Electrostatic Potential and Capacitance

Electrostatic Potential :- Electrostatic potential at a point is equal to the workdone in moving a unit positive test charge from ∞ to that point with uniform and negligible speed.

$q_0 \rightarrow$ Test charge(+ve)

$$V = \frac{W_{ext}}{q_0}$$

or $$\boxed{V = \frac{W}{q}}$$

$V \longrightarrow$ scalar quantity

Unit $\longrightarrow$ S.I $\longrightarrow$ Valt $\left[1V = \frac{1J}{1C}\right] \rightarrow JC^{-1}$

Potential Difference :- Electrostatic potential difference between two points is equal to the work done in moving unit positive test charge from one point to another in the electric field with uniform and negligible speed.

$$V_A - V_B = \frac{W_{ext\,(B \rightarrow A)}}{q_0}$$

or $$\Delta V = \frac{\Delta W}{q}$$

* This workdone is independent from path.
* The workdone is against the electrostatic force.
* This workdone is equal to the potential energy.

Expression for potential due to a point charge:

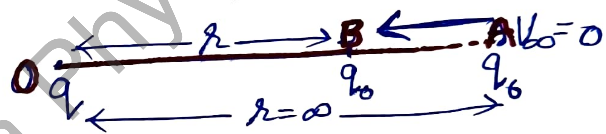
We know electric field due to a point charge is given by

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$ [q is a point charge, r is distance from point charge

Now work done in moving the charge over a small distance dr against the electric field

$$dW = -E \cdot dr$$ [for unit charge $q_0 = 1$]

$$dW = -\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot dr$$



To find the total work done, we integrate from $r = \infty$ ($V_\infty = 0$) to $r = r$

For unit charge this work done is equal to potential, then

$$V = \int_\infty^r \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\, dr$$

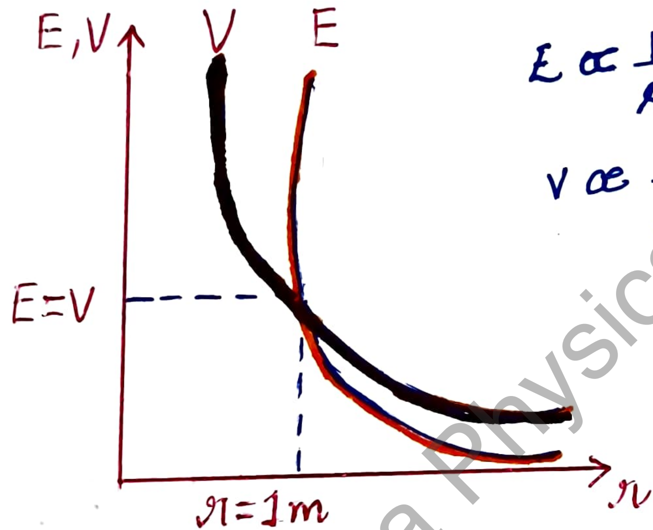
$$= \frac{-q}{4\pi\epsilon_0}\int_\infty^r r^{-2}\, dr$$

$$= \frac{-q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_\infty^r$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}}$$

$$\boxed{V_P = \frac{KQ}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

- For '+Q', V is +ve and for '-Q', V is -ve.
- Comparision of graph of 'V' and 'E' with 'r' -

