Gravitation

Chapter-7

Formula Sheet

1. Kepler's Law

(I) Law of Areas: Areal velocity

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m} = \text{Constant}$$

$\Delta A \rightarrow$ Area

$L \rightarrow$ Angular momentum

(II) Law of periods

$$T^2 \propto r^3$$

or $T^2 = K r^3$

also $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

2. Law of Gravitation

Force b/w two masses

$$F = \frac{G m_1 m_2}{r^2}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$G \rightarrow$ Gravitational constant

$G = 6.67 \times 10^{-11} Nm^2/kg^2$

Vector form

$$\vec{F} = \frac{G m_1 m_2}{r^2} \cdot \hat{r}, \quad \hat{r} = \frac{\vec{r}}{r}$$

3. Acceleration due to gravity (g)

Acceleration produced in a body due to Earth's gravity

$$g = \frac{GM}{R^2}$$

$G \rightarrow$ Gravitational constant

$M \rightarrow$ Mass of earth

$R \rightarrow$ Radius of earth

$g = 9.8 \ m/s^2$

4. Mass and density of Earth

Mass, $M = \frac{gR^2}{G}$

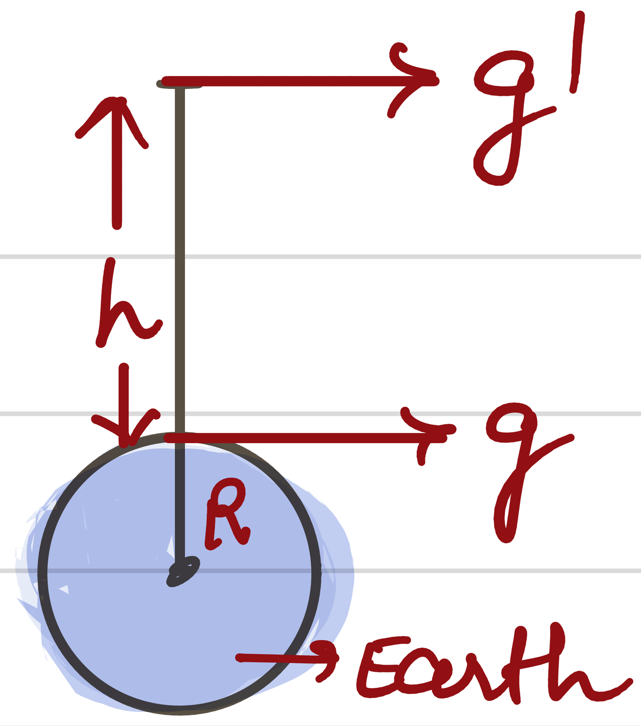
Density, $\rho = \frac{3g}{4\pi G R}$ $\quad \left[\rho = \frac{M}{V}\right]$

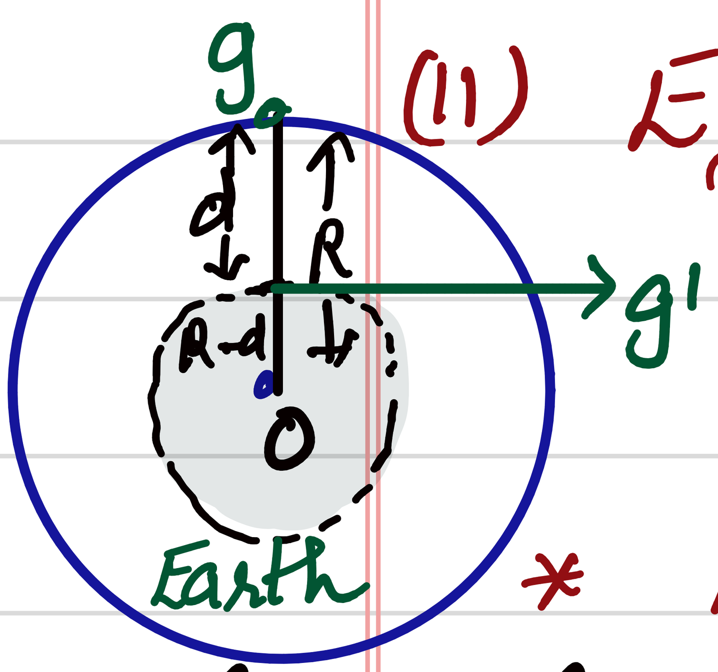
5. Variation of 'g'

(I) Effect of height $\rightarrow$ At some height 'h' acc^n due to gravity, $g' = \frac{g}{(R+h)^2} g$

For $h << R$

$$g' = \left(1 - \frac{2h}{R}\right) g$$





(ii) Effect of depth : Inside the Earth at depth 'd'

$$g' = \left(\frac{R-d}{R}\right) g = \left(1 - \frac{d}{R}\right) g$$

* At the centre of Earth $g' = 0$ [weightlessness]

6. Gravitational field Intensity (I)

Intensity of gravitational field at any point due to mass M

$$I = \frac{GM}{r^2}$$

$r \rightarrow$ distance of point from mass M.

