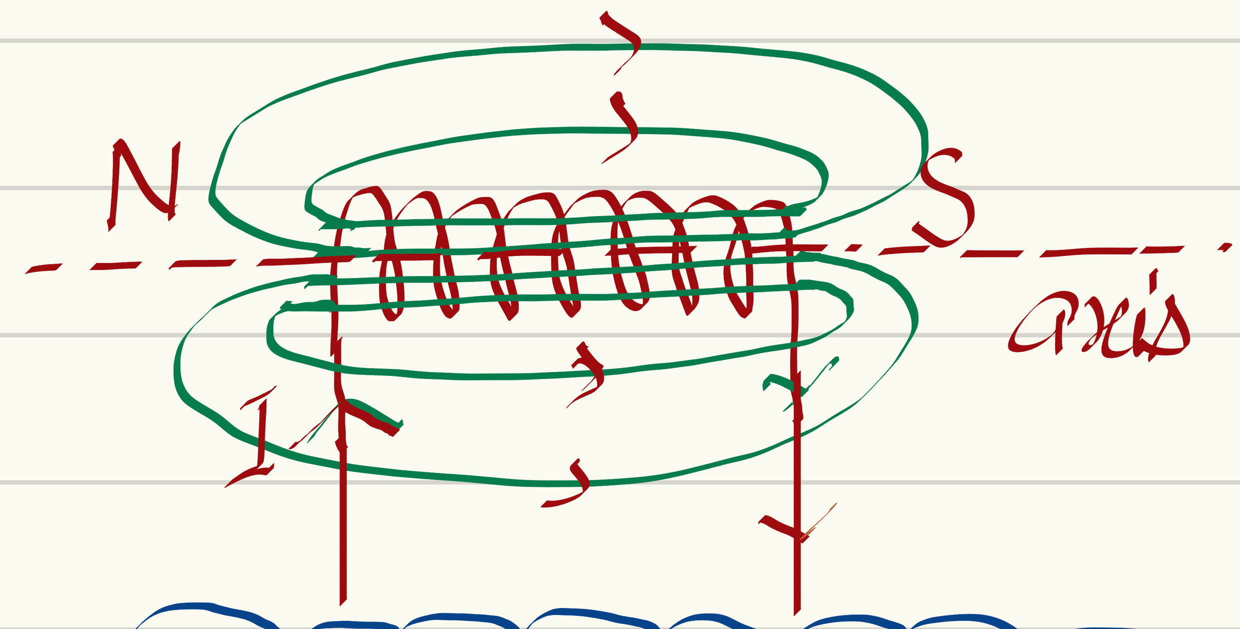


Solenoid

It is a coil which has length and used to produced magnetic field of long range.
It consists a conducting wire tightly wound over a cylindrical frame in the form of helix.

Magnetic field due to a long solenoid





Solenoid behaves like a bar magnet

Formula

$$B = \mu_0 n I$$

$$= \mu_0 \frac{N}{l} I$$

$n \rightarrow$ No. of turns per unit length
$N \rightarrow$ Total turns
$l \rightarrow$ Length of solenoid

* Magnetic field of a solenoid can be increased by inserting an iron rod.

* Magnetic field at both axial end points of solenoid is half.

$$B = \frac{1}{2} \mu_0 n I$$

Magnetic field due to Toroid

mag. field produces within the toroid

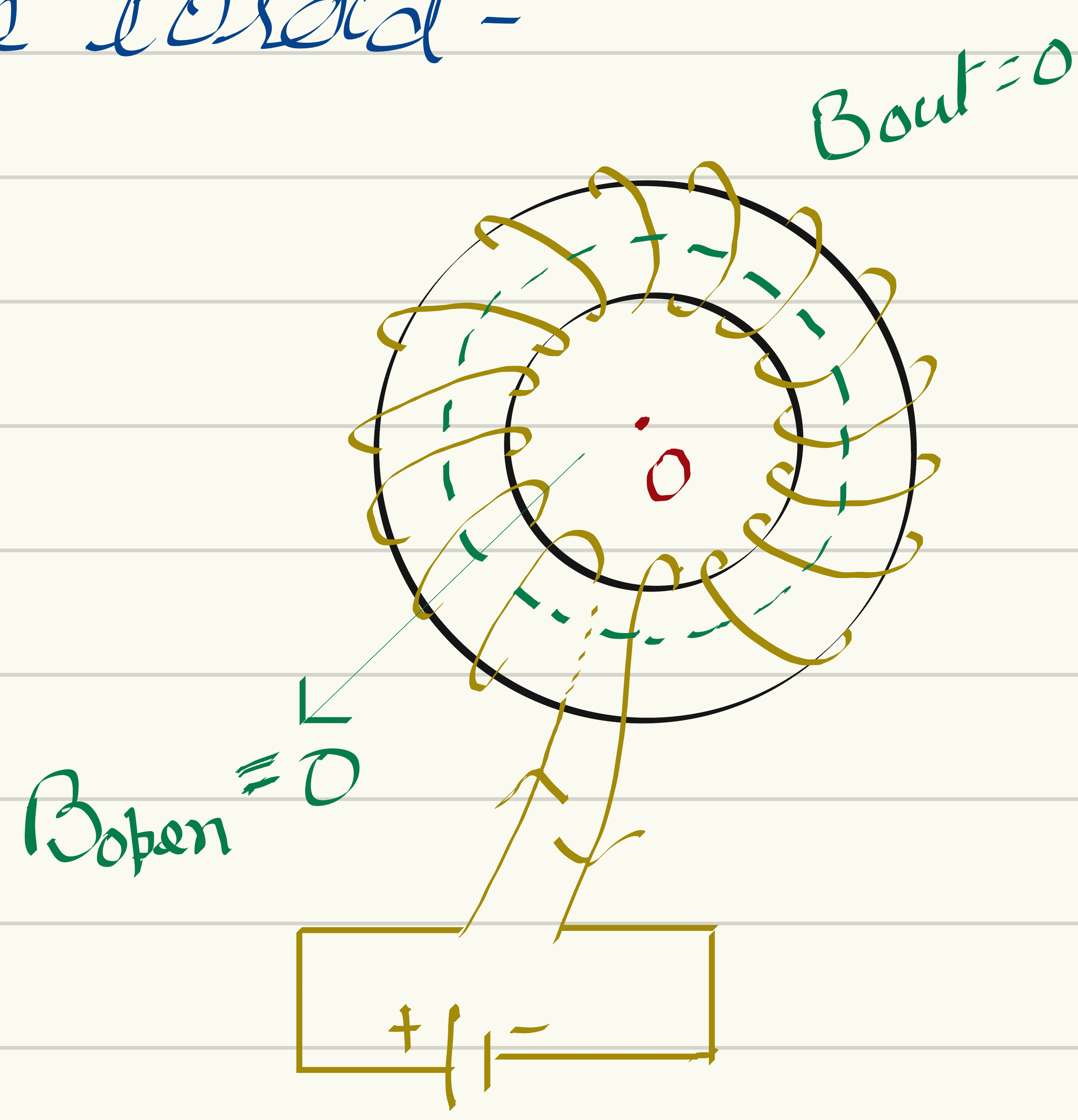
A toroid can be considered as a ring shaped closed solenoid also called endless solenoid.