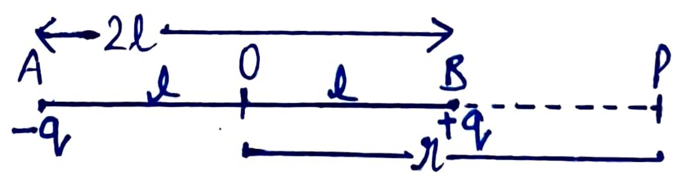


$E \propto \frac{1}{r^2}$

$V \propto \frac{1}{r}$

Potential due to dipole on its axis :-

Potential at point 'P' due to '-q' charge



$$V_- = \frac{-Kq}{(r+l)}$$

AB → dipole
AB = 2l
AP = r+l
BP = r-l

Potential at 'P' due to '+q' charge

$$V_+ = \frac{Kq}{(r-l)}$$

Total potential $V = V_+ + V_-$

$$V = \frac{Kq}{r-l} + \left(-\frac{Kq}{r+l}\right)$$

$$V = kq\left[\frac{1}{r-l} - \frac{1}{r+l}\right]$$

$$= kq\left[\frac{r+l-r+l}{r^2-l^2}\right]$$

$$= kq\left[\frac{2l}{r^2-l^2}\right]$$

$$= \frac{kp}{r^2-l^2} \qquad [\because 2ql = p]$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\cdot\frac{p}{r^2-l^2}}$$

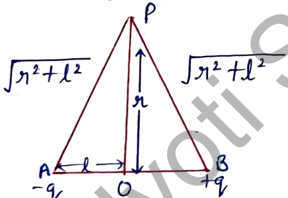
* for dipole $V \propto \frac{1}{r^2}$

Potential due to dipole on equatorial plane-

Potential at point P due to charge '+q'

$$V_+ = \frac{kq}{(r^2+l^2)^2}$$

$$= \frac{kq}{r^2+l^2}$$



Potential due to charge '-q'.

$$V = \frac{-kq}{(\sqrt{r^2+l^2})}$$

$$V = -\frac{kq}{r^2+l^2}$$

AB → dipole

AB = 2l

Total potential $V = V_+ + V_-$

$$V = \frac{kq}{r^2+l^2} - \frac{kq}{r^2+l^2}$$

$$V = 0$$

* At equatorial plane of a dipole net potential is zero. [i.e constant]

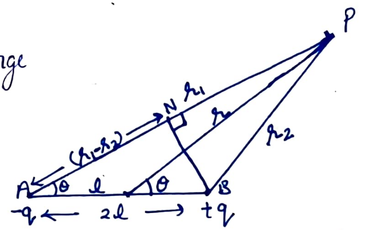
Potential due a dipole at any point :- (For short dipole $l << r$)

Potential at point P due to charge '-q'.

$$V = -\frac{kq}{r_1}$$

Potential due to charge '+q'

$$V_+ = \frac{kq}{r^2}$$



$$V = V_+ + V_-$$

$$= \frac{kq}{r_2} - \frac{kq}{r_1}$$

$$= kq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = kq\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

AB → dipole

AB = 2l

BN is ⊥ on AP

for short dipole

$r_1 = r_2 = r$ (i.e $r_1 r_2 = r^2$)

Also we have $r_1 - r_2 = 2l\cos\theta$ $\left[\because \text{In } \triangle BNA \; \cos\theta = \frac{r_1-r_2}{2l}\right]$

Hence,

$$V = Kq\,\frac{2l\cos\theta}{r^2}$$

$$V = \frac{Kp\cos\theta}{r^2} \qquad [\because\ p = 2ql]$$

or $\boxed{V = \dfrac{1}{4\pi\epsilon_0}\cdot\dfrac{p\cos\theta}{r^2}}$

or $\boxed{V = \dfrac{1}{4\pi\epsilon_0}\cdot\dfrac{\vec{p}\cdot\hat{r}}{r^2}}$

Cases →

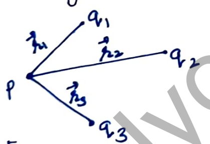
(I) If $\theta = 0°$ or $180°$ [Axial line]

$$V = \pm\frac{1}{4\pi\epsilon_0}\cdot\frac{p}{r^2}$$

(II) If $\theta = 90°$ [Equatorial line]

$V = 0$ $[\because \cos 90° = 0]$

Potential due to multiple charges -

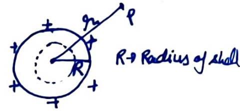


$$\text{Total potential } V = K\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$$

* Remember for -ve charge potential will be -ve.

Potential due to uniformly charged spherical shell :-

(a) Inside the shell -



Inside the shell $q = 0$

i.e $E = 0$

therefore no work is done in moving a charge inside the shell.

Thus potential is constant inside the shell and equals to its value at the surface of shell.

