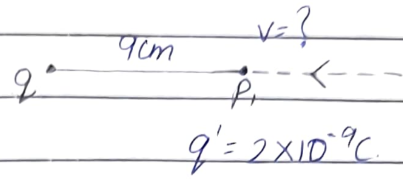
CHAPTER-2 ELECTROSTATIC POTENTIAL AND CAPACITANCE

Example 2.1

Given.

a) $q = 4 \times 10^{-7} C$

$r = 9\,cm = 9 \times 10^{-2} m$



$$V = \frac{kq}{r}$$

$$= \frac{9 \times 10^{9} \times 4 \times 10^{-7}}{9 \times 10^{-2}}$$

$$= 4 \times 10^{4} \text{ Volt}$$ Ans

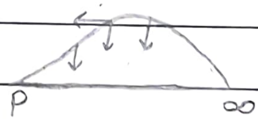
b) $q' = 2 \times 10^{-9} C$

$W = Vq'$

$= 4 \times 10^{4} \times 2 \times 10^{-9}$

$W = 8 \times 10^{-5} J$ Ans

Since electrostatic force is Conservative force therefore Workdone is independent of path.



Example 2.2

Given

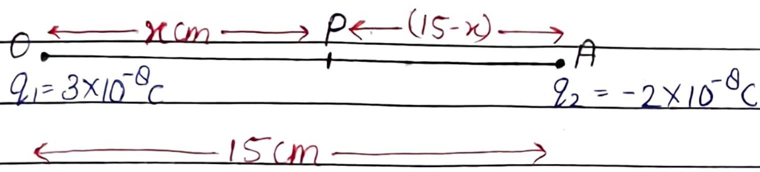
$$V = \frac{kq}{r}$$

$q_1 = 3 \times 10^{-8} C$

$q_2 = -2 \times 10^{-8} C$

$r = 15\,cm \Rightarrow 15 \times 10^{-2} m$

Case I



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At point 'P'

$$V_{q_1} + V_{q_2} = 0$$

$$V_{q_1} = -V_{q_2}$$

$$\frac{Kq_1}{x \times 10^{-2}} = \frac{-Kq_2}{(15-x)\times 10^{-2}}$$

$$\frac{3\times 10^{-8}}{x} = \frac{2\times 10^{-8}}{(15-x)}$$

or $$3(15-x) = 2x$$

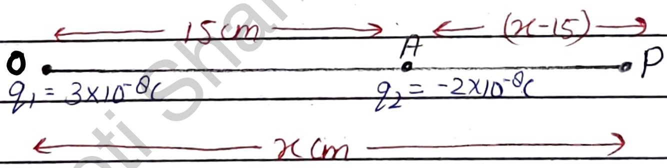
$$45-3x = 2x$$

$$5x = 45$$

$$x = \frac{45}{5} = 9$$

$$\boxed{x = 9\,cm}$$

Case II



At point 'P'

$$V_{q_1} = -V_{q_2}$$

$$\frac{K\times 3\times 10^{-8}}{x\times 10^{-2}} = +\frac{K\times 2\times 10^{-8}}{(x-15)\times 10^{-2}}$$

$$\frac{3}{x} = \frac{2}{x-15}$$

or $$3x-45 = 2x$$

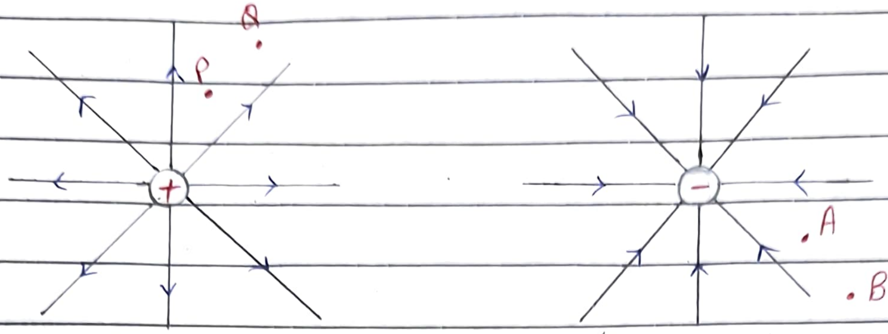
$$\boxed{x = 45\,cm}$$

i.e potential at point 'P' right side of 'O' is also zero.

