Work Energy And Power

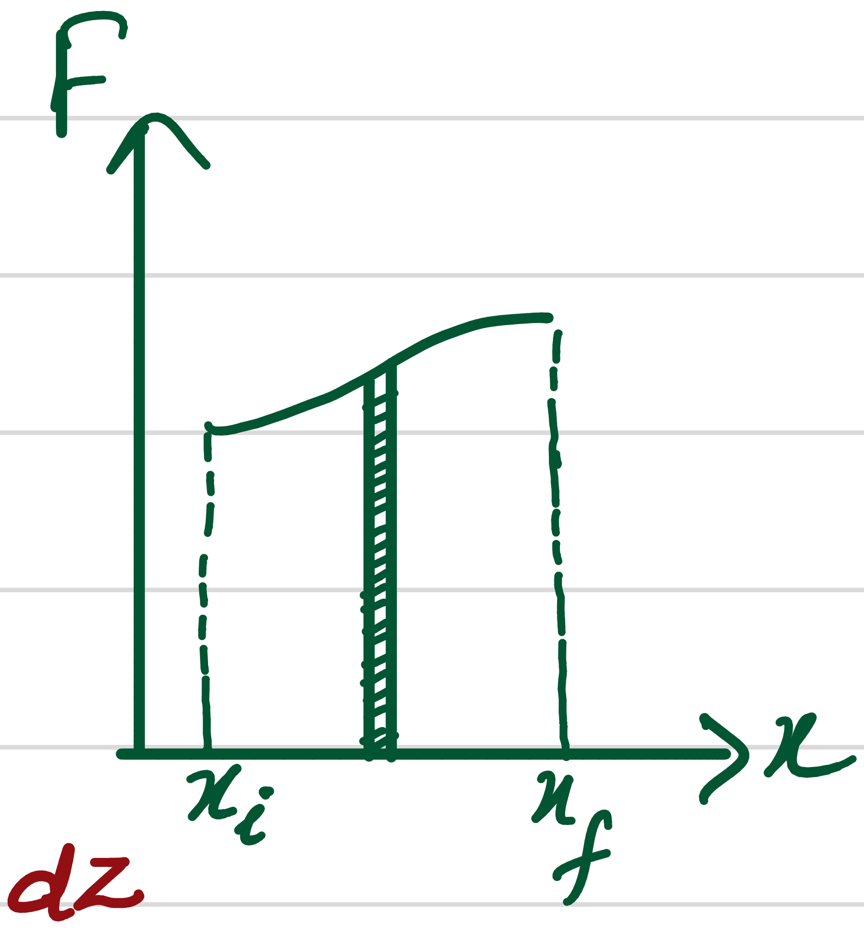
Formula Sheet

1 Work

(1) Work done by constant force

$$W = \vec{F} \cdot \vec{d} \cos\theta = \vec{F} \cdot \vec{d}$$

(II) Work done by variable force



$$W = \int \vec{F} \cdot \vec{dS} = \int F\, dS \cos\theta$$

$$W_{total} = \int_{x_1}^{x_2} F_x\, dx + \int_{y_1}^{y_2} F_y\, dy + \int_{z_1}^{z_2} F_z\, dz$$

2. Units of work

SI unit → joule (J) = N-m

CGS unit → erg

Gravitational unit

1 kg-m = 9.8 J

1 g-cm = 980 erg

Other units

Electron volt (eV) = 1 eV = 1.6×10^{-19} J

Kilo Watt hour (KWH) = 1 KWh = 3.6×10^{6} J

Calorie (cal) 1 cal = 4.18 J = 4.2 J

3 Work done ($= Fd\cos\theta$)

- Positive work: $0^\circ \leq \theta < 90^\circ$
- Negative work: $90^\circ < \theta \leq 180^\circ$
- Zero work: $\theta = 90^\circ$ or $F = 0$ or $d = 0$

4. Work done in terms of rectangular components

$$W = xF_x + yF_y + zF_z$$

5. Kinetic energy (K)

$$K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$m \rightarrow$ mass, $v \rightarrow$ velocity
$p \rightarrow$ linear momentum