$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R}$$

(b) Outside the shell -

Potential outside the shell is as if all charge is concentrated at the centre. Thus,

$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r} \quad [r \geq R]$$

Equipotential surface :-

The surface at which potential is same at each point :

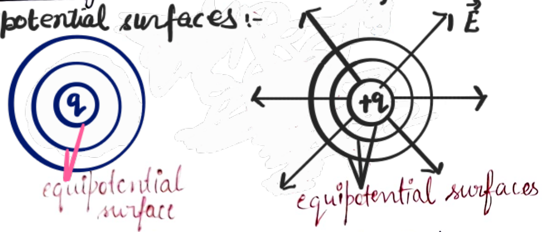
OR

The surface with a constant value of potential at all the points of surface.

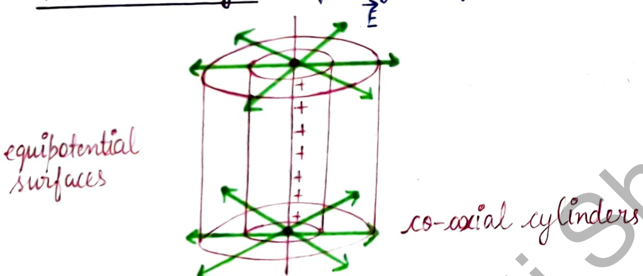
$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}$$

for constant r the value of V is also constant hence equipotential surfaces are the concentric spherical surfaces.

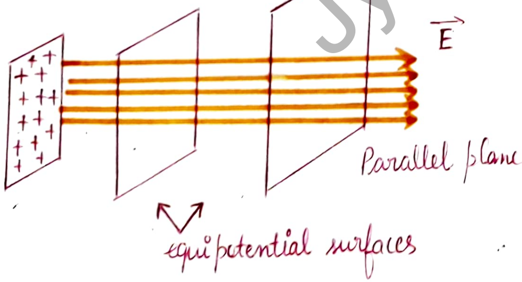
* For point charge - Equipotential surfaces are the equipotential surfaces :-



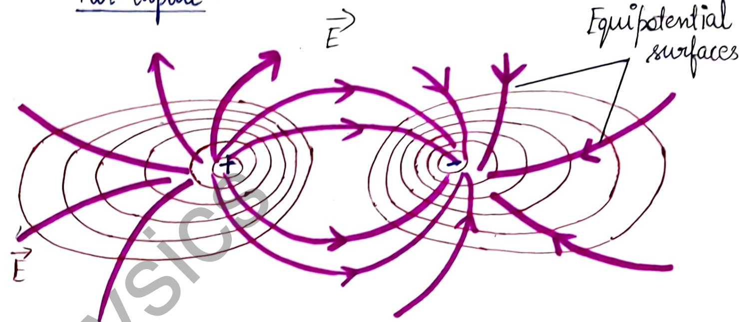
* for line charge - (Uniformly charged wire)



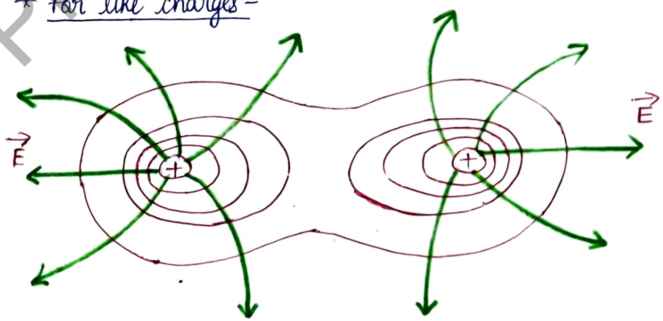
* For uniformly charged plane sheet -



* For dipole -



* For like charges -

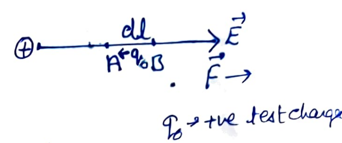


Relation between Electric field and Potential :-

Let the charge q_0 is moved from B to A

force on charge, $q_0 = q_0 E$

Now,



$$V_A - V_B = V_{AB} = \frac{W_{AB}}{q_0}$$

$$V_{AB} = dV = \frac{F \times \text{displacement}}{q_0}$$

$$= \frac{F.dl\cos\theta}{q_0} = \frac{Fdl\cos 180^\circ}{q_0} \quad [\theta = 180^\circ \text{(against the } F_e)]$$