$x = 9\,cm$ (b/w OA)

$x = 45\,cm$ (outside the OA in right side)

Example 2.3



a) $\because V_P > V_Q$

so

$V_P - V_Q$ is +ve

and $V_B > V_A$

$\therefore V_B - V_A$ is +ve

b) $\because U_Q > U_P$

$\therefore U_Q - U_P$ is +ve

and

$U_A > U_B$

$U_A - U_B$ is +ve

c) $\because \vec{F_e} = -\vec{F_{ext}}$

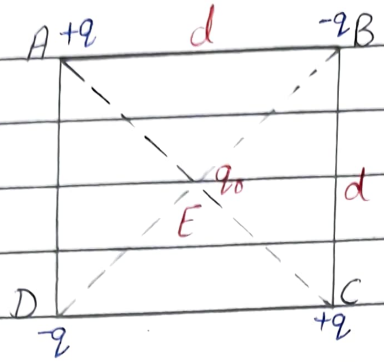
So workdone by the field in moving +ve charge from Q to P is −ve.

d) Workdone by Fext is positive as Fext and displacement of charge are in the same dir^n

e) Since P.E increases from B to A, therefore K.E decreases from B to A.

[$\because$ K.E + P.E = Constant]

Example 2.4



$AC = BD = \sqrt{d^2+d^2}$
$= \sqrt{2d^2}$
$= d\sqrt{2}$

Workdone in bringing a charge '+q' to A

$W_1 = 0$ [∵ no other charge is present]

Workdone in moving a charge '-q' to B

$$W_2 = -\frac{kq^2}{d} \qquad \left[\because W = \frac{kq_1q_2}{r}\right]$$

-ve shows that $q_1q_2 < 0$ [$\because q_1 = q,\ q_2 = -q$]

Workdone in moving a charge '+q' to C

$$W_3 = -\frac{kq^2}{d} + \frac{kq^2}{d\sqrt{2}} \qquad \left[\because AC = \sqrt{d^2+d^2} = d\sqrt{2}\right]$$

+ve sign of 2nd term shows $q_1q_2 > 0$ [$\because q_1 = q,\ q_2 =$

Now workdone in moving charge '-q' to D

$$W_4 = -\frac{kq^2}{d} + \frac{kq^2}{d\sqrt{2}} - \frac{kq^2}{d} = -\frac{2kq^2}{d} + \frac{kq^2}{d\sqrt{2}}$$

Net Workdone

$W = W_1 + W_2 + W_3 + W_4$

$$= 0 + \left(-\frac{kq^2}{d}\right) + \left(-\frac{kq^2}{d} + \frac{kq^2}{d\sqrt{2}}\right) + \left(-\frac{2kq^2}{d} + \frac{kq^2}{d\sqrt{2}}\right)$$

$$= -\frac{4kq^2}{d} + \frac{2kq^2}{d\sqrt{2}}$$

$$= -\frac{kq^2}{d}\left(4 - \frac{2}{\sqrt{2}}\right)$$

$$W = -\frac{kq^2}{d}(4-\sqrt{2}) \qquad [\because 2 = \sqrt{2}\times\sqrt{2}]$$

put $k = \frac{1}{4\pi\epsilon_0}$, then

$$W = \frac{-q}{4\pi\epsilon_0}(4-\sqrt{2})$$

Ans

b) Net potential on q_0 at E.

$$V_E = V_A + V_B + V_C + V_D$$
$$= 0 \qquad [\because V_A = -V_B,\ V_C = -V_D]$$

Therefore the electrostatic potential energy at E

$$W = V_E \times q_0 = 0 \qquad [\because V_E = 0]$$

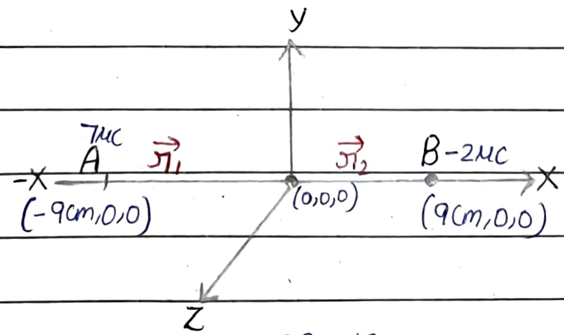
Hence no work is required to bring any charge at 'E' Ans

