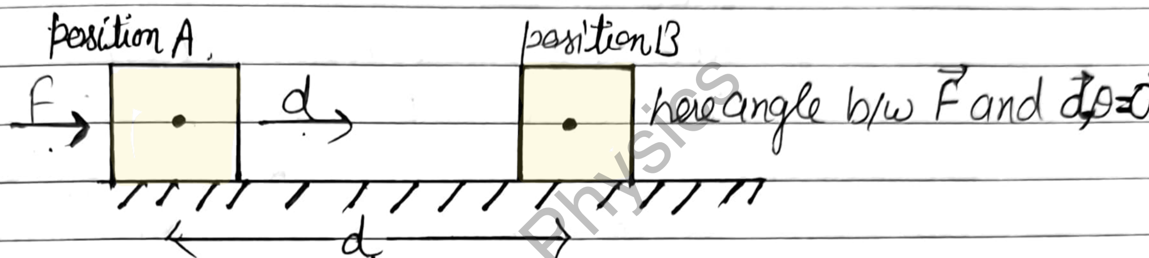
Work, Energy And Power

Chapter-5

Work - The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.

OR

Work is said to be done when a force applied on the body diplaces the body through a certain distance in the direction of applied force.



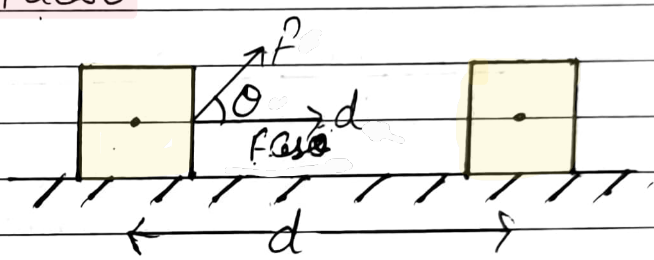
$$W = (F\cos\theta)\, d = Fd\cos\theta$$

for $\theta = 0°$

$$W = Fd\cos 0°$$

or $\boxed{W = Fd}$

for angle θ



$$\boxed{W = Fd\cos\theta}$$

or $\boxed{W = \vec{F}\cdot\vec{d}}$ W is scalar quantity.

Unis of work -

SI unit → joule (J)

CGS unit → erg

1 Joule → Work is said to be 1 joule if 1 newton force displaces a body through 1 m in its direction.

1 Joule = 1 newton × 1 m

Gravitational Unit -

In SI system - Kg-m

1 Kg-m = 9.8 J [1 Kgf × 1m = 9.8 J]

(Amount of work done in lifting up 1 Kg weight vertically by 1m)

In C.G.S system - g-Cm

1 g-Cm = 980 erg

Other Units -

Electron volt (eV) : $1\ eV = 1.6 \times 10^{-19} J$

Kilo Watt-hour (KWh) : $1\ KWh = 3.6 \times 10^{6} J$

Calorie (cal) : $1\ cal = 4.18 J = 4.2 J$

Dimensions - $[ML^2T^{-2}]$ (Work and Energy have same dim)

Nature of Work Done in Different Situations
(Positive, negative and zero work done)

(I) Positive Work done : (When $0° \leq \theta < 90°$, ie $\cos\theta$ is +ve)

If $\theta = 0°$, ie the force and displacement are in the same direction, then

$W = FS\cos\theta$

$W = FS\cos 0°$

or $W = FS$

This type of work done is said to be 'positive work'.

Examples -

- When a spring is stretched, the work done by the stretching force is positive.
- When a lawn roller is pulled or pushed by applying a force, then the work done is positive in both cases.
- In a tug of war, work done by the winning team is positive.

(II) Negative Work done : (When $90° < \theta \leq 180°$, ie $\cos\theta$ is -ve)

If $\theta = 180°$, ie the force and displacement are in opposite direction, then,

$W = FS\cos 180°$

$= FS(-1)$ $[\because \cos 180° = -1]$

or $W = -FS$

This type of work is said to be negative work.

Examples -

- When a body is raised vertically upward from one point to another, then work done by the force of gravity is '-ve'.

$W = mgh\cos 180°$

or $W = -mgh$

- When brakes are applied to stop a moving car the work done by the braking force is '-ve'.
- Work done by a friction force is -ve.
- In tug of war work done by losing team is '-ve'.

(III) Zero Work done :

No work is done if :

(a) $\theta = 90°$, ie force and displacement are mutually perpendicular.

$W = FS\cos 90°$

or $W = 0$ $[\because \cos 90° = 0]$

example - When someone carrying a bag on a horizontal road, the work done is zero.

(b) If displacement $d = 0$

$$W = F d \cos\theta$$
$$W = F \times 0 \cos\theta$$
or $$W = 0$$

i.e work done by a force is zero if there is no displacement in the body.

example – A weightlifter is holding 150 kg mass steadily on his soulder for 30 s does no work during this time.

Also no work is done in pushing a wall or rock.