$$= -\frac{Fdl}{q_0} \quad [\cos 180^\circ = -1]$$

$$\text{or } dV = -\frac{q_0 E dl}{q_0} \quad [qE = F]$$

$$\text{or } \boxed{E = -\frac{dv}{dl}}$$

-ve sign shows that the potential difference decreases in the direction of E.

Also we can write $\left|\frac{dV}{dr}\right|$ = Potential gradient

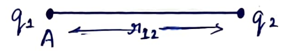
i.e change in potential per unit distance.

Properties of equipotential surfaces :-

1. Work done $W = 0$ for any motion along any equipotential surface.
2. The electric field is alway ⊥ to the equipotential surface.
3. The electric field points in the direction of decreasing potential.
4. The surface of a conductor is always equipotential.
5. All the points of a conductor have same potential.
6. On moving away the equipotential surfaces look more and more like spheres because conductor looks like a point charge.
7. Equipotential surfaces never intersect each other because at the point of intersection there will be two directions of electric field which is not possible.

Potential Energy of a system of charges: (Without external field)

(i) For a system of two charges-



Work done in bringing a charge q_1, $W_1 = 0$ because initially there is no field, $E = 0$

Workdone in bringing a charge q_2 from ∞ to B, $W_2 = V_B q_2$

Here, $V_B = \frac{kq_1}{r_{12}}$

$$W_2 = \frac{kq_1q_2}{r_{12}}$$

Total work done $W = W_1 + W_2$

$$= 0 + \frac{kq_1q_2}{r_{12}}$$

$$= 0 + \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1q_2}{r_{12}}$$

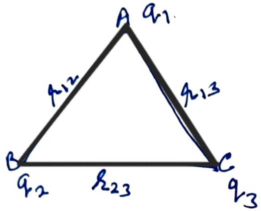
or $\boxed{W = U \text{ (Potential energy)} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r_{12}}}$

(ii) For system of three charges :-

$W_1 = 0 \quad (\because V_A = 0)$

$$W_2 = \frac{K q_1 q_2}{r_{12}}$$

$$W_3 = K\left[\frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right]$$



Total work done

$$W = W_1 + W_2 + W_3$$
$$= K\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

* In absence of electric field, there is no workdone in bringing first charge.
* In bringing second charge, work is done due to first charge and in bringing third charge work is done to I and II both the charges and so on.

Workdone or potential energy of system of charges (In presence of external field).

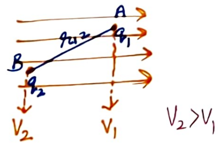
When a charge is brought at a point in presence of external field, the workdone done is given by

$$W_1 = V_1 q_1$$

where q_1 is the charge, bringing from ∞ to A and V_1 is the potential at 'A'.

Now workdone in bringing of charge q_2 from ∞ to B.

$$W_2 = V_2 q_2 + \frac{K q_1 q_2}{r_{12}}$$



Net work done

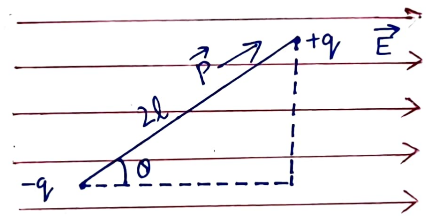
$$W = W_1 + W_2$$
$$= V_1 q_1 + V_2 q_2 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}$$

this workdone is equal to the potential energy i.e

$$\boxed{U = V_1 q_1 + V_2 q_2 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}}}$$

Potential energy of a dipole in external field:

Let an electric dipole AB is placed in uniform electric field at an angle θ with field.



