FORMULA SHEET

Chapter-6

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Rigid body: Body which is not deformed on applying an external force is called rigid body.

Centre of mass: The point at which total mass of the body is supposed to be concentrated.

It can be inside and outside of the body.

1. **Position Vector of centre of mass**

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots m_n\vec{r}_n}{m_1 + m_2 + \cdots m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

Coordinates of COM

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots m_nx_n}{m_1 + m_2 + \cdots m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + \cdots m_ny_n}{m_1 + m_2 + \cdots m_n}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + \cdots m_nz_n}{m_1 + m_2 + \cdots m_n}$$

2. **Concept of mass density**

→ Linear mass density, $\lambda = \frac{\text{mass}}{\text{length}}$

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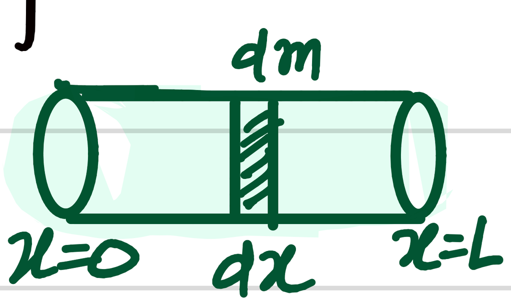
→ Surface or area mass density, $\sigma = \frac{\text{mass}}{\text{area}}$

→ Volume mass density, $\rho = \frac{\text{mass}}{\text{volume}}$

3. **Centre of mass of uniform length L**

$$x_{cm} = \frac{\int_0^L x\,dm}{\int dm}$$



$$= \frac{\int_0^L x \frac{M}{L} dx}{M}$$

$$= \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

Continuous mass system

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$z_{cm} = \frac{\int z\,dm}{\int dm}$$

4. <u>Velocity of COM</u>

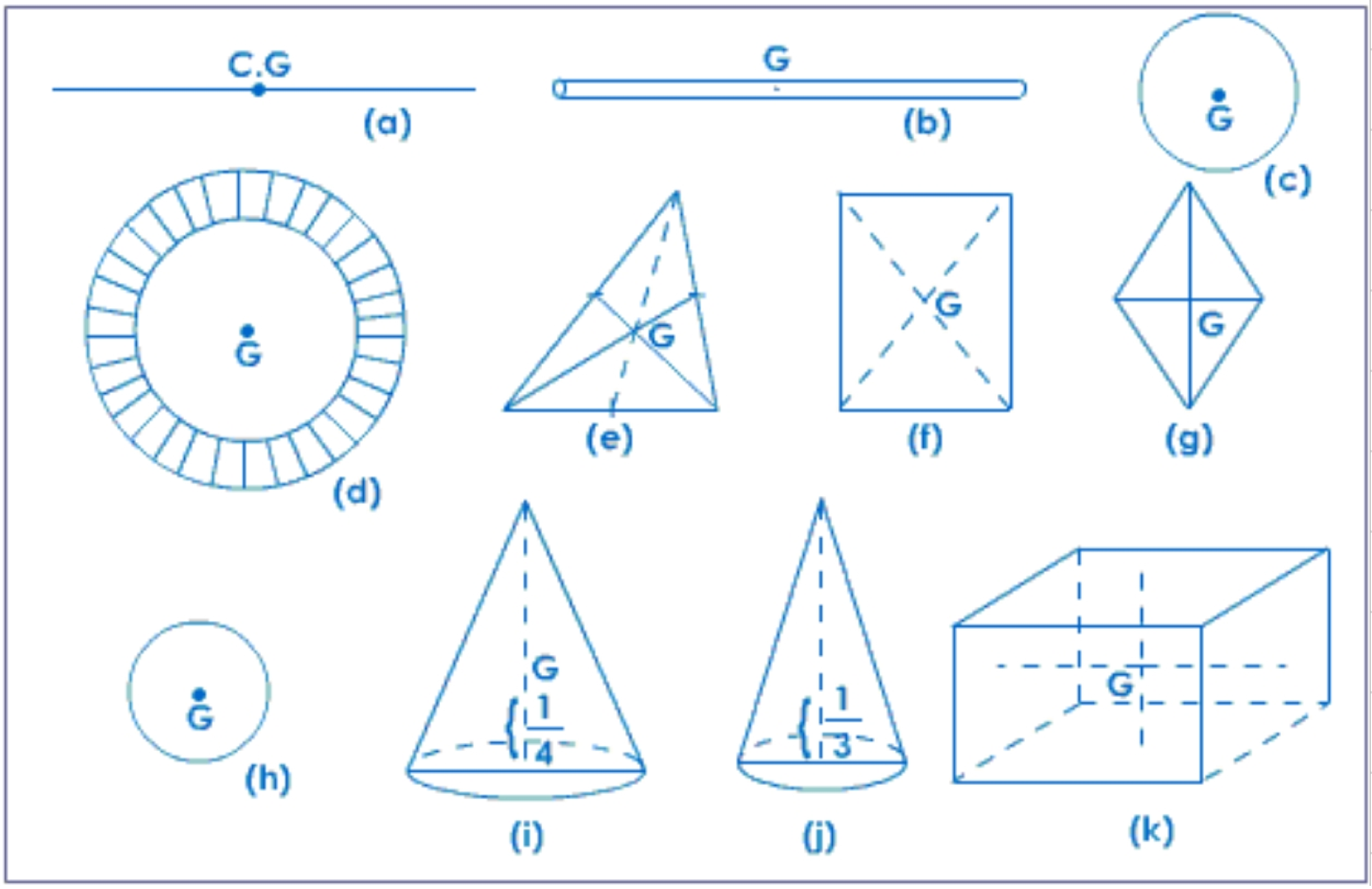
$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots m_n\vec{v}_n}{m_1 + m_2 + \cdots m_n}$$