(c) If force is zero ($F = 0$)

$$W = F d \cos\theta$$

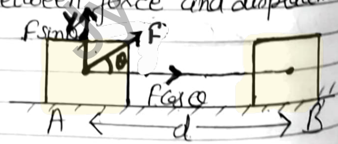
If $F = 0$ $\quad W = 0$

example – A block moving on a smoth horizontal table ($F = 0$ since friction is zero) undergo a large displacement but work done is zero.

Work done by a constant Force.

Consider a body on horizontal surface. Let force F is applied on it. Angle between force and displacement is θ.

Resolve force F into two components



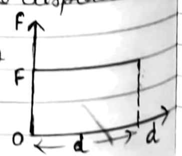
(i) $F\cos\theta$, in the direction of displacement

(ii) $F\sin\theta$, in perpendicular direction to the displacement

then

$$W = (F\cos\theta)d = F d \cos\theta$$

or $$\boxed{W = \vec{F}\cdot\vec{d}}$$



Thus work done is equal to the scalar product or dot product of the force and displacement.

In terms of rectangular components

$$\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

then by

$$W = \vec{F}\cdot\vec{d}$$

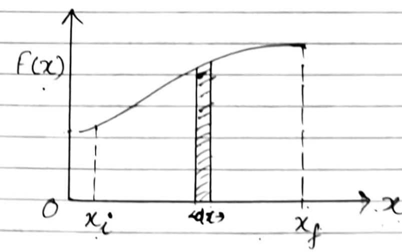
$$W = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(x\hat{i} + y\hat{j} + z\hat{k})$$

or $$\boxed{W = xF_x + yF_y + zF_z} \qquad [\because \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1]$$

Work done by a Variable force :-

Let us consider a variable force acting on a body. Let the body is displaced in the direction of applied force.

The graph between the variable force $F(x)$ and the displacement (x) is shown in fig.



$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\,\Delta x$$

or $$\boxed{W = \int_{x_i}^{x_f} F(x)\,dx}$$

or Work done = Area under force and displacement graph

Energy - The energy of a body is its capacity for doing work.

It is measured by the total amount of work that a body can do.

It is scalar quantity.

Units - The units and dimensions of energy are the same as of the work.

SI unit - joule (J)

Forms of Energy - There are many forms of energy such as mechanical energy, heat energy, light energy, sound energy, electrical energy, magnetic energy, chemical energy and nuclear energy etc.

Mehanical Energy - The energy possessed by an object due to its motion and position is called mechanical energy.

Mechanical Energy

- Energy due to motion → Kinetic Energy
- Energy due to position → Potential Energy

Mechanical energy is the sum of kinetic energy and potential energy.

$M.E = K.E + P.E.$

Kinetic Energy -

The energy possessed by a body by virtue of its motion is called kinetic energy.

Examples -

- Flowing water (water mills)
- Moving vehicles
- Moving air (wind mills)
- Bullet fired from the gun.

* All moving bodies are example of kinetic energy.

Expression -

Consider a body of mass m, initially at rest. A constant force f is applied on it and the body starts moving with acceleration a.

Now by $v^2 = u^2 + 2aS$

here $u = 0 \Rightarrow v^2 = 2aS$

or

$$a = \frac{v^2}{2S}$$

By $F = ma$

$$F = \frac{mv^2}{2S}$$

We know

$$W = F \times S$$

so $W = \frac{mv^2}{2S} \times S$

or $\boxed{W = \frac{1}{2} mv^2}$

Relation b/w K.E and linear momentum (p)

$$K = \frac{1}{2} mv^2 = \frac{1}{2} mv^2 \times \frac{m}{m}$$

$$\Rightarrow K = \frac{(mv)^2}{2m}$$

$$\Rightarrow \boxed{K = \frac{p^2}{2m}}$$

or $K.E = W = \frac{1}{2} \times \text{mass} \times \text{velocity}^2$

Units -
SI unit - joule , C.G.S unit - erg
dimensions - $[ML^2T^{-2}]$
* K.E of electrons is usually expressed in electron-volt (eV)
$1 eV = 1.6 \times 10^{-19} J$
$1 MeV = 1.6 \times 10^{-13} J$ $[\because 1 MeV = 10^6 eV]$

Work-Energy Theorem - (For constant force)

This theorem states that the work done by a force on a body is equal to the change in kinetic energy of the body.

We know for rectilinear motion.

$$v^2 = u^2 + 2as$$

where u and v are initial and final speeds and s is the distance.

Multiply both sides by $\frac{1}{2}m$, we get-

$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}m(2as)$$

or $$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mas$$

or $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

or Final K.E − Initial K.E = F.S $[\because F = ma]$

or $$K_f - K_i = W \quad [\because W = F.S]$$

or $$\boxed{\Delta K = W}$$

or change in kinetic energy = Work done
This is known as work-energy theorem.