In uniform electric field dipole experiences a torque

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$\tau = pE\sin\theta$$

The amount of workdone by this external torque

$$W = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

or $W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$

$= pE\,[-\cos\theta]_{\theta_1}^{\theta_2}$

$= -pE\,[\cos\theta_2 - \cos\theta_1]$

this work done is stored as potential energy of the system.

$U = -pE[\cos\theta_2 - \cos\theta_1]$

$pE[\cos\theta_1 - \cos\theta_2]$

for $\theta_1 = 90^\circ$ and $\theta_2 = \theta$

$U = -pE\cos\theta$

$\boxed{\text{or } U = -\vec{p}\cdot\vec{E}}$

* At $\theta = 0^\circ$ [p and E are ∥]

  $U = -pE$ [Max^m negative]

  i.e stable equilibrium

* At $\theta = 180^\circ$ [p and E are anti ∥]

  $U = +pE$ [max^m positive]

  i.e unstable equilibrium

* At $\theta = 90^\circ$

  $U = 0$

i.e net work done is zero.

OR

* The expression $(U = -\vec{p}\cdot\vec{E})$ can also be calculated by applying the expression for two charges system in external field.

$U(\theta) = qV_1 - qV_2 - \dfrac{q^2}{4\pi\epsilon_0 \times 2a}$

$= q(V_1 - V_2) - \dfrac{q^2}{4\pi\epsilon_0 \times 2a}$

$= q(-E \times 2l\cos\theta) - \dfrac{q^2}{4\pi\epsilon_0 \times 2l}$ [by $V = E.dr$]

the second term is constant and insignificant for potential energy so we drop it then

$U(\theta) = -2qlE\cos\theta$

$-pE\cos\theta$

or $\boxed{U = -\vec{p}\cdot\vec{E}}$ same result

Electrostatic of conductors –

Conductors contain mobile charge carriers. In metals these are electrons. In a metal the valence electrons are free to move within the metal but not free to leave the metal. Electrons collide with each other and move randomly in different direction in absence of electric field. In an external field they drift against the direction of electric field.

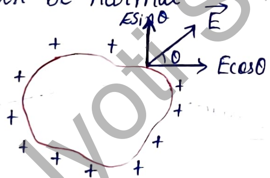
The important results regarding the electrostatics of conductors are -

1. Inside a conductor, electrostatic field is zero -

In the static situation, the overall flow of the charge is zero, so electric field inside the conductor must be zero. If electric field is not zero, the current will flow which is not possible in the static situation.

2. At the surface of a charged conductor the electrostatic field must be normal to the surface at every point -

In the static situation E should have no tangential component. Thus electrostatic field at the surface of a charged conductor must be normal



for static condⁿ

$E\cos\theta = 0$

or $\cos\theta = 0$

i.e $\theta = 90$

3. The interior of a conductor can have no excess - On a closed surface, the electrostatic field is zero. So from Gauss's law there is no net charge enclosed by the surface.

On the closed surface $E = 0$

i.e $\phi = E.A = 0$

According to Gauss's law

$$\phi = \frac{q}{\epsilon_0}$$

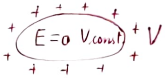
or $\frac{q}{\epsilon_0} = 0$

i.e $\boxed{q = 0}$ ⟹ Net charge inside the conductor is zero.

4. Electrostatic Potential is constant throughout the volume of the conductor and has the same value on its surface ⟶

We know

$$dV = -E\,dr$$



but $E = 0$ inside the conductor, then

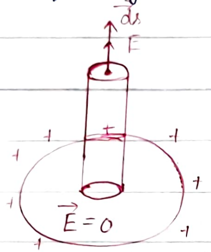
$$dV = 0$$

or $V = \text{constant}$

i.e potential will be the same at the surface as well as whole of the interior of the conductor.