5. <u>Acceleration of COM</u>

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots m_n\vec{a}_n}{m_1 + m_2 + \cdots m_n}$$

6. <u>Centre of mass of different shapes</u>

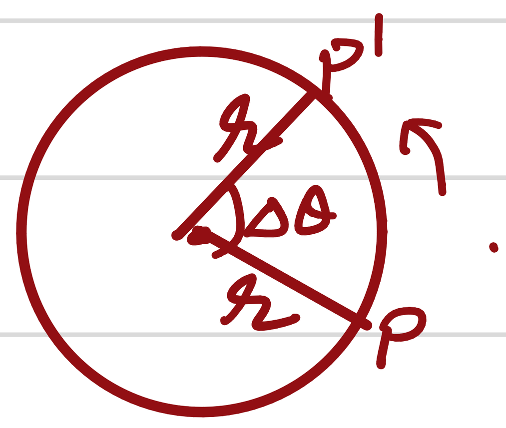
SHAPE	AREA	$\bar{x}$	$\bar{y}$
RECTANGLE	bh	$\frac{1}{2}b$	$\frac{1}{2}h$
TRIANGLE	$\frac{1}{2}bh$		$\frac{1}{3}h$
CIRCLE	πr^2	0	0
SEMICIRCLE	$\frac{\pi}{2}r^2$	0	$\frac{4r}{3\pi}$
QUARTER CIRCLE	$\frac{\pi}{4}r^2$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
ELLIPSE	πab	0	0



7. Kinematics of Rotational motion

→ **Angular displacement (θ)**

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}}, \quad \Delta\theta = \frac{PP'}{r}$$



→ **Angular velocity (ω)**

$$\omega = \frac{\Delta\theta}{\Delta t}$$

SI unit → Rad s^{-1}

Dim^n → $[T^{-1}]$

vector quantity

At any instant,

$$\omega = \frac{d\theta}{dt}$$

Average angular velocity

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

→ **Angular acceleration (α)**

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

SI unit → Rad s^{-2}

Dim^n → $[T^{-2}]$

vector quantity

At any instant, $\alpha = \frac{d\omega}{dt}$

Average angular acceleration

$$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

Moment of Inertia (I)

It is the measure of an object's resistance to change in its rotational motion

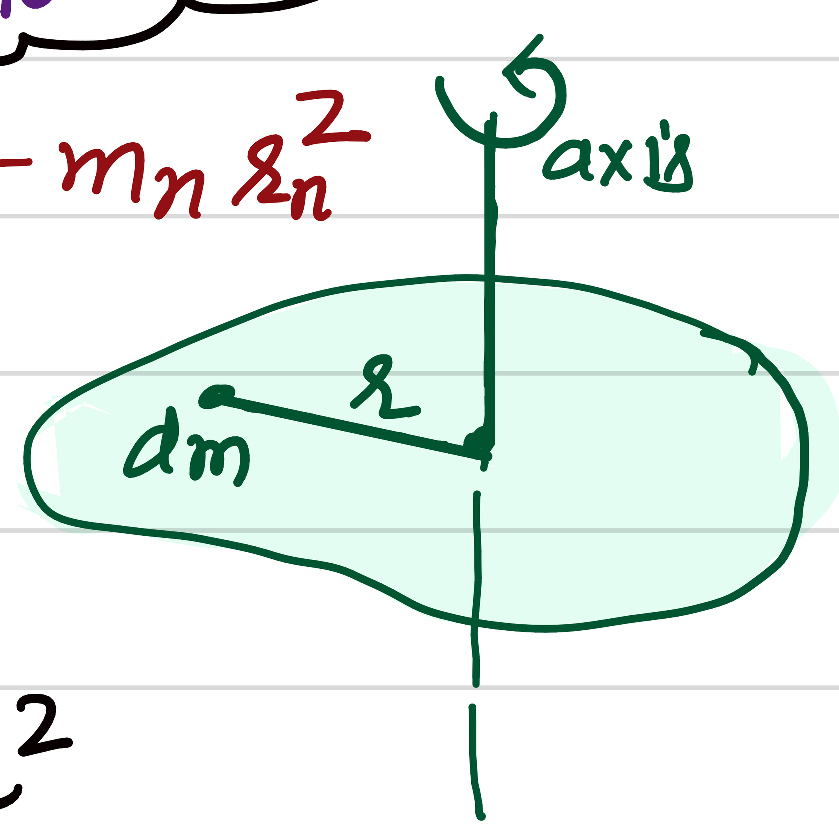
$$I = mr^2$$

For a system of n particles

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots m_n r_n^2$$

For continuous body

$$I = \int r^2\, dm$$



SI unit → Kg m^2, Dim^n → ML^2

It is scalar quantity.