Magnetic field inside the toroid -

$$B = \mu_0 n I$$

where

$$n = \frac{N}{2\pi R}$$



* Loop encloses no current.
ie $I_{enclosed} = 0$, $B_{open} = 0$

* $B_{outside} = 0$, since $I_{en} = 0$
* In an ideal toroid the coils are circular. In reality the turns form helix and there is always a small magnetic field external to the toroid

Uses of solenoid and toroid:

- In television to generate magnetic field.
- Synchrotron uses both, solenoid and toroid to generate high magnetic field.

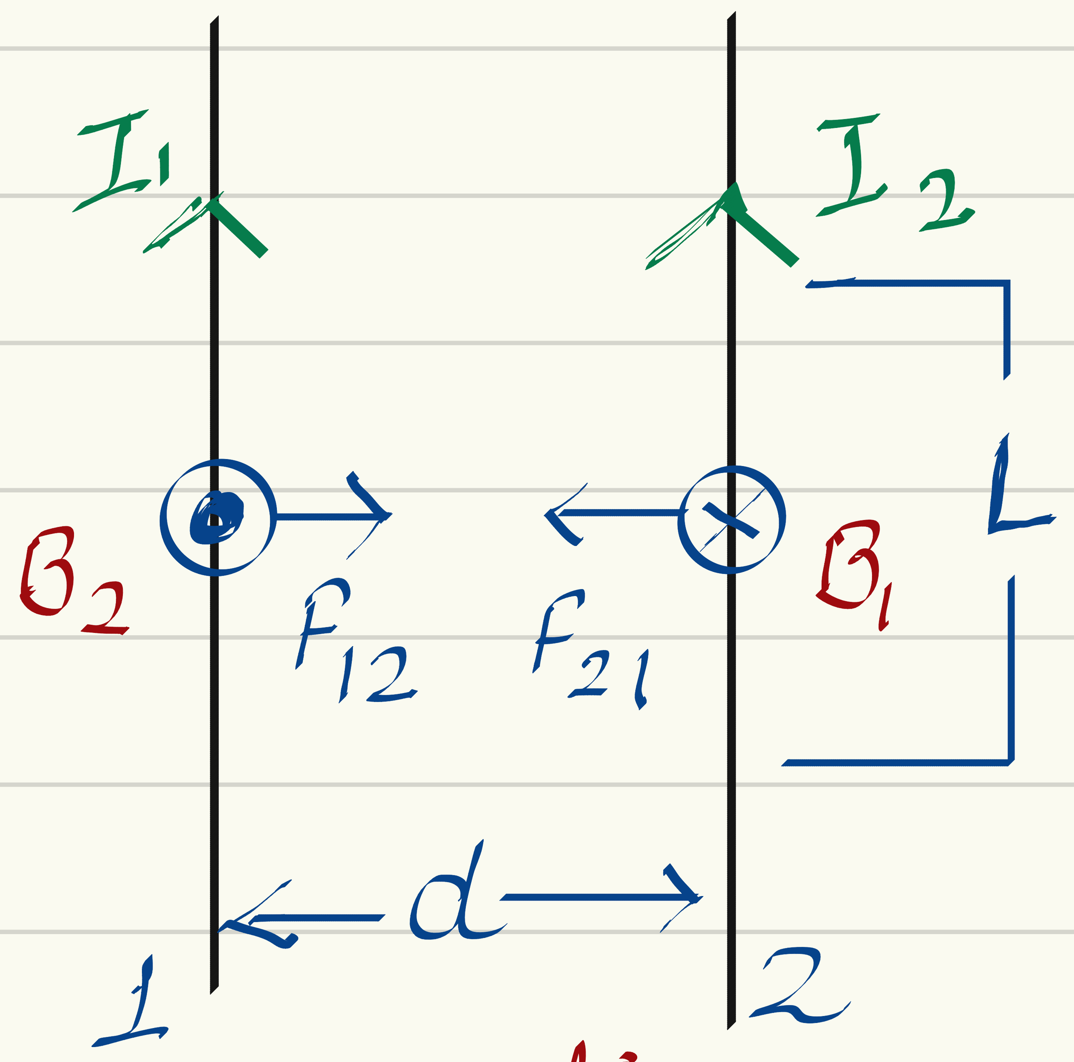
Difference b/w solenoid and electromagnet

Solenoid is just a coil of wire but when current flows through the coil, it is called an electromagnet.

Force Between Two Parallel currents:

Fig. shows two long parallel conductors 1 & 2 separated by a distance d and carrying (parallel) currents I_1 and I_2 respectively.
Let I_1 produces mag. field B_1 and I_2 produces B_2 mag field.

Like currents



Attraction

L → wire's length segment

Force on wire 1 due the field of wire 2 (B_2)

$$F_{12} = I_1 L B_2 \qquad [\theta = 90°]$$

$$= I L \left(\frac{\mu_0 I_2}{2\pi d}\right)$$

$$= \frac{\mu_0 I_1 I_2}{2\pi d} L$$

$$F_{12} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d} . L$$

Similarly.

$$F_{21} = I_2 L B_1$$

$$= I_2 L \left(\frac{\mu_0 I_1}{2\pi d} \right)$$

$$= \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{d} \cdot L$$

$$F_{21} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d} \cdot L$$

force per unit length

$$\frac{F_{12}}{L} = \frac{F_{21}}{L} = \frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{d}$$