Work Energy theorem for a variable force -

The time rate of change of kinetic energy is -

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$= m \times \frac{1}{2} \times 2v\frac{dv}{dt}$$

$$= m\frac{dv}{dt}v$$

$$= (ma)v \quad \left[\because a = \frac{dv}{dt}\right]$$

$$= Fv \quad [\because F = ma]$$

or $$\frac{dK}{dt} = F\frac{dx}{dt} \quad \left[\because v = \frac{dx}{dt}\right]$$

or $$dK = F\,dx$$

on integrating -

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F\,dx$$

or $$K_f - K_i = W \quad \left[\because W = \int_{x_i}^{x_f} F\,dx\right]$$

or $$\Delta K = W$$

This is Work-Energy theorem by variable force.

* If there is no change in velocity.
i.e $u = v$, then
by $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$W = 0$

e.g. A particle moves in circular path with zero work done.
* If there is decreasing speed
i.e $v < u$
by WE theorem
$W = \frac{1}{2}m(v^2 - u^2)$

as $v < u$
W = negative
e.g. Work done by gravitional force is -ve, when an object is projected upward
* If velocity of the particle is increasing.
i.e $v > u$
then $W = \frac{1}{2}m(v^2 - u)$

or W = positive [$\because v > u$]
e.g. Work done by gravitational force when an object is dropped, is +ve.

The concept of potential Energy.
The energy possessed by a body by virtue of its position or configuration is called ~~pocl~~ potential energy.
Potential energy is a stored energy.
Examples -
- Water stored in a dam.
- An object at certain height from earth surface.
- A stretched spring.

Types of Potential Energy -
(i) Gravitational potential Energy
(ii) Elastic potential Energy

Gravitational potential Energy.
Energy possessed by a body due to its height from the earth surface is known gravitational potential energy.

Elastic Potential Energy
Energy possessed by a body by virtue of its deformed shape. (i.e. streched or compressed)
Units
SI unit - Joule
CGS unit - erg
dimensions - $[ML^2T^{-2}]$

Gravitational Potential Energy
Consider a block of mass m raised at height h above the ground.
Work done by applied force $\vec{F}$ against the gravitational force
$W = \vec{F} \cdot \vec{h}$
$= Fh\cos\theta$
$= Fh\cos 0^\circ = Fh$ [$\because \theta = 0$]
Now work done by gravitational force
$W = \vec{F_g} \cdot \vec{h} = F_g \cdot h \cos 180^\circ$ [$\theta = 180$]
or $W = -F_g h$ [$\cos 180^\circ = -1$]
or $W = -mgh$ [$F_g = -mg$]

When object falls from height h

$W = mgh$

or $U_g = mgh$ $\quad$ $U_g \rightarrow$ gravitational P.E

Change in gravitational potential energy
Consider a body at height h_1 from the earth surface. Let the body raised to height h_2 $(h_2 > h_1)$, then work done by gravitational force

$$\Delta W = -mg(h_2 - h_1)$$
$$= -mgh_2 + mgh_1$$
$$= -U_2 + U_1$$
$$= -(U_2 - U_1)$$

or $\Delta W = -\Delta U$

-ve sign shows that work done by the gravitational force is negative.

* At the surface of earth gravitational potential energy is zero. $(\because h = 0)$
* It is independent of path. It means depends only initial and final position.

The conservation of Mechanical Energy -

I Method If the total work done by external forces and all internal force is zero

$$\Delta E = 0$$

$\Delta KE + \Delta U = 0 \Rightarrow K_f - K_i + U_f - U_i = 0$

or $K_f + U_f = K_i + U_i$

i.e Initial M.E = Final M.E

i.e Mechanical Energy is conserved.

II Method (NCERT) Under the action of conservative force, by using W E theorem

$$\Delta K = \Delta W = F(x)\Delta x \quad -①$$

and potential energy

$$\Delta W = -\Delta U = F(x)\Delta x \quad -②$$

from eqn ① and ②

$\Delta K = -\Delta U$

or $\Delta K + \Delta U = 0$

or $\Delta(K + U) = 0$

i.e $K + U = $ constant

which means that the sum of kinetic and potential energy of the body is constant.

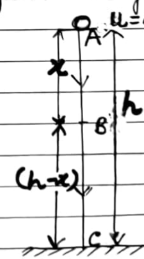
For the whole path x_i to x_f

$$\boxed{K_i + U_i = K_f + U_f}$$

The total M.E of a system is conserved if the forces doing work on it are conservative.

Example - Conservation of Mehanical Energy of A freely falling body

Consider a body of mass 'm' initially at position A. The point A is at height h from the ground.