8. <u>Moment of Inertia of symmetrical mass distribution</u>

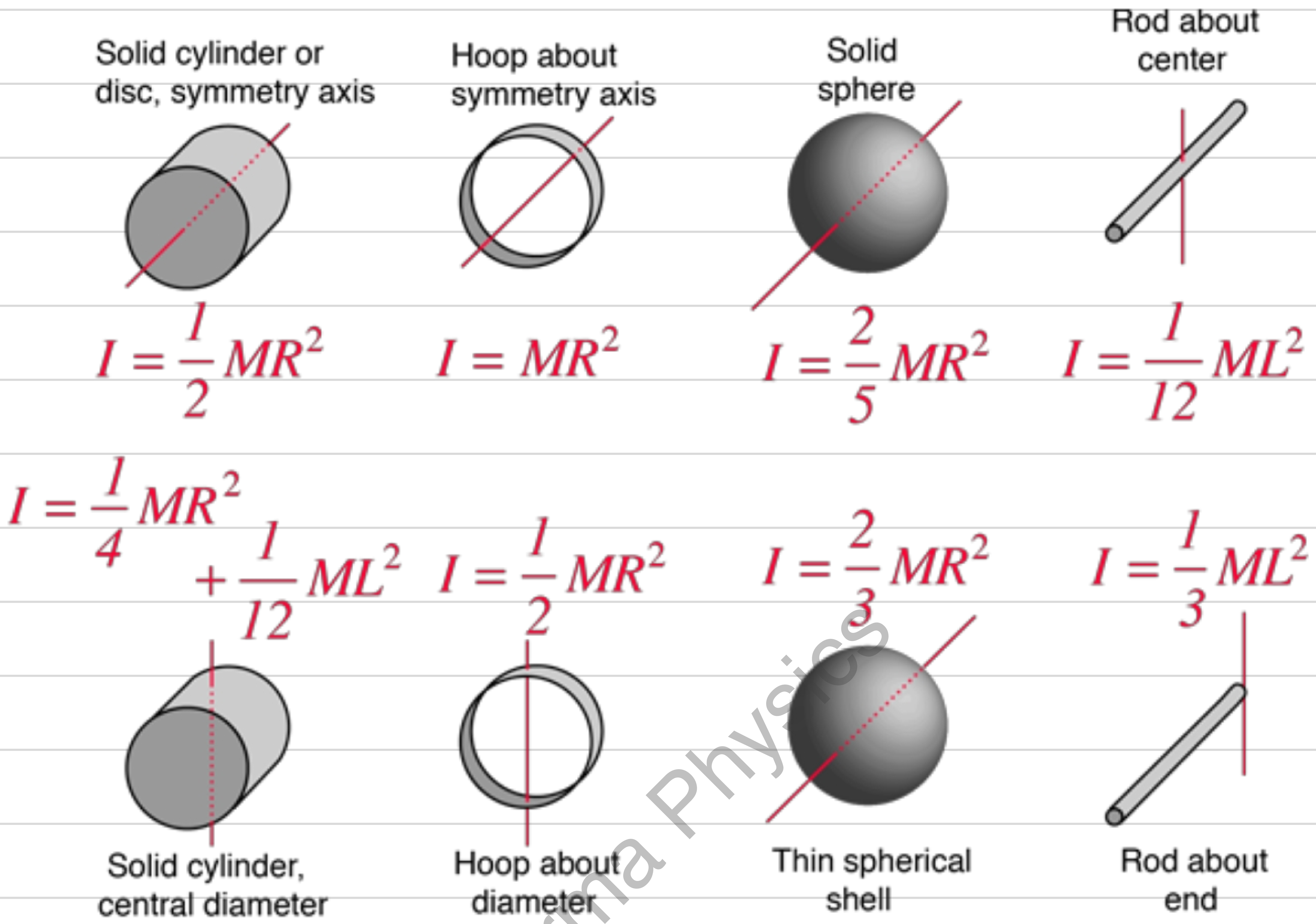


Table 7.1 **Moments of Inertia of some regular shaped bodies about specific axes**

Z	Body	Axis	Figure	I
(1)	Thin circular ring, radius R	Perpendicular to plane, at centre		MR^2
(2)	Thin circular ring, radius R	Diameter		$MR^2/2$
(3)	Thin rod, length L	Perpendicular to rod, at mid point		$ML^2/12$
(4)	Circular disc, radius R	Perpendicular to disc at centre		$MR^2/2$
(5)	Circular disc, radius R	Diameter		$MR^2/4$
(6)	Hollow cylinder, radius R	Axis of cylinder		MR^2
(7)	Solid cylinder, radius R	Axis of cylinder		$MR^2/2$
(8)	Solid sphere, radius R	Diameter		$2MR^2/5$

It is the distance from the axis at which the entire mass of the body can be assumed to be concentrated to give the same M.I.