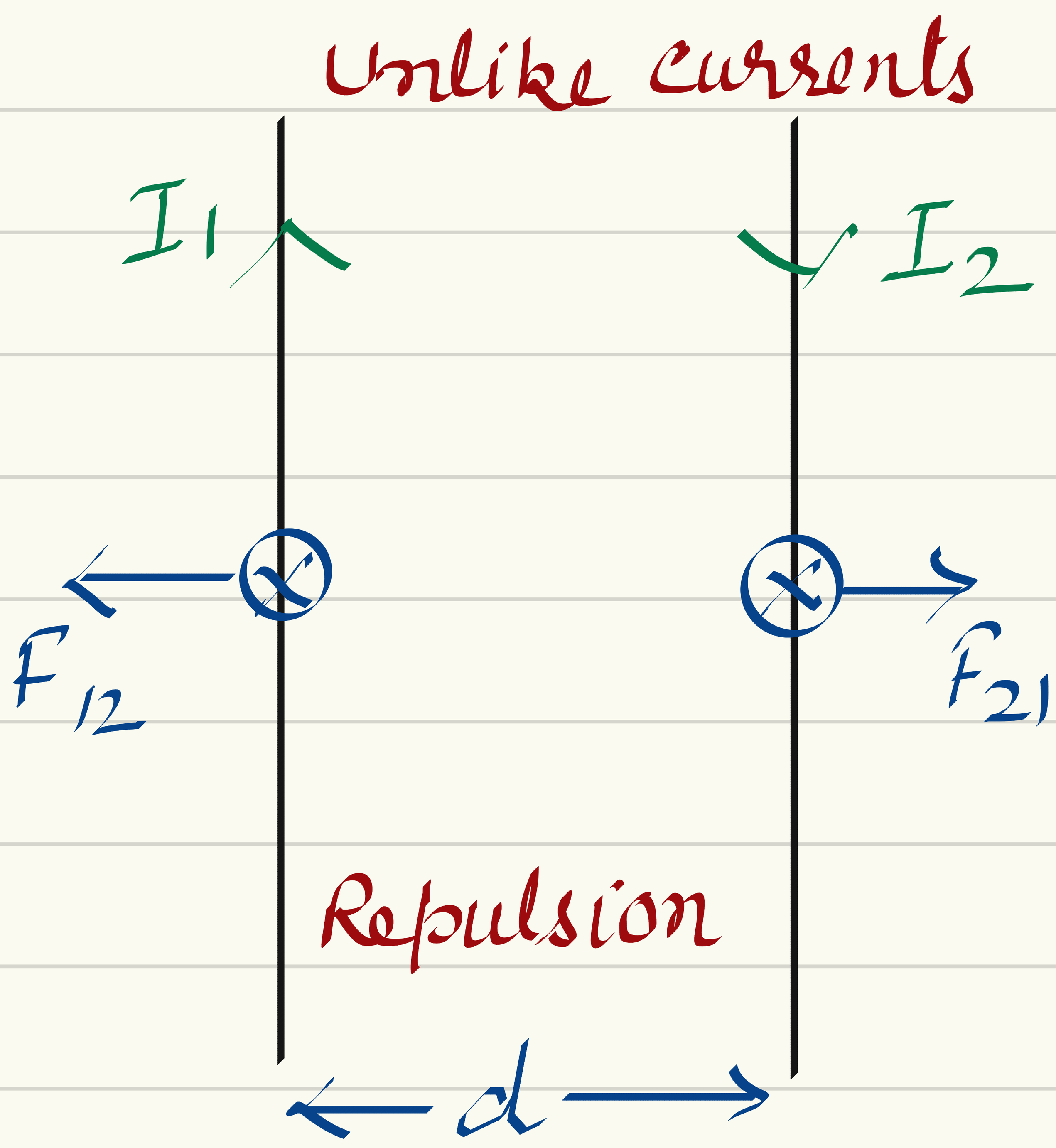
Forces $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude but opposit in dirn.

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

this verifies Newton's III Law.

*Parallel currents → Attract

Antiparallel currents → Repel



* Force per unit length is used to define the Ampere (A)

<u>One Ampere</u>: Ampere is the current which passes through each of two parallel infinite long straight conductor paced in free space

at a distance of 1 m produces a force of 2×10^{-7} N/m.

$$f = \frac{F_{12}}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

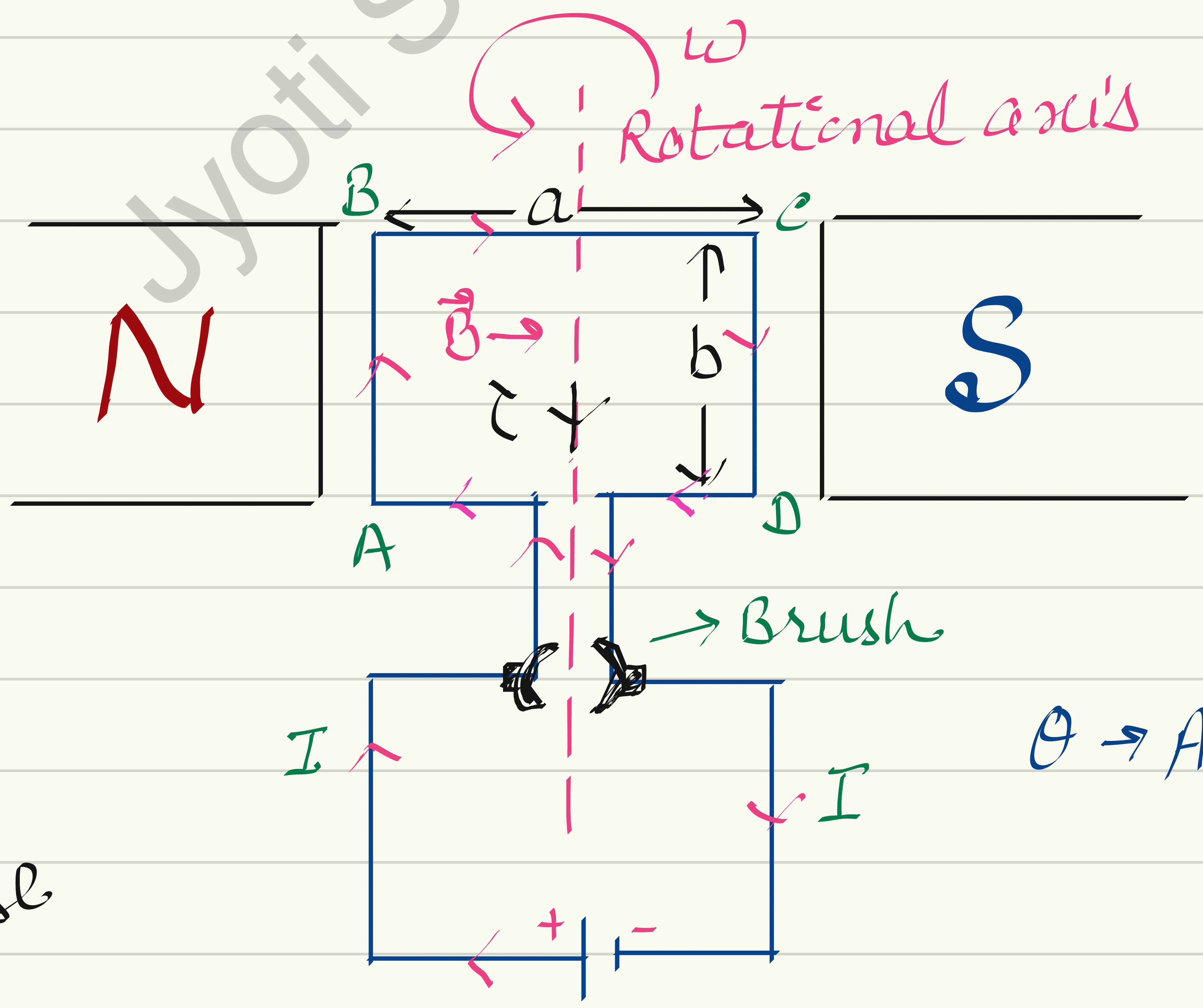
$$f = \frac{2\times10^{-7}(1)\times(1)}{1}$$

or $$f = 2\times10^{-7}\ N/m$$

[* An instrument called current balance is used to measure this mechanical force]