Example 2.5

Given,

$q_1 = 7\mu C = 7\times10^{-6} C$

$q_2 = -2\mu C = -2\times10^{-6} C$



$AB = r = 18 cm = 18\times10^{-2} m$

AB = 18cm

$|\vec{r}_1| = |\vec{r}_2| = r = 9cm$
$= 9\times10^{-2} m$

a) P.E of the System

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

$$U = \frac{9\times10^{9}\times7\times10^{-6}\times(-2)\times10^{-6}}{18\times10^{-2}}$$

$$= \frac{-14\times10^{-3+2}}{2}$$

$$U = -7\times10^{-1}$$

$$U = -0.7\ J$$

b) Amount of workdone to separate the two charges (at infi

W = −(workdone in assembling the two charges)

= −(P.E stored in the system)

= −(−0.7)

$W = 0.7\ J$

OR

$W = U_f - U_i$

$= 0 - (-0.7)$

$W = 0.7\ J.$

c) Given,

External electric field $E = \frac{A}{r^2}$

Where $A = 9\times10^{5}\ C\,m^{-2}$

we know

$$V(r) = E.r$$

$$= \frac{A}{r^2}.r = \frac{A}{r}$$

$$V(r_1) = \frac{A}{r_1}, \quad V(r_2) = \frac{A}{r_2}$$

Now the net electrostatic energy

$$U_{net} = q_1V(r_1) + q_2V(r_2) + \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

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$$= 7\times10^{-6}\frac{A}{r_1} + (-2)\times10^{-6}\frac{A}{r_2} + (-0.7)$$

$$= A\times10^{-6}\left[\frac{7}{r_1} - \frac{2}{r_2}\right] - 0.7$$

$$= 9\times10^{5}\times10^{-6}\left[\frac{7}{9\times10^{-2}} - \frac{2}{9\times10^{-2}}\right] - 0.7$$

$$= \frac{9\times10^{-1}}{9\times10^{-2}}\left[7-2\right] - 0.7$$

$$= 10\times5 - 0.7$$

$$U_{net} = 50 - 0.7$$

$$= 49.3\ J \quad \underline{Ans}$$

Example 2.6

Given,

Dipole moment of 1 molecule $= 10^{-29}$ C-m

total dipole moment of one mole of the Substance

$$p = 6\times10^{23}\times10^{-29} \Rightarrow 6\times10^{-6}\ Cm$$

[1 mole $= 6\times10^{23}$ molecules]

$E = 10^{6}\ Vm^{-1}$

Initial P.E (at $\theta = 0°$)

$$U_i = -pE\cos\theta$$

$$= -6\times10^{-6}\times10^{6}\cos0°$$

$$\boxed{U_i = -6\ J}$$

$$U_f = -pE\cos60°$$

$$= -6\times10^{-6}\times10^{6}\times\frac{1}{2}$$

$$\boxed{U_f = -3\ J}$$

Change in P.E $= U_f - U_i$

$$= -3-(-6)$$

$$= 3\ J$$

This loss of P.E is equal to the heat released by the substance in aligning the dipoles along the new direction.

Example 2.7

Ans a) When someone comb dry hair, friction b/w the comb and hair makes the comb electrically charged. This charged comb attracts bits of paper because the paper gets slightly polarised (charges rearrange inside the paper) causing an attraction.
But if the hair is wet or its a rainy day the friction is reduced because water conduct electricity.
Therefore comb does not attract the paper bits.

b) Because when an aircraft lands a lot of static charge builds up due to friction with air and ground. So the aircraft tyres are make slightly conducting so they can release the charge safely to the ground.

c) When such vehicles move, friction creates static electricity which can cause a fire in enflommable materials. So they have metal chains or ropes touching ground to release the charge safely.

d) When a bird sits on the wire, both its feet are on the same wire so no voltage difference and hence no current flows.
But when a person standing on the ground touches wire there is a potential difference, so current flows through the body which can cause a fetal shock

Example 2.8

Sol^n Given,

$$t = \frac{3}{4} d$$

We have, $C_0 = \frac{A\epsilon_0}{d}$ [without dielectric]