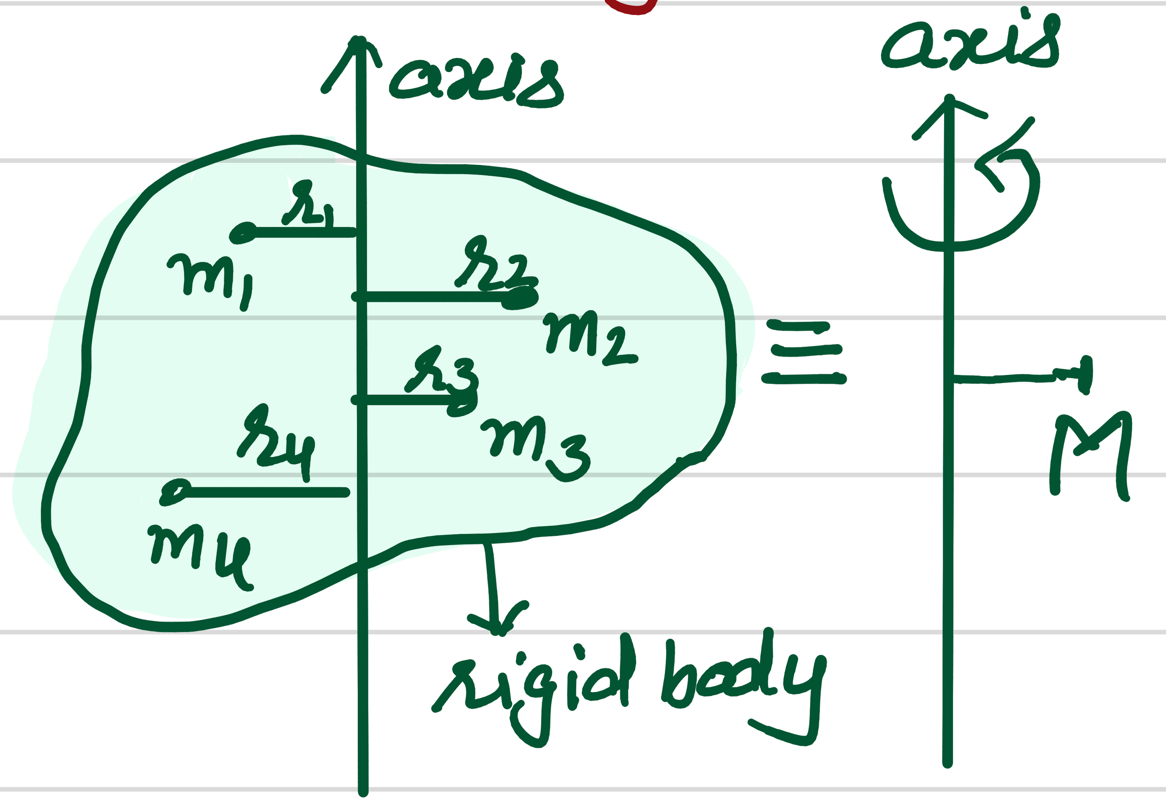
9. **Radius of Gyration (K)**

$$I = mK^2$$

$m \rightarrow$ mass of the body

$I \rightarrow$ momentum of inertia

$$\text{or} \quad K = \sqrt{\frac{I}{m}}$$



$$\text{Also}, \quad K = \sqrt{\frac{m_1r_1 + m_2r_2 + \cdots m_nr_n}{m_1 + m_2 + \cdots m_n}}$$