Torque On Current Loop, Magnetic Dipole

Fig shows a rectangular loop carrying a steady current I, placed in uniform mag. field $\vec{B}$ and experiences a torque.



$\theta \rightarrow$ Angle b/w $\vec{B}$ and $\vec{A}$
Area vector

General case

Case I When mag. field is in the plane of the loop. $(\theta = 90°)$

$\vec{F}_{AB} = -\vec{F}_{CD}$ [by F.L.H. Rule]

i.e. $F_{net} = 0$

and $F_{AD} = F_{BC} = 0$ [∥ and anti ∥ to $\vec{B}$]

There will a torque on the loop as $F_{net} = 0$

We know $\tau = |Force| \times \perp$ distance

$$= F_1 \times \frac{a}{2} + F_2 \times \frac{a}{2}$$

$$= IbB \cdot \frac{a}{2} + IbB \cdot \frac{a}{2} \quad \left[F_1 = F_2 = IbB \atop l \to b \right]$$

$$= I(ab)B$$

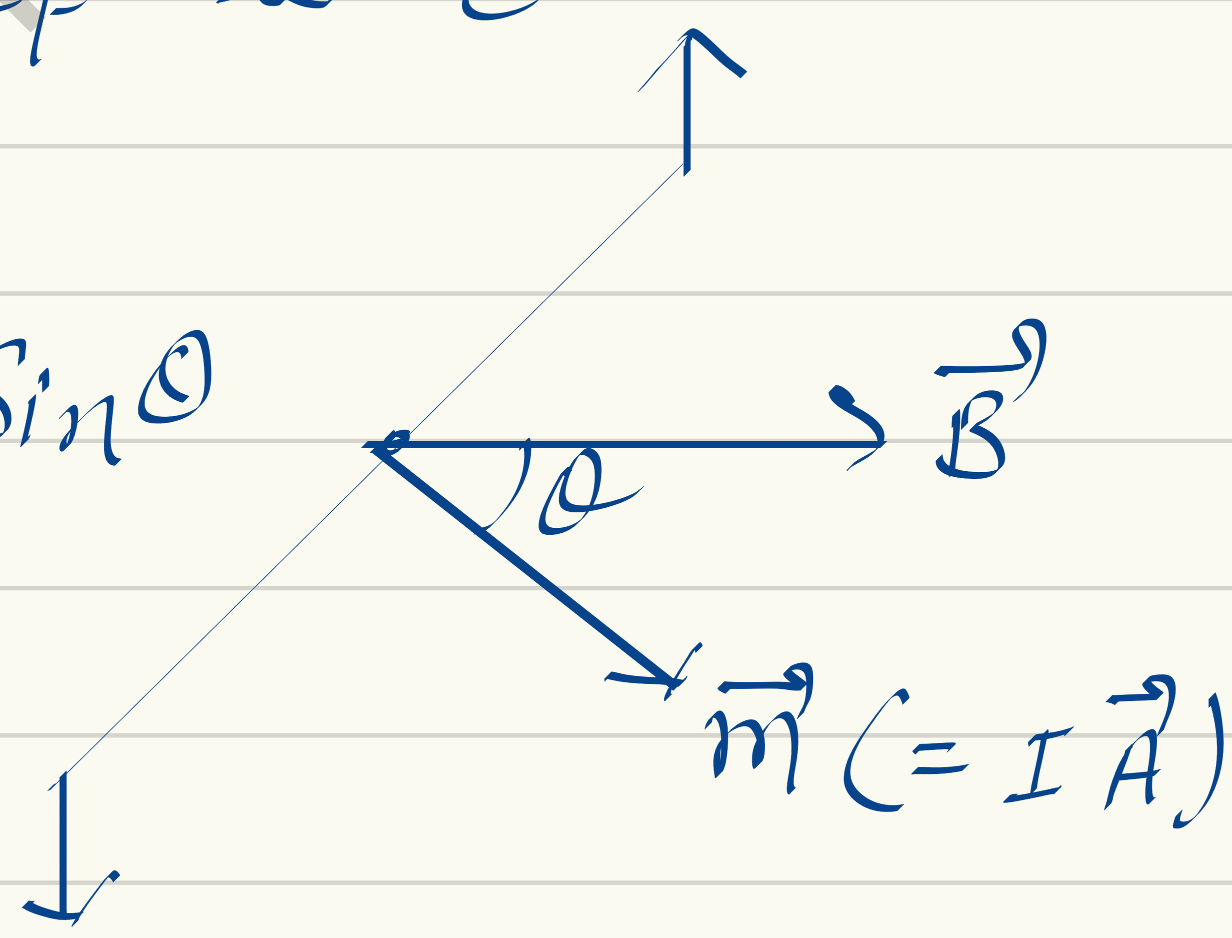
$$\boxed{\tau = IAB} \quad [ab = \text{Area } A]$$

Case II : When mag. field is not in the plane of the loop

Let angle b/w mag. field B and normal to the plane of loop is θ

here,

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$



$$= I(ab)B\sin\theta$$

$$\boxed{\tau = IAB\sin\theta}$$

for N turns

$$\boxed{\tau = NIAB\sin\theta}$$

Vector form, $\vec{\tau} = NI(\vec{A} \times \vec{B})$

area vector ($\vec{A}$), mag. field ($\vec{B}$)

here $I\vec{A} = \vec{m} \rightarrow$ magnetic moment

* dirⁿ of $\vec{m}$ is same to $\vec{A}$.

Magnetic Moment (m) - It is the product of current I and area vector $\vec{A}$.

$$\vec{m} = I\vec{A}$$

$\vec{m}$ is a vector in the dir^n of $\vec{A}$.