At point A

$K.E = \frac{1}{2}mv^2 = 0$

and $P.E = mgh$

The total energy i.e mechanical energy at point A

$M.E = K.E + P.E$

$= 0 + mgh$

$M.E = mgh$ —(1)

At point B

$v^2 = u^2 + 2aS$, here $a \to g$, $S \to x$ and $u = 0$, so

$v^2 = 2gx$

Now

$K.E = \frac{1}{2} m (2gx)$

$= mgx$

and

$P.E = mg(h-x)$

Hence

$M.E = K.E + P.E$

$= mgx + mg(h-x)$

$= \cancel{mgx} + mgh - \cancel{mgx}$

$M.E = mgh$ —— (2)

At point C

$v^2 = u^2 + 2gh$

here $u = 0$ and $h = h$, so

$v^2 = 2gh$

Now $K.E = \frac{1}{2} m (2gh)$

$K.E = mgh$

and $P.E = mgh = 0$ [$\because$ from ground $h = 0$]

Hence

$M.E = K.E + P.E$

$= mgh + 0$

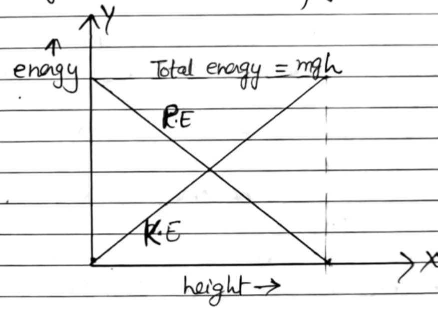
or $M.E = mgh$ — (3)

from eqn (1), (2) and (3), at point A, B and C

$M.E = mgh$

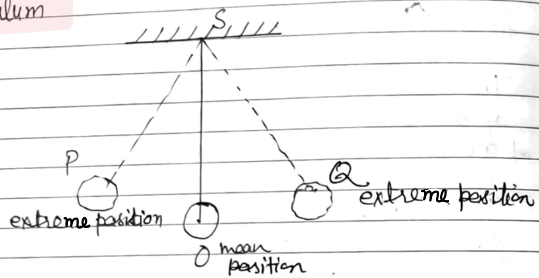
Therefore we can see that mechanical energy of a body during the free fall under the action of gravity remains constant.

The variation of total M.E., K.E and P.E with the height is shown in fig. as



* At maximum height K.E is zero. As object starts falling K.E energy increases and at the time of hitting ground K.E is maximum.
* At maximum height P.E is maximum. As object starts falling P.E. decreases ($\because$ h is decreasing) and at the ground P.E becomes zero.
* In the journey towards ground K.E increases and P.E. decreases.

Conversion of Mechanical energy of a Simple pendulum



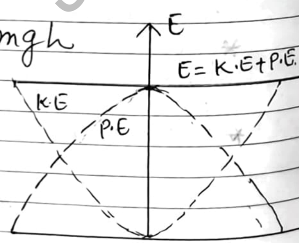
The motion of a simple pendulum is a good example of conservation of mechanical energy.

* At point P, P.E is maximum and K.E is minimum. When pendulum ~~stat~~ moves towards O, K.E increases and P.E decreases.
* At point O K.E becomes maximum. It is clear P.E is converted in K.E.
* When pendulum moves O to P K.E decreases and potentential energy increases. At point Q, K.E becomes zero and P.E becomes maximum.

So. K.E at O = P.E at P or at Q

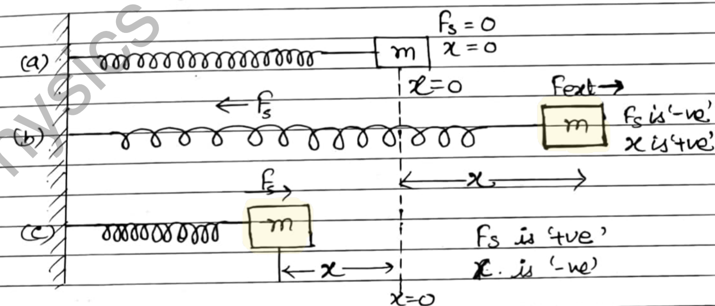
i.e $\frac{1}{2}mv^2 = mgh$

The variation of K.E & P.E of a simple pendulum is shown in figure



Elastic Potential Energy of a Spring

When a spring is stretched or compressed from its equilibrium position ($x=0$) by a small distance x, then a restoring force is developed in the spring to bring it back.



According to force law of spring (Hooke's law)

$$F_s = -kx$$

where k is called spring constant.
when block is pulled outward, the work done by the spring force is

$$W_s = \int_0^x F_s\,dx$$

$$= -\int_0^x kx\,dx$$

$$\boxed{W_s = -\frac{1}{2}kx^2}$$

The same is true when spring is compressed.
Also the work done by applied force

$$\boxed{W = \frac{1}{2}kx^2}$$

Graphical method -

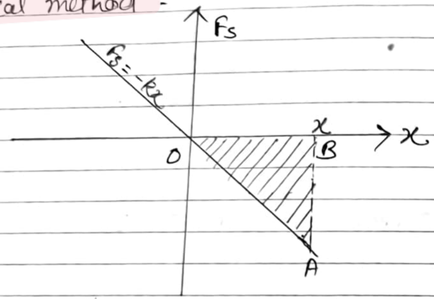


Fig. shows the graph between spring force and the displacement.