5. Electric field at the surface of charge conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$, where σ is surface charge density

Consider a charged conductor of irregular shape
Acc. to Gauss theorem



charged conductor

$$\oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

or $\oint E\,ds\cos 0^\circ = \frac{q}{\epsilon_0}$

or $E\oint ds = \frac{q}{\epsilon_0}$ $[\cos 0^\circ = 1]$

but $\oint ds = ds$ and $q = \sigma ds$, then

$$E\,ds = \frac{\sigma ds}{\epsilon_0}$$

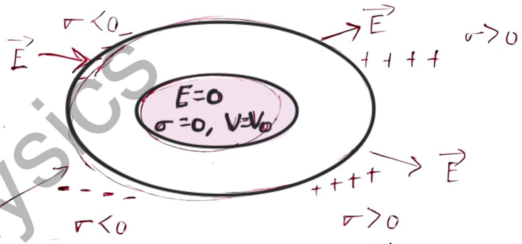
or $E = \frac{\sigma}{\epsilon_0}$

Vector form,

or $\boxed{\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}}$ [$\because$ E is $\perp$ to the surface]

* for $\sigma > 0$, E is normal to the surface outward.
* for $\sigma < 0$, E is normal to the surface inward.

Electrostatic shielding $\rightarrow$ The field inside the cavity of a conducting body is always zero.
This is known as electrostatic shielding.



[here no charge is placed inside the cavity]

Dielectrics — The non-conducting materials (insulators) are called dielectrics.

e.g $\rightarrow$ air, glass, mica, wax

Types
- Polar
- Non-polar

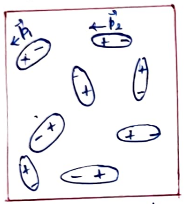
Polar dielectrics — Each molecule has non-zero dipole moment.
It is made of polar molecules (com [centre of mass] of +ve and -ve charge does not concide).
e.g $\rightarrow$ HCl, NH_3, H_2O, CO_2, etc..

Non-polar dielectrics → Each molecule has net zero dipole moment.

It is made of non-polar molecules (COM of +ve and -ve charge concides)

e.g H_2, O_2, N_2, CO_2 etc.

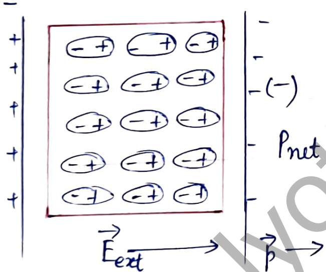
• Polar dielectric (a) without external electric field:-



$P_{net} = 0$

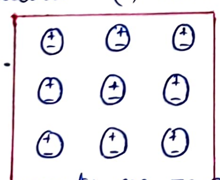
* Individual dipole moments are not zero.

(b) With field -



$P_{net} \neq 0$

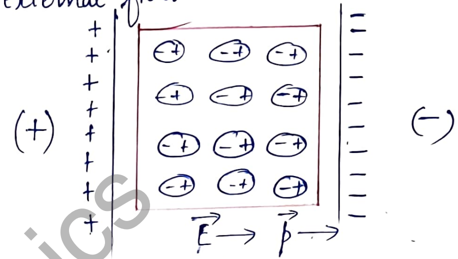
• Non-Polar dielectric - (a) without external field



$P_{net} = 0$

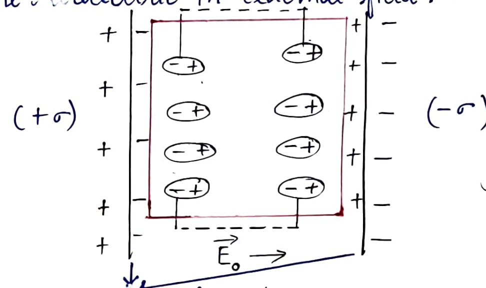
* Individual moments are zero.

(b) In external field -



$P_{net} \neq 0$

Polarisation of dielectrics - It is the process of inducing equal and opposite charges on the two opposite faces of the dielectric in external field.



|| plates with opposite charge

Polarisation Vector: Net dipole moment per unit volume is called polarisation vector.

$$\vec{P} = \frac{P_{net}}{Volume}$$

→ SI unit c/m^2

→ It is vector quantity (dir^n along net dipole moment)

Susceptibility (χ_e) $\rightarrow$

$$\vec{P} \propto \epsilon_0 \vec{E}$$
$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$
or $$\boxed{\chi_e = \frac{\vec{P}}{\epsilon_0 \vec{E}}}$$

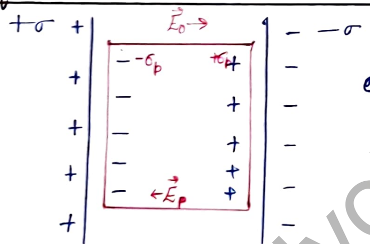
$\rightarrow$ χ_e is called electric susceptibility.