S.I unit - Am^2

Now by $\tau = NIAB\sin\theta$

Case(I) If $\theta = 0°$

$\Rightarrow \tau = 0$ [Equilibrium position]

(II) If $\theta = 90°$

$\Rightarrow \tau = NIAB$ [Max. torque τ]

When $\theta = 90°$ τ is max.

This is called radial field ($\theta = 90°$)

Torque in terms of magnetic moment

$$\tau = NIAB\sin\theta$$

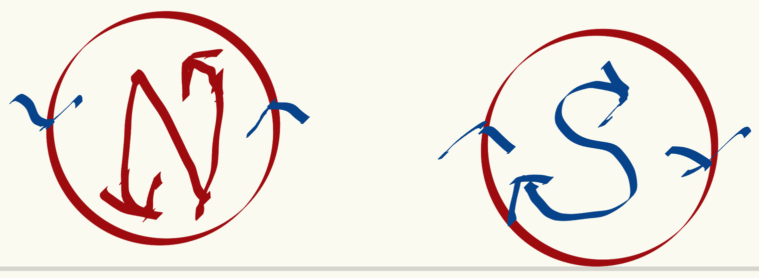
$$\tau = mB\sin\theta \qquad [m = NIA]$$

$$\boxed{\vec{\tau} = \vec{m} \times \vec{B}}$$

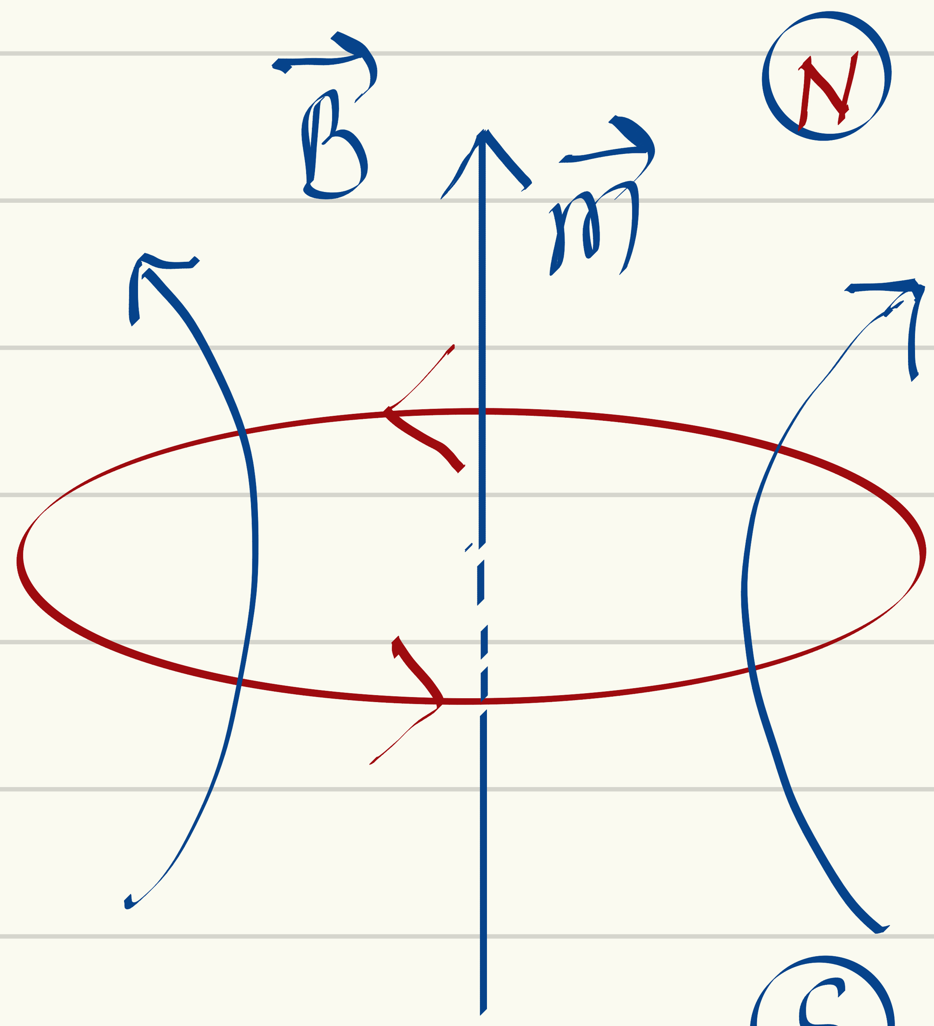
Dir^n of torque is $\perp$ to the plane of $\vec{m} \times \vec{B}$ and given by right hand thumb rule.

Circular Current Loop as a Magnetic Dipole

Magnetic field on the axis of a circular loop, of radius R, carrying current I is given by -



$$B = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}$$



For $x >> R$

$$B = \frac{\mu_0 I R^2}{2 x^3}$$

$$= \frac{\mu_0 I (\pi R^2)}{2\pi x^3}$$

$$= \frac{\mu_0 I A}{2\pi x^3} \qquad [A = \pi R^2]$$

$$\boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2m}{x^3}} \qquad [\because m = IA]$$

* This expression is similar to $\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{2p}{x^3}}$

$\mu_0 \rightarrow \frac{1}{\epsilon_0}$, $m \rightarrow p$ and $B \rightarrow E$

ϵ_0 ↓ permittivity; p ↓ electric dipole moment; E ↓ electric field

* B for a point in the plane of the loop at a distance x from the centre –

$$B = \frac{\mu_0}{4\pi} \frac{m}{x^3} \qquad [x >> R]$$

* Magnetic monopoles are not known to exist.