For identical particles

$$K = \sqrt{\frac{r_1 + r_2 + \cdots r_n}{n}}$$

SI unit → m, Dim^n → [L], scalar quantity

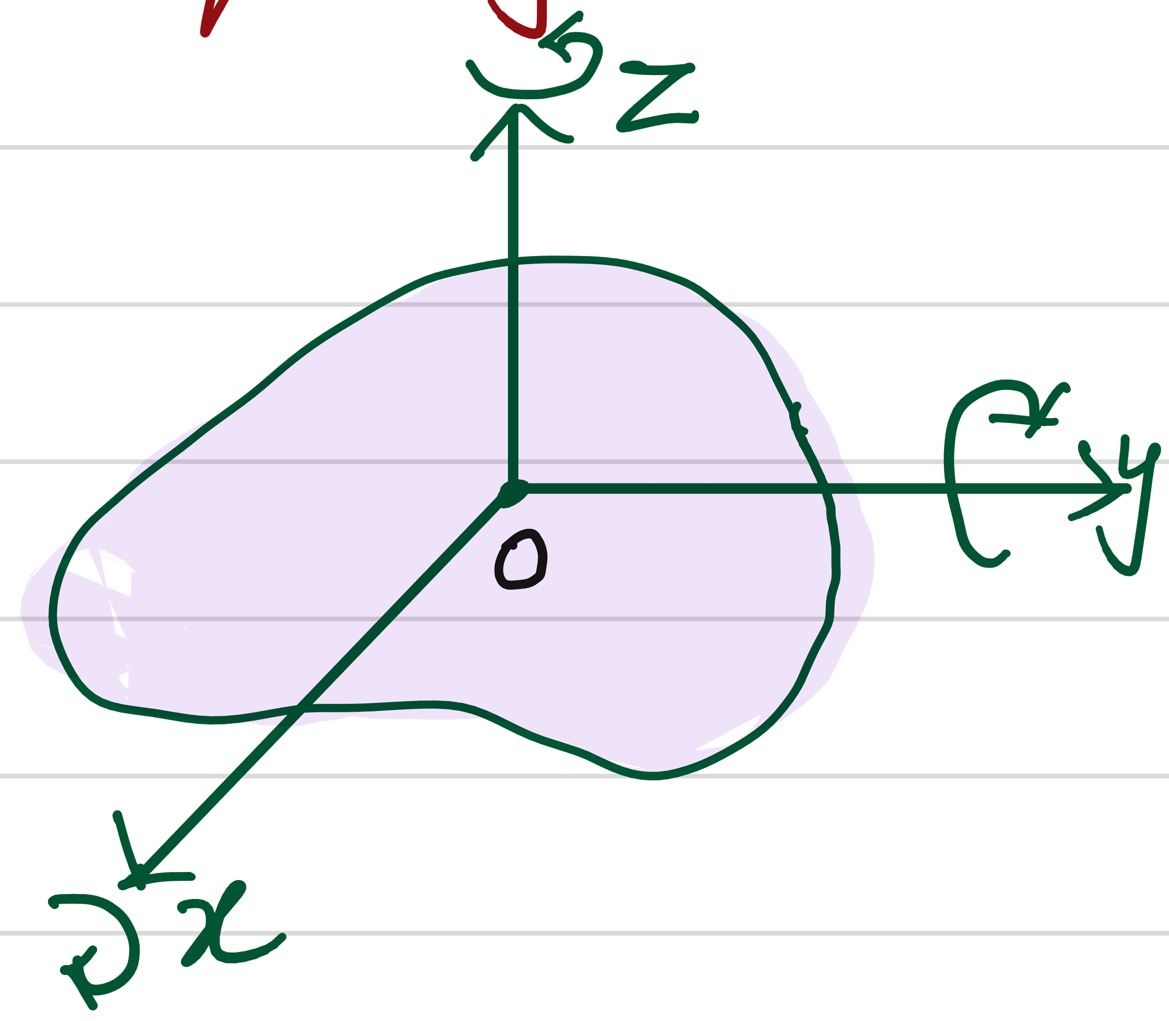
10 **Theorem of perpendicular axis**

$$I_z = I_x + I_y$$

Where I_x = M.I about x-axis

I_y = M.I. about y-axis

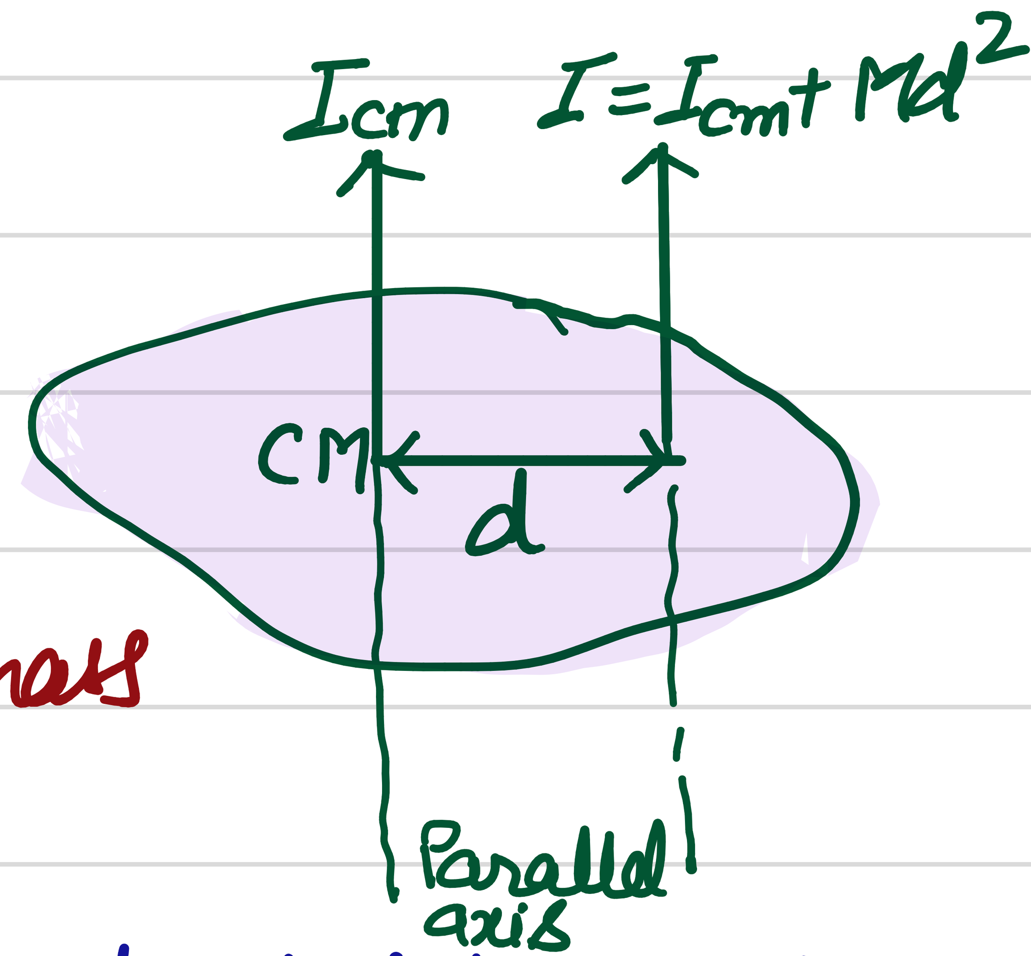
I_z = M.I about Z axis



11. **Theorem of parallel axis**

$$I = I_{cm} + Md^2$$

here I_{cm} = M.I about the axis axis passing through the centre of mass



12. **Torque (τ)** The turning effect of force about a point or axis

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\theta$$

SI unit → N-m

Dim^n → $[ML^2T^{-2}]$

vector quantity

Torque = $|\vec{F}|$ × ⊥ distance of the line of action of force from the axis of rotation