Work done in compressing or stretching the spring is given by

$$W = \text{area under force displacement graph}$$
$$= \text{area of shaded portion}$$
$$= \text{area of } \Delta OAB$$
$$= \frac{1}{2} OB \times AB$$
$$= -\frac{1}{2} x \times kx$$
$$W = -\frac{1}{2} kx^2$$

This work done is '-ve' by spring force and '+ve' by applied force.

This work done is stored in the spring as elastic potential energy.

* Elastic potential energy is always '+ve'. because x^2 is always +ve and k is also +ve.

Spring Constant -

The restoring force per unit displacement is called force constant.

$$k = \frac{F}{x}$$

SI unit is $N m^{-1}$

CGS unit is dyne cm^{-1}

Dimension[k] = $[M T^{-2}]$

* Spring is said to be stiff if k is large and soft if k is small.

Spring force is conservative -

If a block is moved from x_i to x_f, the work done by the spring

$$W_s = -\int_{x_i}^{x_f} kx\,dx = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}$$

i.e. the work done by the spring force depends only on the ends point.

If the block is pulled from x_i and allowed to return to x_i

$$W_s = \int_{x_i}^{x_i} kx\,dx$$
$$= \frac{kx_i^2}{2} - \frac{kx_i^2}{2}$$
$$= 0$$

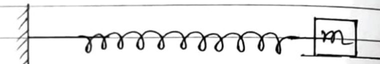
The work done by a spring force in a cyclic process is zero.

As the work done by spring force depends only initial and final position therefore spring force is conservative force.

Maximum speed of the spring-

Potential energy of the spring is zero when block and spring are in equilibrium.

ie $U=0$



For an extension and compression x.

$$U = \frac{1}{2}kx^2$$

or $$\frac{dU}{dx} = kx$$

If the spring is stretched to x_m and released, the its total M.E at any point x will be

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

[x lies b/w $+x_m$ to $-x_m$]

from this result it is clear that- speed and K.E will be maximum at $x=0$, i.e.

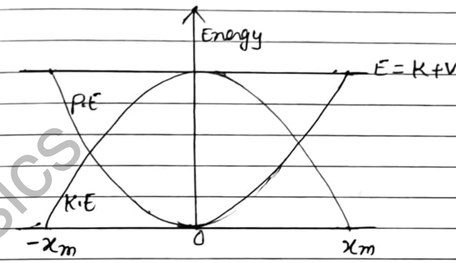
$$\frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$ $v_m \rightarrow$ max. velocity

or $$v_m^2 = \frac{kx_m^2}{m}$$

or $$v_m = \sqrt{\frac{k}{m}}\,x_m$$

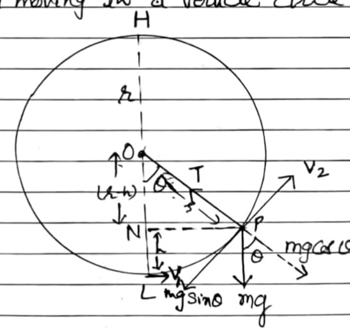
The K.E gets converted to potential energy and vice-versa.

Graphical representation-



Motion of a body in verticle circle-

Consider a stone of mass m tied to one end of the string and moving in a verticle circle of radius r.



H → heighest point
L → lowest point
v_1 → velocity at L
v_2 → velocity at P

Kinetic energy of stone at L $= \frac{1}{2}mv_1^2$

Potential energy at L $=0$

Total energy $= \frac{1}{2}mv_1^2 + 0$

$= \frac{1}{2}mv_1^2$

K.E at P = $\frac{1}{2}mv_2^2$

P.E at P = mgh

Total energy = $\frac{1}{2}mv_2^2 + mgh$ —(1)

According to the law of conservation of energy

$$\frac{1}{2}mv_2^2 + mgh = \frac{1}{2}mv_1^2$$

or $$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - mgh$$

or $$v_2^2 = v_1^2 - 2gh \quad —(2)$$

it is clear that $v_2 < v_1$

Tension in the string –

Various forces on the stone at point P

(i) Tension T, along PO

(ii) Weight mg ↓

(iii) mg Cosθ and mg Sinθ are the components of weight mg

Now $$T - mg\cos\theta = \frac{mv_2^2}{r}$$

or $$T = \frac{mv_2^2}{r} + mg\cos\theta$$

put the value of v_2 from eqn (2)

$$T = \frac{m}{r}(v_1^2 - 2gh) + mg\cos\theta$$

from fig $\cos\theta = \frac{r-h}{r}$

so. $$T = \frac{m}{r}\left[v_1^2 - 2gh\right] + mg\left(\frac{r-h}{r}\right) \quad — (3)$$