Important Results

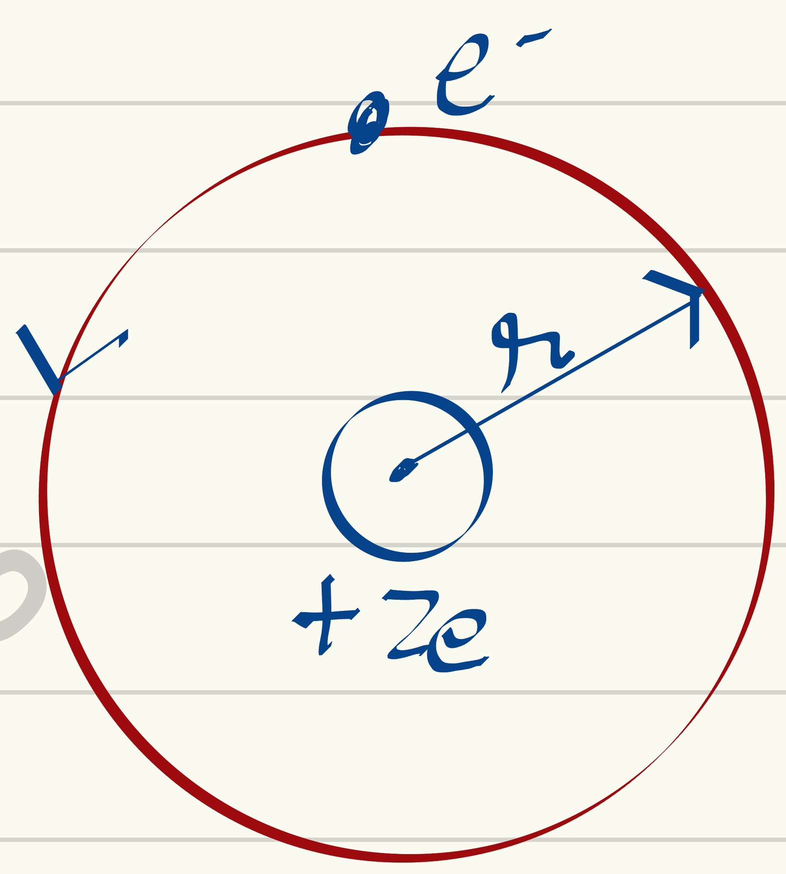
(1) Current loop produces a magnetic field and behaves like a magnetic dipole at large distances.

(ii) current loop is subjected to torque like a magnetic needle.

(iii) Elementry particles such as electron, proton also carry intrinsic magnetic moment.

The Magnetic Dipole Moment of a Revolving Electron:

In fig. Bohr picture of electron is shown.



Current due to the circular motion of electron,

$$I = \frac{e}{T} \qquad e = 1.6\times10^{-19}\,C$$

here $\qquad T \rightarrow$ Time period

$$T = \frac{2\pi r}{v}$$

$\otimes\ \mu_l$ ↓ into the page

So $$I = \frac{ev}{2\pi r}$$

The magnetic moment associated with electron's current

$$\mu_l = I\pi r^2$$

[dirn of μ_l is into the page]

$$= \left(\frac{ev}{2\cancel{\pi}\cancel{r}}\right)\times\cancel{\pi} r^{\cancel{2}}$$

$$\mu_l = \frac{evr}{2}$$

or $$\mu_l = \frac{e}{2m_e}(m_e v r)$$

$$= \frac{e}{2m_e}\cdot l$$

[$l \rightarrow$ angular momentum, $l = mvr$]

Vector form $\qquad \boxed{\vec{\mu_l} = -\frac{e}{2m_e}\,\vec{l}}$

−ve sign shows the opposite dir^n of μ_e and l.
For +ve charge μ_e and l are in same dir^n.

$$\frac{\mu_e}{l} = \frac{e}{2m_e}$$

This ratio $\frac{\mu_e}{l}$ is called gyromagnetic ratio.

It is constant and $\frac{\mu_e}{l} = 8.8 \times 10^{10}$ C/kg for an electron.

According to Bohr model

$$l = \frac{nh}{2\pi} \qquad n = 1, 2, 3 - - - .$$

$h \rightarrow$ Planck's constant

This discreateness is called - Bohr quantisation condition

Now $\mu_e = \frac{e}{2m_e} . l$

$$= \frac{e}{2m_e} . \frac{nh}{2\pi}$$

Put $n = 1$, $h = 6.626 \times 10^{-34}$ Js^{-1}
$m_e = 9.11 \times 10^{-31}$ kg

we get

$$(\mu_e)_{min} = 9.27 \times 10^{-24} \, Am^2$$

This value is called the Bohr magneton.

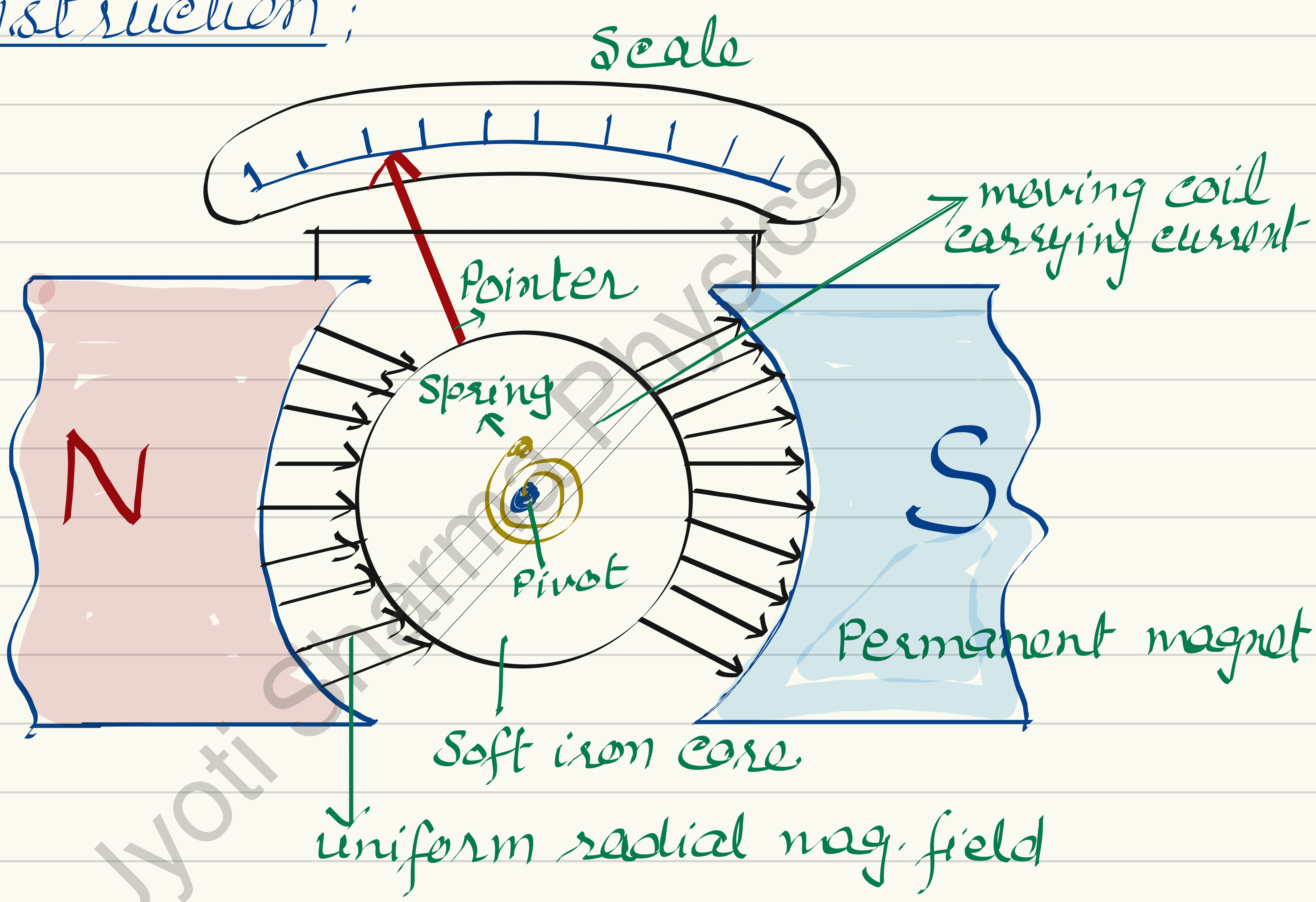
* Besides the orbital moment, the electron has an intrinsic magnetic moment which is called spin magnetic moment and has the same numerical value.