$$\text{Also}, \quad \vec{\tau} = I\vec{\alpha}$$

$$\text{and} \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

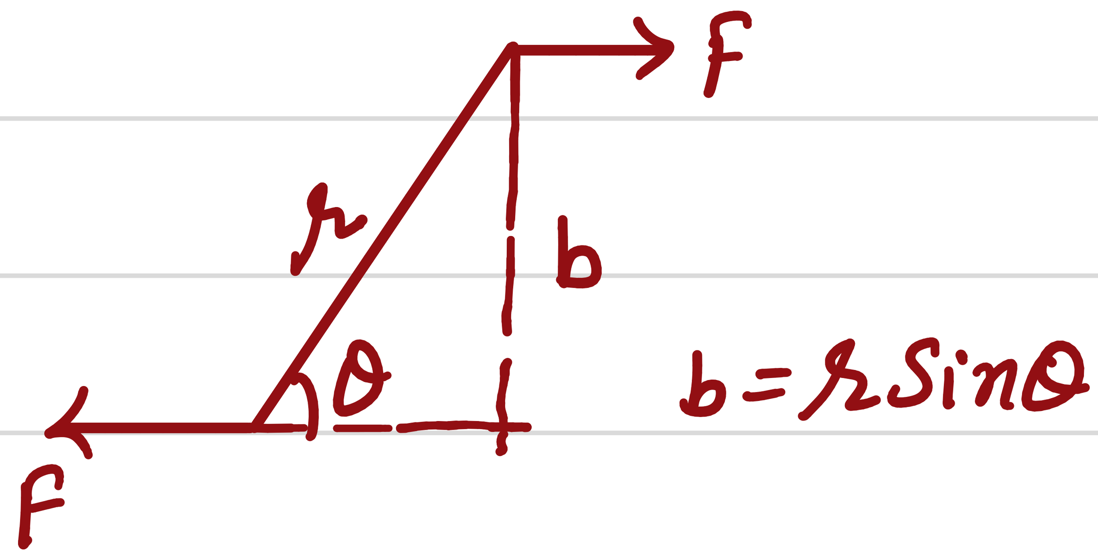
$I \rightarrow$ moment of inertia

$\alpha \rightarrow$ angular acceleration

$L \rightarrow$ angular momentum

13. <u>Couple of forces</u>

$$\tau = Fr\sin\theta$$
$$\tau = Fb$$



For constant torque

$$Fr\sin\theta = \text{Constant}$$

or $F = \dfrac{\text{Constant}}{r\sin\theta} \Rightarrow F \propto \dfrac{1}{r}$, $F \propto \dfrac{1}{\sin\theta}$

14. <u>Equilibrium in mechanics</u>

Translational Equilibrium	Rotational Equilibrium
• Net force on the body (in rest or motion) is zero. i.e $F_{net} = 0$ $(a = 0)$	• Net torque on the body (rest or rotating) is zero. i.e $\tau_{net} = 0$ $(\alpha = 0)$