Numerically $I = g$

7. Gravitational Potential (V)

$$V = -\frac{GM}{r}$$

8. Gravitational Potential Energy (U)

$$U = -\frac{GMm}{r}$$

*U is maximum (zero) at infinity

-ve sign shows that object is bound to gravitational field and require +ve energy to escape to infinity.

9. Escape Speed (v_e)

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

For earth $v_e = 11.2$ Km/s

For moon $(v_e)_{moon} = 2.38$ Km/s

10. Orbital speed of Satellite (v_o)

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gr}$$

For earth $v_o = 7.8\ km s^{-1}$

$$\boxed{v_e = \sqrt{2}\, v_o}$$

11. Time period of a Satellite

$$T = \frac{2\pi r}{v_o} = \frac{2\pi(R+h)}{v_o} = \frac{2\pi(R+h)^{3/2}}{\sqrt{GM}}$$

or $\quad T^2 = K(R+h)^3$

where $K = \frac{4\pi^2}{GM}$

12. Energy of an orbiting Satellite

(I) Kinetic Energy, $K = \frac{GMm}{2(R+h)} = x$ (say)

(II) Potential Energy, $U = -\frac{GMm}{(R+h)} = -2x$

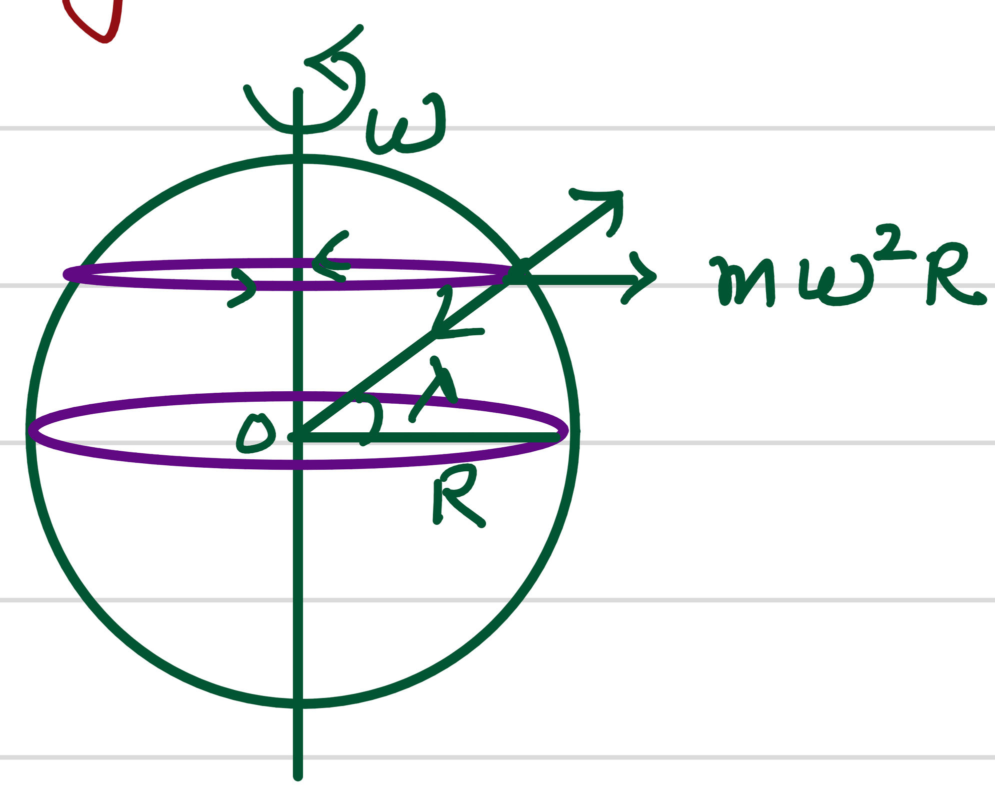
(III) Total Energy, $E = K + U$

$$E = -\frac{GMm}{2(R+h)} = -x$$

E is also called binding energy. -ve sign shows satellite is bound to gravitation field.

* Variation of g with latitude

$$g' = g - \omega^2 R\cos^2\lambda$$



$m\omega^2 R \rightarrow$ centrifugal force

$\lambda \rightarrow$ latitude

→ at equator $\lambda = 0 \Rightarrow g' = g - \omega^2 R$ [minimum]

→ at poles, $\lambda = 90° \Rightarrow g' = g$ [Maximum]