Now, $C = \dfrac{A\epsilon_0}{(d-t)+\dfrac{t}{K}}$

Put $t = \dfrac{3}{4}d$

$$C = \frac{A\epsilon_0}{\left(d-\frac{3}{4}d\right)+\frac{3}{4}\frac{d}{K}}$$

$$C = \frac{A\epsilon_0}{\frac{d}{4}+\frac{3}{4}\frac{d}{K}}$$

$$= \frac{4A\epsilon_0}{d\left(1+\frac{3}{K}\right)}$$

$$= \frac{4KA\epsilon_0/d}{K+3} \qquad \left[\because C_0 = \frac{A\epsilon_0}{d}\right]$$

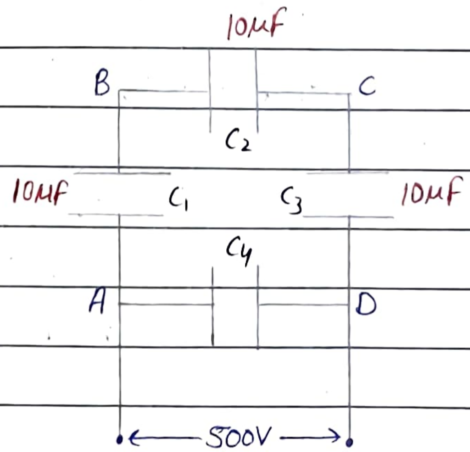
$$C = \frac{4KC_0}{K+3} \quad \text{Ans}$$

Example 2.9

Sol^n Given,

$C_1 = C_2 = C_3 = C_4 = 10\mu F$
$= 10\times10^{-6} F$

$V = 500V$



here C_1, C_2 and C_3 are in series.
The effective Capacitance is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Or $$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

or $C' = \frac{10}{3} \mu F \Rightarrow \frac{10 \times 10^{-6}}{3} F$

Now C' and C_4 are in parallel.
Equivalent Capacitance.

$C_{eq} = C' + C_4 = \frac{10}{3} + 10 \Rightarrow \frac{40}{3} \mu F$

or $C_{eq} = \frac{40}{3} \times 10^{-6} F$

$= 13.3 \times 10^{-6} F$ Ans

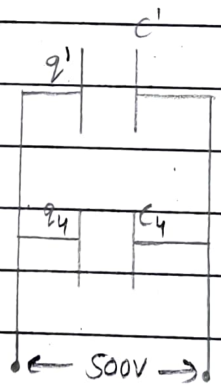
b) We know charge is same in series. Hence
Charge at C' ($= C_1 = C_2 = C_3$)

$q' = C' \times V$ $[\because q = CV]$

$= \frac{10}{3} \times 10^{-6} \times 500$

$= \frac{5}{3} \times 10^{-3} C$ Ans

$= 1.67 \times 10^{-3} C$



Now,

$q_4 = C_4 \times V$

$= 10 \times 10^{-6} \times 500$

$= 5 \times 10^{-3} C$ Ans

Example 2.10

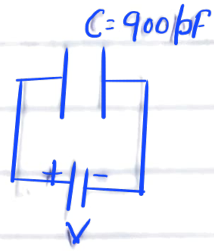
Solⁿ Given,

$C = 900 pF$

$= 900 \times 10^{-12} F$

$= 9 \times 10^{-10} F$

$V = 100 V$



a) Energy stored by Capacitor

$U = \frac{1}{2} CV^2 = \frac{1}{2} qV$ —— (1)

$= \frac{1}{2} \times 9 \times 10^{-10} \times 100^2$

$U = 4.5 \times 10^{-6}$ J

b) As $C_1 = C_2$ by the Conservation of Charge at steady situation the charge at each Capacitor

$q' = \frac{q}{2}$

and hence $V' = \frac{V}{2}$

Now energy of the system

$U' = \left(\frac{1}{2} q'V'\right) \times 2$

$= \frac{2}{2}\left(\frac{q}{2} \times \frac{V}{2}\right) \Rightarrow \frac{1}{2}\left(\frac{1}{2} qV\right)$

from eq^n(1) $U' = \frac{1}{2} U = \frac{4.5 \times 10^{-6}}{2}$

$U = 2.25 \times 10^{-6}$ J

There is no loss of charge but energy becomes half of the initial. There is some loss of energy during the process of charging due to transient current. Energy is lost in the form of heat and electromagnetic radiation.

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Loss of energy when a Capacitor is charged

for battery

$U = qV$ [$W = Vq$]

Now energy of Capacitor

$U_c = \frac{1}{2} qV$

Loss of energy $= qV - \frac{1}{2} qV$

$= \frac{1}{2} qV$

50% energy is lost.