15. <u>Angular momentum (L)</u> SI unit → $kg\,m^2 s^{-1}$ Dimⁿ → $[ML^2T^{-1}]$

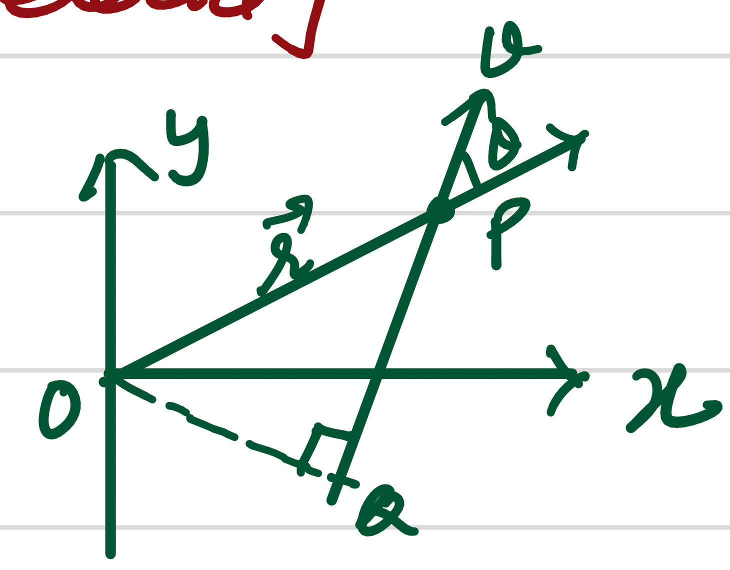
It represents amount of rotational motion of a body

$L = rp\sin\theta = mvr\sin\theta$ $[p = mv]$

$\vec{L} = \vec{r} \times \vec{p}$ p → linear momentum

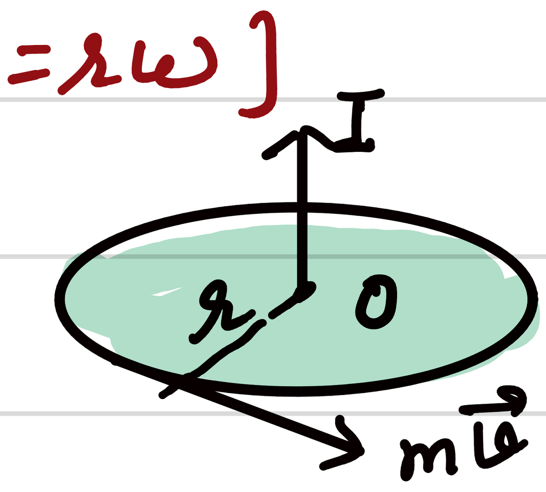
also $L = I\omega$ ω → angular velocity

and $L = mr^2\omega$ $[I = mr^2]$



→ For a system of n particles

$$L = m_1 v_1 r_1 + m_2 v_2 r_2 + \cdots$$
$$= m_1 r_1^2\omega + m_2 r_2^2\omega + \cdots \quad [v = r\omega]$$
$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$



It is vector quantity.

16. <u>Relation between torque and angular momentum</u>

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

17. <u>Conservation of angular momentum</u>

If $\tau_{ext} = 0 \Rightarrow \dfrac{dL}{dt} = 0 \Rightarrow L = \text{Constant}$

ie $L = I\omega = mr^2\omega = \text{Constant}$

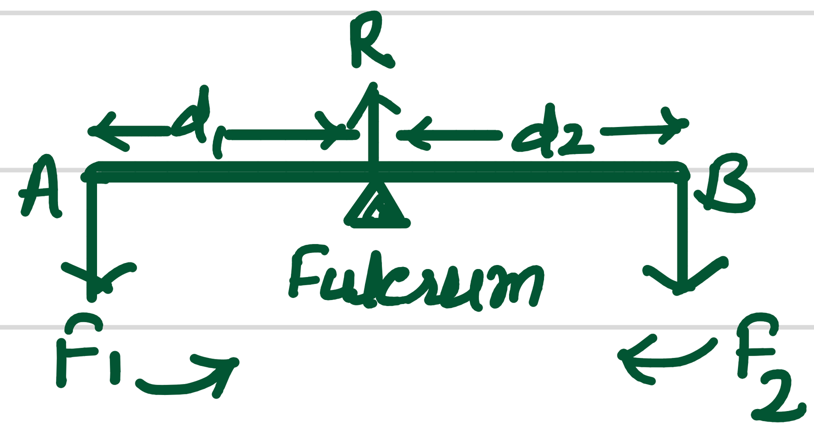
if $\omega \uparrow$, $I \downarrow$

18. **Centre of gravity** (A point where the whole body weight appears to act, regardless its position)

The total torque about the centre of gravity is zero.

$$\Sigma \tau = 0$$

* In uniform gravity, it concides with COM.