Tension at lowest point L

at L, h=0

then

$$T_L = \frac{mv_1^2}{r} + mg$$

$$= \frac{m}{r}(v_1^2 + rg)$$

Tension at highest point H

At highest point, h = 2r

then from eqn (3)

$$T_H = \frac{m}{r}\left[v_1^2 - 2g(2r)\right] + mg\left[\frac{r-2r}{r}\right] \quad [\because h = 2r]$$

$$= \frac{m}{r}(v_1^2 - 4gr) - mg$$

or $$T_H = \frac{m}{r}(v_1^2 - 5gr)$$

Difference in tension

$$T_L - T_H = \frac{m}{r}(v_1^2 + rg) - \frac{m}{r}(v_1^2 - 5gr)$$

$$T_L - T_H = 6mg$$

Speed of the stone

(1) At lowest point L

In order to keep moving in circular path

$$T_H \geq 0$$

i.e $\frac{m}{r} + (v_1^2 - 5gr) \geq 0$

or $v_1^2 \geq 5gr$

or $v_1 \geq \sqrt{5gr}$

hence the minimum speed at L so that it may go round

$$v_L = v_1 = \sqrt{5rg}.$$

At height 2r (At highest point H)

from eqn (2)

$$v_2^2 = v_1^2 - 2gh$$

put $v_1 = \sqrt{5rg}$ and $h = 2r$,

$$v_H^2 = v_2^2 = 5rg - 2g(2r)$$

or $v_H^2 = rg$

or $v_H = \sqrt{rg}$

Which is the velocity of the stone at the highest point for looping.

Practical application -

(i) Water in a bucket does not spill if it rotated in a verticle circle.

(ii) A motor cyclist in a circus drives the motor cycle along a verticle circle in a cage.

(iii) A piolet loops a loop without falling at the top.

Different forms of Energy -

We have discussed one form of energy i.e mechanical energy which is the sum of K.E and M.E.
The other forms of energy are -

(i) **Heat or Thermal Energy** - An object possesses heat energy due to the disorderly motion of the molecules. e.g. friction produces heat. In winter to feel warmer we rub our hands together.

(ii) **Chemical Energy** - Chemical energy is due to the chemical bonding of the atoms of the chemical compound. It's produced by chemical reaction.
A chemical reaction is said to be exothermic if heat is released during chemical reaction.
If heat is absorbed in chemical reaction, it is called endothermic reaction.

(iii) **Electrical Energy** - The work done against the electric forces appears as the electrical energy. This energy is used to glow bulbs, to run machines etc.

(iv) **Nuclear Energy** - The energy required to hold the protons and neutrons in the nucleus of an atom is called nuclear energy.
The nuclear energy is released in nuclear fusion and nuclear fission.
Nuclear fusion is a process in which light nuclei combin (or fuse) together to form a relatively heavy nucleus.

The energy released during nuclear fusion is given by

$$E = \Delta mc^2$$

where Δm is the difference between the mass of light nuclei and heavy nucleus; $c = 3\times10^8$ m/s is the speed of light in vacuum.

Nuclear fission is a process in which a less stable nucleus (heavy) like $_{92}U^{235}$ breaks up into two stable nuclei along with the release of energy.

The equivalence of Mass and Energy -

Albert Einstein in 1905 showed that mass and energy are equivalent and are related by

$$E = mc^2$$

where m is mass and c is speed of light in vac.

Thus for m = 1 kg the energy associated is

$$E = 1\times(3\times10^8)^2$$

or $$E = 9\times10^{16}\,J$$

which a very large amount of energy.

Principle of conservation of Energy -

If forces are conservative M.E remains constant but if forces are non-conserved some part of energy is transferred into another form of energy like heat, light and sound. But the total energy remains constant. i.e.

For an isolated system, total energy is constant.

Principle of conservation of energy cannot be proved but no violation has been observed.

Power -

Power is defined as the rate of doing work.

If an amount of work ΔW is done in the interval of time Δt, then average power is given by

$$\bar{P} = \frac{\Delta W}{\Delta t}$$

The instantaneous power P is defined as the instantaneous rate of doing work.

$$P = \lim_{\Delta t\to 0} \frac{\Delta W}{\Delta t}$$

or $$P = \frac{dW}{dt}$$

Units -

SI unit - Watt (W)

1 Watt = 1 J/s

Power is said to be one watt if one joule work is done in one second by any agent.

Other units are -

$1\,KW = 10^3\,W$

$1\,MW = 10^6\,W$

1 horse power (hp) = 746 W

Dimension - $[ML^2T^{-3}]$

It is scalar quantity.

Power and Energy -

$$\text{Power} = \frac{\text{Work done}}{\text{Time}}$$

Since work done = Energy supplied or Energy consumed or $W = E$, then

$$P = \frac{\text{Energy}}{\text{Time}}$$

or Energy = Power x time

Power in terms of velocity -

We have

$$P = \frac{dW}{dt}$$

but $W = \vec{F}\cdot\vec{S}$

so. $$P = \frac{d(\vec{F}\cdot\vec{S})}{dt}$$

or $$P = \vec{F}\cdot\frac{d\vec{S}}{dt}$$

or $$\boxed{P = \vec{F}\cdot\vec{V}}$$

In vector form -

$$P = \vec{F}\cdot\vec{V} = FV\cos\theta$$

where θ is the angle b/w $\vec{F}$ and $\vec{V}$.