Moving coil Galvanometer (MCG)

This is an instrument used for detection and measurement of small electric current.

<u>Principle</u>: When a current carrying coil is placed in magnetic field it experience a torque.

<u>Construction</u>:



A Weston galvanometer consist of a rectangular coil of fine insulated copper wire wound on a light non magnetic (Al) frame. The two ends of circle of this frame are pivoted b/w two jewelled bearings. The motion is controlled by a pair of hair springs of phosphor bronze. The spring provided restoring torque. A pointer is attached to the coil to measure its deflection on a suitable scale.

A soft iron core is mounted b/w magnet. This makes the mag. field radial. Soft iron core increases the strength of magnetic field.

Working: When current flows in the coil a torque is set up. In equilibrium –

Deflecting torque τ_m = Restoring torque τ_R

$$NIABSin\theta = K\phi$$

here

$K \rightarrow$ Torsional constant

$\phi \rightarrow$ twist produced

for $\theta = 90°$

$$NIAB = K\phi$$

$$\phi = \left(\frac{NAB}{K}\right) I$$

or $\phi \propto I$

and $I = \left(\frac{K}{NAB}\right)\phi = G\phi$

where

$G = \frac{K}{NBA}$ = Galvanometer constant (current reduction factor)

Figure of Merit:

The ratio of current and the angle of deflection is called figure of merit.

$$G = \frac{I}{\alpha} = \frac{K}{NBA}$$

Sensitivity of Galvanometer:

A galvanometer is said to be sensitive if it shows large scale deflection even for small current.

Current Sensitivity: It is defined as the deflection per unit current.

$$I_S = \frac{\alpha}{I} = \frac{NAB}{K}$$

Voltage Sensitivity: It is defined as the deflection per unit voltage.

$$V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NAB}{KR}$$

Clearly $V_S = \frac{I_S}{R}$

* current sensitivity can be increased by -
  - increasing N, A and B
  - decreasing K

* Increasing I_S may not necessarily increase the voltage sensitivity. because

if $N \to 2N$

$I_S \to \frac{\phi}{I} \to 2\frac{\phi}{I}$ [If no. turns doubled, I_S also gets doubled]

but if $N \to 2N$
$R \to 2R$

thus $V_S \to \frac{\phi}{V} \to \frac{\phi}{V}$ $\left\{ V_S = \frac{2NAB}{K(2R)} = \frac{NAB}{R} \right.$

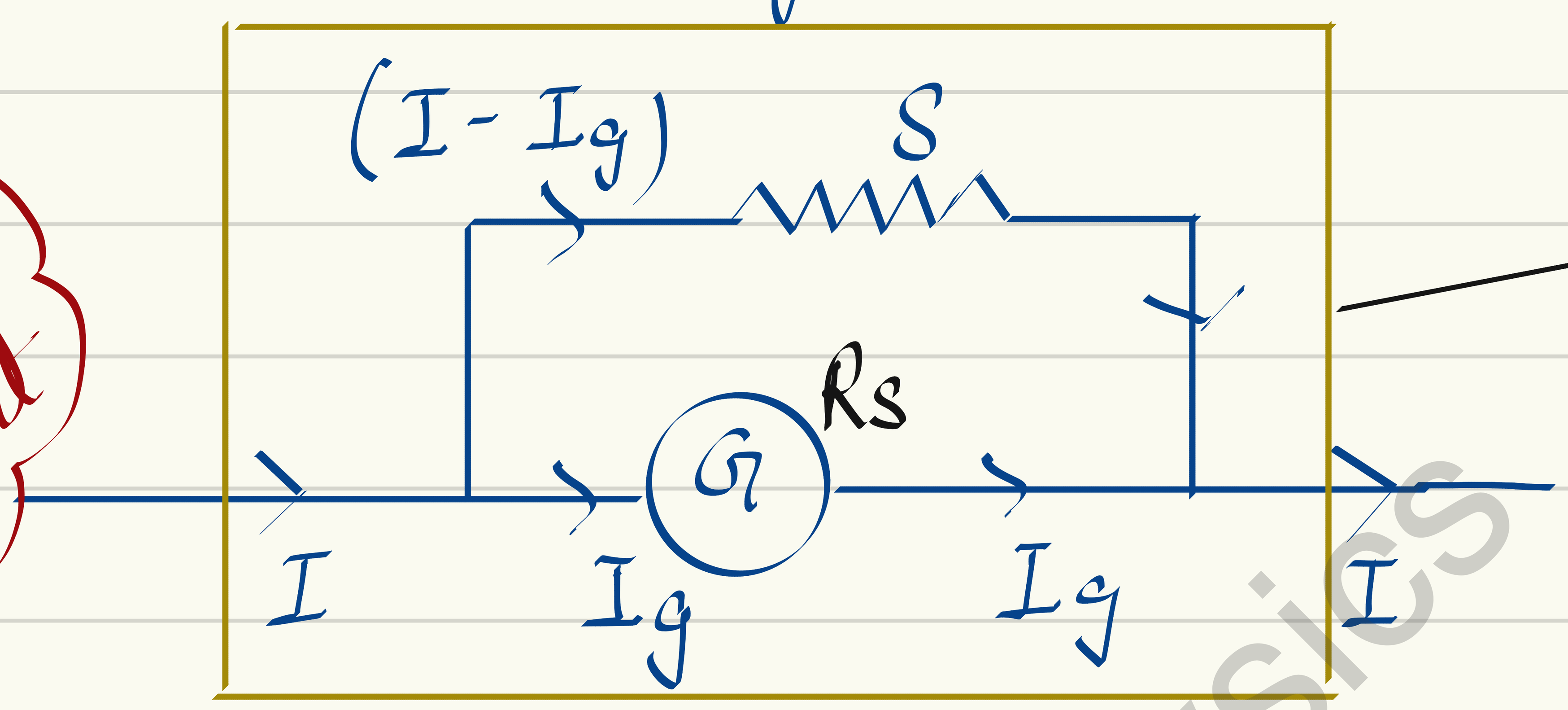
V_S remain unchanged on increasing N. but can be increase by increasing A & B and by decreasing K.