$\rightarrow$ It is unitless and scalar quantity.

Relation b/w electric susceptibility χ_e and dielectric constant K

$$\boxed{K = 1 + \chi_e}$$

Electric field inside the dielectrics $\rightarrow$



$\sigma_p \rightarrow$ surface charge density due to polarisation

$$E_{ext} = \frac{\sigma}{\epsilon_0} = E_0$$

$$E_{in} = \frac{\sigma_p}{\epsilon_0} = E_p$$

$$E_{net} = E_{ext} - E_{in}$$
$$= \frac{1}{\epsilon_0}(\sigma - \sigma_p)$$

put $E_{net} = \dfrac{E_0}{K}$

$E_0 \rightarrow$ air/Vac.

$E_{net} \rightarrow$ medium

$K \rightarrow$ Dielectric constant

$$\frac{E_0}{K} = \frac{1}{\epsilon_0}(\sigma - \sigma_p)$$

or $$\frac{\sigma}{K\epsilon_0} = \frac{1}{\epsilon_0}(\sigma - \sigma_p)$$

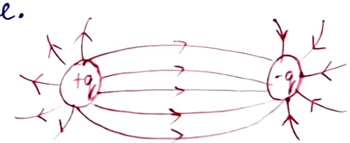
or $$\boxed{K = \frac{\sigma}{\sigma - \sigma_p}}$$

i.e $K > 1$

- * For metals $K = \infty$
- * For air/vac. $K = 1$
- * For water $K = 80$
- * For mica $K = 6$

Capacitor: A capacitor is a system of two conductors separated by an insulator.

* A capacitor is charged by a battery and the two conductors get equal and opposite charge.



The potential difference between the two conductors

$V = V_1 - V_2$

Electrical capacitance: When some charge is given to a conductor it gains some potential.

and $q \propto V$

$q = cV$

$c = \frac{q}{V}$

where c is called electrical capacitance or capacity of the conductor.

It is defined as the charge required to raise the potential through one unit.

SI unit of c is Farad (F).

Other small units are

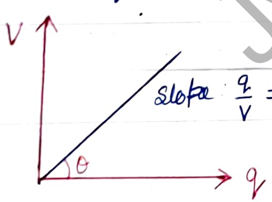
$1\,\mu F = 10^{-6} F$, $1\,nF = 10^{-9} F$; $1\,PF = 10^{-12} F$

* For a capacitor $C = \frac{q}{V}$, so the graph –

$Q = CV \Rightarrow y = mx$

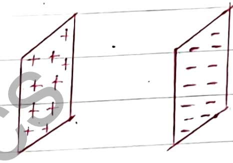
$\Rightarrow m = c$ (slope)

$\Rightarrow \tan\theta = C$



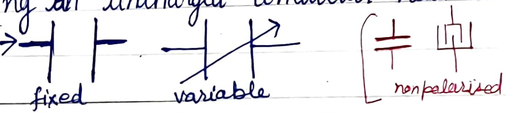
* c is always +ve.

Parallel Plate Capacitor (PPC): A parallel plate capacitor consists two parallel conducting thin plates separated by a small distance.



Principle – Principle of PPC is electrostatic induction. i.e capacitance of a charged conductor is increased by bringing an uncharged conductor near it.

symbol →

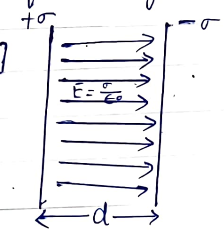


Capacitance of parallel plate capacitance.

Let d is the distance b/w two plates of capacitor.

Let '$+\sigma$' and '$-\sigma$' are the surface charge densities of the plates.