* If force acts in the direction of motion i.e if $\theta = 0$, then

$$P = FV\cos 0^\circ \quad [\cos 0^\circ = 1]$$

or $P = FV$

* If $\theta = 90^\circ$, then

$$P = 0 \quad [\cos 90^\circ = 0]$$

i.e Perpendicular to motion → no work → no power.

$$\boxed{P = \vec{F}\cdot\vec{v} = Fv\cos\theta}$$

Conservative and Non conservative forces

	Conservative forces	Non conservative forces
1.	Work done is independent of the path.	• Work done depends upon the path
2.	Work done in a round trip is zero	• Work done in a round trip is not zero.
3.	Total energy remains constant	• Energy is dissipated as heat energy.
5.	Force is -ve gradient of potential energy	• No such relation exists.
6.	e.g. gravitation force, magnetic force, spring force.	• e.g Friction and air drag

Collisions – The interaction between two bodies or particles due to which the direction and the magnitude of the velocity changes.

Types -

1. Perfectly Elastic collision
2. Inelastic collision
3. Perfectly inelastic collision

Perfectly Elastic collision – If linear momentum and kinetic energy both remain conserved.

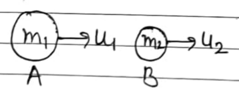
e.g. It is a rare event but collisions b/w atomic particle, billiard balls are nearly elastic collisions.

Inelastic collisions - If linear momentum remains conserved but kinetic energy is not conserved.
e.g. A stone smashes a window pane. then collision between the stone and window pane is inelastic.
* loss of K.E. appears in the form of heat, sound and light energy etc.

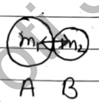
Perfectly Inelastic collision - If the two bodies after collision stick together and move as one body, collision is called perfectly inelastic collision.
In this collision linear momentum remains conserved but K.E is not conserved. e.g. bullet embedded into a block of wood.

Elastic One-Dimensional Collision -

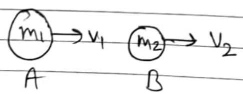
Consider two bodies A and B of masses m_1 and m_2 moving in a straight line with velocities u_1 and u_2 respectively ($u_1 > u_2$). After collision, let their velocities change to v_1 and v_2 respectively.



Before collision



during collision



After collision

Total linear momentum of the system before collision $= m_1u_1 + m_2u_2$
total momentum after collision $= m_1v_1 + m_2v_2$

As the collision is elastic linear momentum and and kinetic energy are conserved.
According to the law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad -(1)$$

$$\text{or} \quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad -(2)$$

According to the conservation of K energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\text{or} \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$\text{or} \quad m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad -(3)$$

divide eq^n (3) by eq^n (2), we get

$$u_1 + v_1 = v_2 + u_2$$

$$\text{or} \quad u_1 - u_2 = v_2 - v_1 \quad -(4)$$

i.e
Relative velocity of approach = Relative velocity of separation

Coefficient of Restitution (e) -

The ratio of relative velocity of separation to the relative velocity of approach is called coefficient of restitution.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

* For perfectly elastic collision $e = 1$ [No loss of K.E]
* for inelastic collision, $0 < e < 1$ [Some loss of K.E]
* For superelastic collision $e > 1$ [e.g. crackers] K.E↑
* For perfectly inelastic collision $e = 0$ [Max^m loss of K.E]

Most of the collisions we see in our daily life are inelastic collisions.

Velocities after collision -

from eqn (6)

$$V_2 = u_1 - u_2 + V_1$$

put this value of V_2 in equation (2), we have

$$m_1(u_1 - V_1) = m_2[u_1 - 2u_2 + V_1]$$

or

$$V_1 = \frac{2m_2u_2 + u_1(m_1 - m_2)}{(m_1 + m_2)}$$

put this value of V_1 in eqn (4)

$$u_1 - u_2 = V_2 - \left[\frac{2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2}\right]$$

or

$$V_2 = u_1 - u_2 + \frac{2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2}$$

$$= \frac{m_1u_1 - m_1u_2 + m_2u_1 - m_2u_2 + 2m_2u_2 + u_1(m_1 - m_2)}{m_1 + m_2}$$

$$= \frac{2m_1u_1 - m_1u_2 + m_2u_2}{m_1 + m_2}$$

$$V_2 = \frac{2m_1u_1 + u_2(m_2 - m_1)}{m_1 + m_2}$$

Special cases

(i) When $m_1 = m_2 = m$

$$V_1 = \frac{2mu_2 + u_1 \times 0}{2m} = u_2$$

i.e $\boxed{V_1 = u_2}$

and

$$V_2 = \frac{2mu_1 + u_2 \times 0}{2m}$$

or $\boxed{V_2 = u_1}$

It is clear that when two bodies of equal masses suffer one dimensional elastic collision, then their velocities are interchanged after collision.