19. **Principle of Moments**

$$\frac{F_1}{F_2} = \frac{d_2}{d_1} \Rightarrow F_1 d_1 = F_2 d_2$$

20. **Work done by a torque**

$$dW = \tau \, d\theta$$

$$W = \int dW = \int \tau \, d\theta$$

21. **Rotational Power (P)** The rate at which work is done by torque.

$$P = \frac{dW}{dt} = \tau\omega \qquad [W = \tau d\theta]$$

SI unit → Watt(W) or JS^{-1}

Dim^n → $[ML^2T^{-3}]$ Scalar quantity

22. **Rotational Kinetic Energy ($K.E_R$)**

$$K.E_R = \frac{1}{2} I\omega^2 = \frac{1}{2}(I\omega)\omega$$

$$K.E_R = \frac{1}{2} L\omega = \frac{1}{2}\left(\frac{L}{I}\right) = \frac{L^2}{2I} \qquad [I\omega = L]$$

23. **Rolling Motion** (When an object rotates and moves forward)

Total energy in rolling = Translation K.E + Rotatory K.E

$= K.E + K.E_R$

$$\text{Rolling Kinetic Energy} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}mk^2\left(\frac{v^2}{R^2}\right)$$

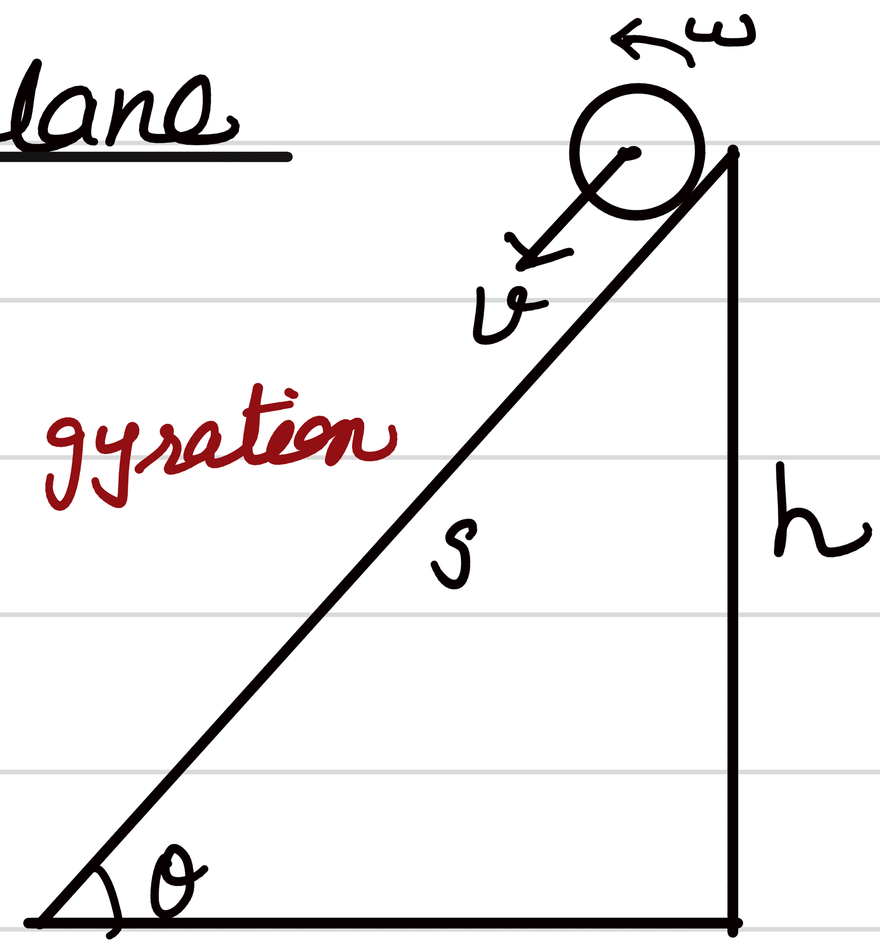
$$= \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

$E_{translational} : E_{rotation} : E_{total} = 1 : \frac{K^2}{R^2} : 1 + \frac{K^2}{R^2}$

24. **Rolling motion on an inclined plane**

$$v = \sqrt{\frac{2gh}{\left(1 + \frac{K^2}{R^2}\right)}}$$

$K \to$ Radius of gyration



$h = s \sin\theta$

If different bodies are allowed to roll down, least $\frac{K^2}{R^2}$ will reach first.

* For ring, disc, hollow sphere and solid sphere

$$v_{sphere} > v_{disc} > v_{hollow\ sphere} > v_{ring}$$

25.

Linear Motion		Rotational Motion	
Position	x	θ	Angular position
Velocity	v	ω	Angular velocity
Acceleration	a	α	Angular acceleration
Motion equations	$x = \bar{v}t$	$\theta = \bar{\omega}t$	Motion equations
	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$	
	$x = v_0 t + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$	
Mass (linear inertia)	m	I	Moment of inertia
Newton's second law	$F = ma$	$\tau = I\alpha$	Newton's second law
Momentum	$p = mv$	$L = I\omega$	Angular momentum
Work	Fd	$\tau\theta$	Work
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	Kinetic energy
Power	Fv	$\tau\omega$	Power