* K.E of electrons is usually expressed in electron-volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J} \qquad [1 \text{ MeV} = 10^6 \text{ eV}]$$

6. Work-energy theorem

$$\Delta K = W$$

Change in kinetic energy = Work done

$$K_f - K_i = \text{Work done.}$$

or

$$W = \frac{1}{2} m (v^2 - u^2)$$

7. Potential energy (U) — SI unit → Joule

(I) Gravitational Potential energy $U = mgh$

(II) Elastic potential energy $U = \frac{1}{2} k x^2$

8. Conservation of mechanical energy

Under the action of conservative force

i.e $K_i + U_i = K_f + U_f$

Initial M.E = Final M.E

$K + U$ = constant

Total mechanical energy

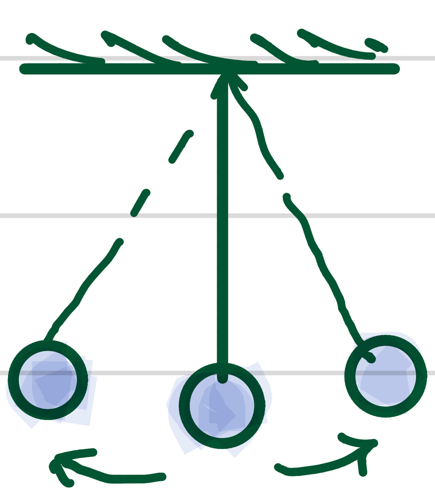
$K_i \rightarrow$ initial K.E
$K_f \rightarrow$ final K.E
$U_i \rightarrow$ initial P.E
$U_f \rightarrow$ final P.E

9. Conservation of mechanical energy for a simple pendulum

At any point of motion,

Kinetic energy = Potential energy

$$\frac{1}{2} m v^2 = mgh$$



10 Elastic potential energy

When a spring is stretched or compressed the energy sto in a spring.

$$U - W = \frac{1}{2} k x^2$$

Elastic P.E. is always true

here, k = spring constant

x = displacement from the mean position.