Conversion of Galvanometer Into Ammeter and Voltmeter

(1) Galvanometer as an Ammeter:

A galvanometer is converted into an ammeter by connecting a low resistance (shunt S) in parallel with galvanometer.

Ammeter is used to measure current so it has low resistance



→ Ammeter

Rs → Resistance of G

$$V_{galvanometer} = V_{shunt}$$

$$I_g R_g = (I - I_g) S$$

Ig → Current in galv.

G → Resistance of galvanometer

or $$\boxed{S = \frac{I_g R_g}{(I - I_g)}}$$

or $$(I - I_g) = I_g \frac{R_g}{S}$$

or $$I = I_g \frac{R_g}{S} + I_g$$

or $$I = I_g \left(\frac{R_g + S}{S} \right)$$

or $$I_g = \left(\frac{S}{R_g + S} \right) I$$

An ammeter has low resistance and it is always connected in series to the circuit

i.e. $$\boxed{I_g \propto I}$$

* To increase the range of ammeter n times shunt S to be connected as $S = G/(n+1)$

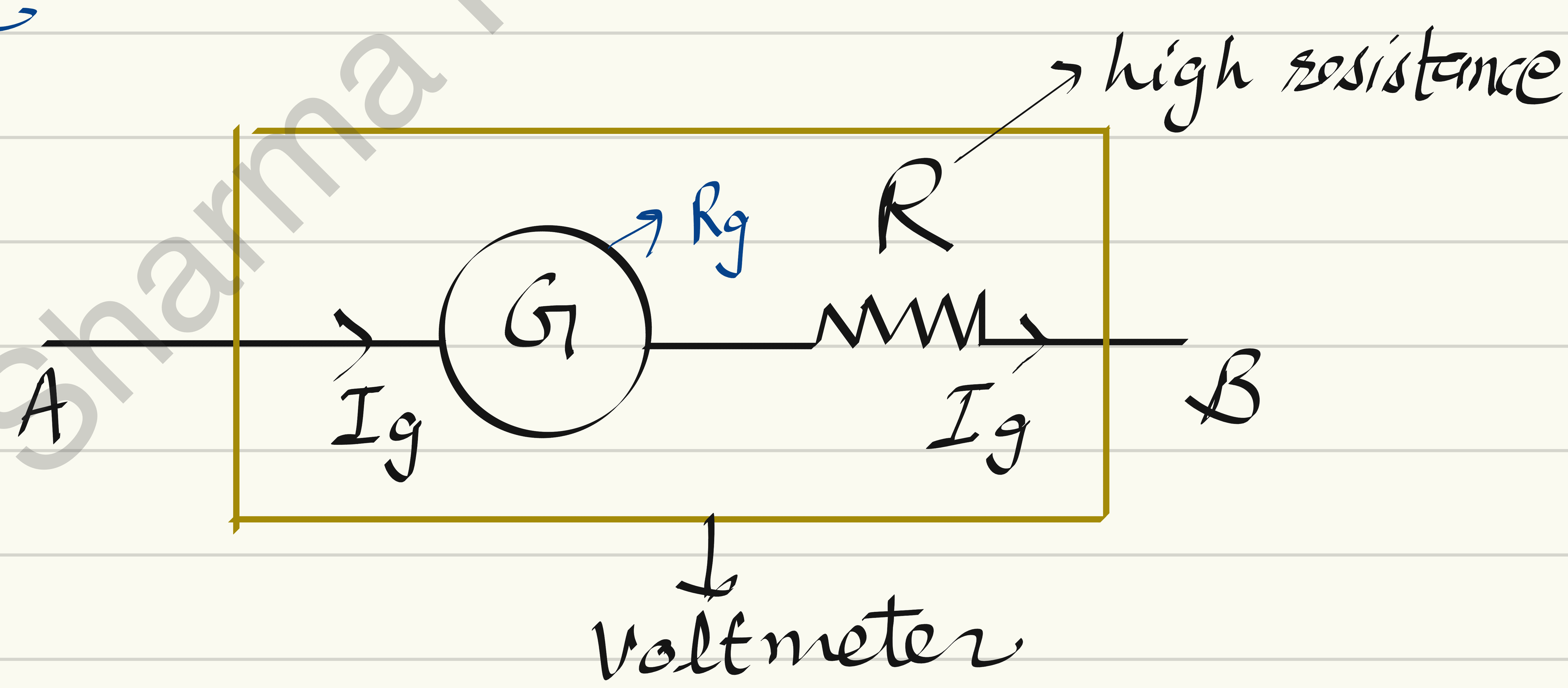
* Shunt S is connected in parallel to galvanometer. Therefore

$$\frac{1}{R_{eff}} = \frac{1}{R_g} + \frac{1}{S} \Rightarrow R_{eff} = \frac{R_g S}{R_g + S}$$

* An ideal ammeter has zero resistance. Therefore to convert the galvanometer into ammeter, it is connected in parallel to reduce the R_{eff}.

Galvanometer as a Voltmeter:

A galvanometer is converted into voltmeter by connecting high resistance R in series with galvanometer.



In series,

$$I = I_g$$

$$= \frac{V}{R_{eff}}$$ $V \rightarrow$ Potential difference

or $$I = \frac{V}{R + R_g}$$ $[R_{eff} = R + R_g]$

or $$R + R_g = \frac{V}{I}$$

or $$\boxed{R = \frac{V}{I} - R_g}$$

here $I_g \propto V$

* Since the galvanometer and high resistance R are connected in series

$$R_{eff} = R + R_g$$

* An ideal voltmeter has infinite resistance.
* Resistance of voltmeter is very high, so it draw least current.
* Voltmeter is always connected in parallel with circuit element through the potential is to be calculated.
* In order to increase the range of voltmeter n times, the resistance to be connected in series with G is

$$R = (n-1) G$$

Difference between Ammeter and Voltmeter:

S.No.	Ammeter	Voltmeter
1	It is a low resistance instrument.	It is a high resistance instrument.
2	Resistance is GS / (G + S)	Resistance is G + R
3	Shunt Resistance is $(GI_g) / (I - I_g)$ and is very small.	Series Resistance is $(V / I_g) - G$ and is very high.
4	It is always connected in series.	It is always connected in parallel.
5	Resistance of an ideal ammeter is zero.	Resistance of an ideal voltmeter is infinity.
6	Its resistance is less than that of the galvanometer.	Its resistance is greater than that of the voltmeter.
7	It is not possible to decrease the range of the given ammeter.	It is possible to decrease the range of the given voltmeter.