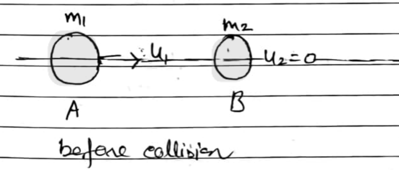
(ii) When $u_2 = 0$

$$V_1 = \frac{u_1(m_1 - m_2)}{m_1 + m_2}$$

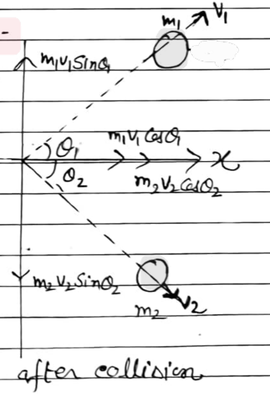
and

$$V_2 = \frac{2m_1u_1}{m_1 + m_2}$$

Collision in two dimensions -



before collision



after collision

As momentum is conserved

$$m_1u_1 = m_1V_1\cos\theta_1 + m_2V_2\cos\theta_2$$

and

$$0 = m_1V_1\sin\theta - m_2V_2\sin\theta_2$$

or

$$m_1V_1\sin\theta = m_2V_2\sin\theta_2$$

If collision is perfectly elastic, then K.E is also conserved.

i.e K.E before collision = K.E after collision

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Completely Inelastic collision in One dimension

from previous fig

$$\theta_1 = \theta_2 = 0$$

$$m_1 u_1 = (m_1 + m_2) V$$ [After collision both the bodies moves together with velocity V

$$V = \frac{m_1}{m_1 + m_2} u_1$$

The loss in K.E

$$\Delta K = \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2) V^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u_1}{m_1 + m_2}\right)^2$$

$$= \frac{1}{2} m_1 u_1^2 - \frac{1}{2} \frac{m_1^2}{m_1 + m_2} u_1^2$$

$$\Delta K = \frac{1}{2} m_1 u_1^2 \left[1 - \frac{m_1}{m_1 + m_2}\right]$$

or $$\Delta K = \frac{1}{2} \frac{m_1 m_2 u_1^2}{(m_1 + m_2)}$$

Special case-

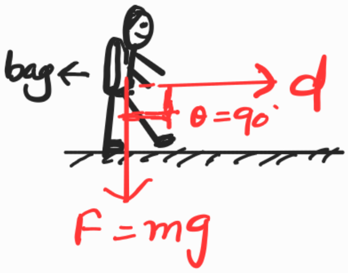
- If $m_1 = m_2$ and $u_2 = 0$, then final velocity

$$V = \frac{u_1}{2}$$

and $$\Delta K = \frac{1}{4} m u_1^2$$

Some important points

1. Zero work done does not mean no force.
   If $\theta = 90^\circ$ ie $\vec{F}$ and $\vec{d}$ are $\perp$ to each other, $W=0$
   e.g. carrying a bag horizontally.



2. Work done by gravity is path independent.
3. Kinetic energy is frame dependent.
   A person has zero kinetic energy relative to moving train. [Energy is not absolute it's relative]
4. Potential energy is stored in the system, not the object alone.
   e.g. gravitational P.E is shared b/w earth and object.
5. Conservation of M.E requires absence of non conservative force.
   If friction or air resistance is present, M.E is not conserved.
6. Work - energy theorem is always valid even when friction acts. Friction reduces K.E because it does negative work.
7. Power does not indicate more work. It indicates work in less time. Power is about rate, not amount.
8. Energy stored in a spring is due to internal energy
9. Elastic collision conserve both momentum and energy but that does not mean velocities are same. Both can be conserved with change in velocities.
10. In inelastic collisions, energy is not lost - it just changes it forms → heat, sound etc.

11. Force constant 'K' is system property. not just of spring. 'K' measures the stiffness of a system. Stiffer system has high K (like steel spring)

12. Conceptual relation between K.E, p and m:

$K = \frac{p^2}{2m}$ $K \to$ Kinetic energy

$p \to$ linear momentum , $m \to$ mass

Case I If two bodies have same mass

$K \propto p^2 \Rightarrow \boxed{\frac{K_1}{K_2} = \frac{p_1^2}{p_2^2}}$

Case II If K.E is same

$p^2 \propto m \Rightarrow \frac{p_1^2}{p_2^2} = \frac{m_1}{m_2} \Rightarrow \boxed{\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}}$

Case III If momentum is same

$K \propto \frac{1}{m} \Rightarrow \boxed{\frac{K_1}{K_2} = \frac{m_2}{m_1}}$

Example If K.E of a body is increased by by 50% what is the percentage change in momentum? (mass is constant)

Solution $K = \frac{p^2}{2m} \Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}}$

here $K_1 = K$, $K_2 = K + 50\%$ of $K = 1.5K$

So, $\frac{p_2}{p_1} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{1.5K}{K}} = \sqrt{1.5} \approx 1.225$

Now % change in $p = \frac{\Delta p}{p} \times 100 = \frac{0.225p}{p} \times 100 = 22.5\%$

Ans