* Maximum speed of the spring

$$v_m = \sqrt{\frac{k}{m}}\, x_m$$

11. Motion of a body in vertical circle

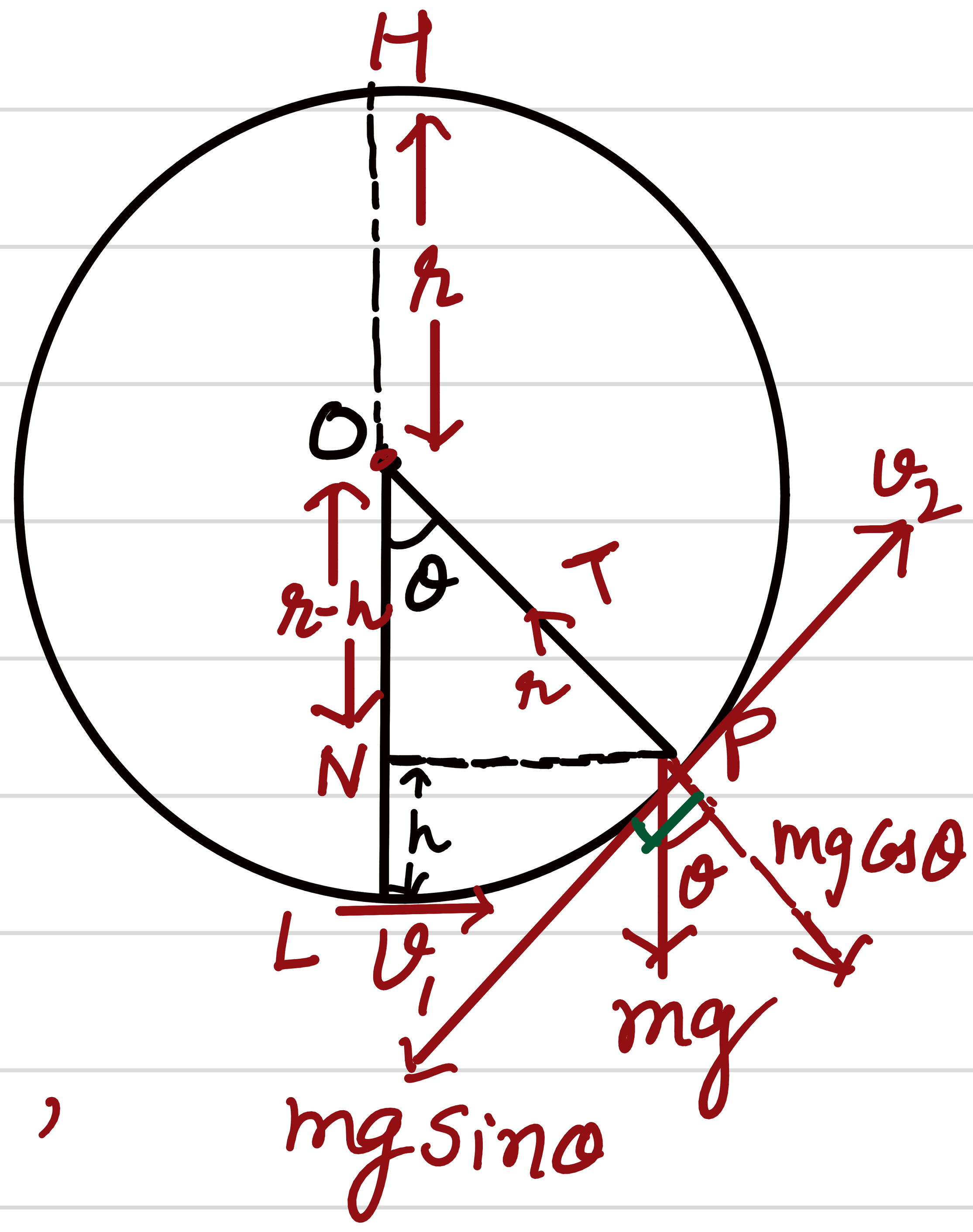
In figure

H = highest point

L = Lowest point

v_1 = Velocity at point 'L'

v_2 = velocity at point 'P'



(i) Tension at the lowest point 'L'

$$T_L = \frac{m v_1^2}{r} + mg$$

(ii) Tension at the highest point 'H'

$$T_H = \frac{m}{r}\left[v_1^2 - 5gr\right]$$

(iii) Speed of the stone

→ At the lowest point 'L'

$$V_L = V_1 \geq \sqrt{5gr}$$

→ At the heighest point 'H'

At $2r$ height

$$v_2 = v_H = \sqrt{rg}$$

Einstein's mass and energy equivalence

$$E = mc^2$$

m = mass , c = speed of light

Power (P)

$$P = \frac{\Delta W}{\Delta t}$$

SI unit → watt or JS^{-1}

$1\,KW = 10^3\,W$

$1\,MW = 10^6\,W$

$1\,hp = 746\,W$

→ Instantaneous power

$$P = \frac{dW}{dt}$$

→ Relation between P, F and v

$$P = \vec{F} \cdot \vec{v}$$

or $$P = F v \cos\theta$$

F → force

v → velocity

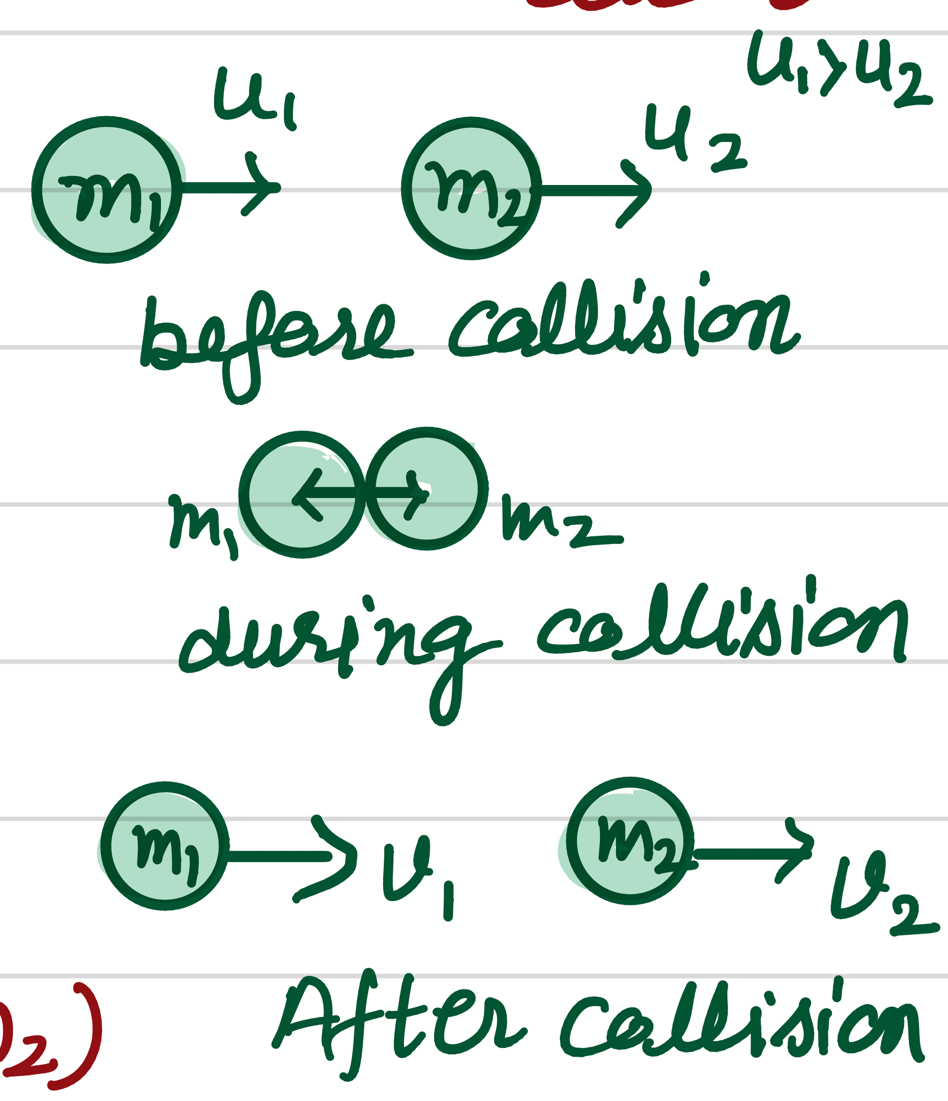
θ → angle b/w $\vec{F}$ & $\vec{v}$

Collisions

Elastic one-dimensional collision

Relative velocity of approach = Relative velocity of separ-ation

$$u_1 - u_2 = v_2 - v_1$$



Coefficient of restitution (e)

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Velocities after collision

$$v_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{(m_1 + m_2)}$$

and $$v_2 = \frac{2m_1u_1 + u_2(m_2 - m_1)}{m_1 + m_2}$$

→ When $m_1 = m_2 = m$, than

$v_1 = u_2$ and $v_2 = u_1$

→ When $u_2 = 0$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

$$v_2 = \frac{2 m_1 u_1}{m_1 + m_2}$$

Completely Inelastic collision in one dimension

(After collision both the bodies move together)

The loss of in K.E

$$\Delta K = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) V^2$$

and

$$\Delta K = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} u_1^2$$

Important conceptual points

- Conservative force → Work done is path-independent e.g. gravity, spring
- Non-Conservative force → Work depends on path. e.g. friction, air resistance.
- In absence of non conservative force - Mechanical energy (K.E + P.E) remains constant
- Spring force (restoring) $F = -kx$ (Hooke's law)
- Linear momentum is always conserved in all type collisions (elastic or inelastic), if no external force is acting.
- Elastic collision in 1D (head on) has special formulas:

→ Velocities are exchanged when masses are equal (if $m_1 = m_2 \Rightarrow u_1 = v_2$ and $u_2 = v_1$)

→ Total energy before and after remain same.