Now $E = \frac{\sigma}{\epsilon_0}$ [inside the plates]



but $E = \frac{dV}{dr} = \frac{V}{d}$

or $V = Ed$

$V = \frac{\sigma d}{\epsilon_0}$ $\left[\because E = \frac{\sigma}{\epsilon_0}\right]$

We know

$C = \frac{q}{V}$

$= \frac{q}{\sigma d/\epsilon_0} = \frac{q\,\epsilon_0}{\sigma d}$

here $q = \sigma A$, then

$$C = \frac{\sigma A \epsilon_0}{\sigma d}$$

$$\boxed{C = \frac{A\epsilon_0}{d}}$$

$C \propto A$,

$C \propto \frac{1}{d}$

* C depends upon shape and size of the plates and also on the material filled b/w the plates.

* If $C = 1F$, $d = 1mm$ then

$$A = \frac{Cd}{\epsilon_0} = \frac{1 \times 1 \times 10^{-3}}{8.86 \times 10^{-12}} \simeq 110\ km^2$$

i.e side of plate $\simeq$ 10 km which shows 1 Farad is very big unit of c.

Effect of dielectric on capacitance:-

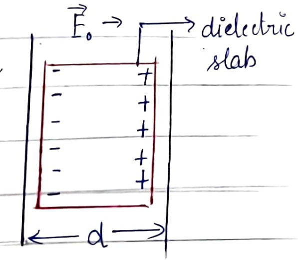
Case I: When dielectric slab filled completely the space between the plates of a capacitor

We know that for air/vac.

$$C_0 = \frac{A\epsilon_0}{d} \quad \text{—(1)}$$

Now for a medium of dielectric constant k.

$$C = \frac{A\epsilon}{d}$$



E is permittivity of any medium

but we know $\epsilon = k\epsilon_0$

from (1) and (2)

$$\frac{C}{C_0} = \frac{kA\epsilon_0/d}{A\epsilon_0/d}$$

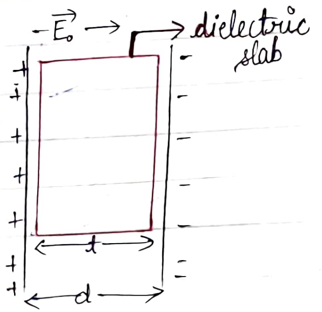
$$\frac{C}{C_0} = k$$

$$\boxed{C = kC_0}$$

i.e capacitance of capacitor becomes k times when filled with dielectric.

* C always ↑ on inserting the medium
* If capacitor filled with metal $(k = \infty)$, $c = \infty$

Case II: When dielectric slab of thickness 't' is filled b/w the plates.



$$V = E_0(d - t) + E.t \quad \text{—(1)}$$

[$\because V = V_1 + V_2$ and $V = Ed$]

[here V_1 is for free space and V_2 is for dielectric]

but $E = \frac{E_0}{k}$, then

$$V = E_0(d-t) + \frac{E_0}{K}t$$

$$V = E_0\left[(d-t) + \frac{t}{K}\right]$$

Now, $C = \frac{q}{V}$

$$C = \frac{q}{E_0\left[(d-t) + \frac{t}{K}\right]}$$

put $E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$, then

$$C = \frac{q}{\frac{q}{A\epsilon_0}\left[(d-t) + \frac{t}{K}\right]}$$

$$\boxed{C = \frac{A\epsilon_0}{\left[(d-t) + \frac{t}{k}\right]}}$$

special cases →

(1) If $d = t$

$$C = \frac{A\epsilon_0}{(\cancel{d}-\cancel{d}) + \frac{d}{k}}$$

or $C = \frac{KA\epsilon_0}{d}$

$\boxed{C = KC_0}$ $\left[\because C_0 = \frac{A\epsilon_0}{d}\right]$

(11) If $t = \frac{d}{2}$, then

$$C = \frac{2KC_0}{K+1}$$

* **Spherical capacitor:-** It consists two concentric spherical shell of different radius capacitance C for spherical capacitor.

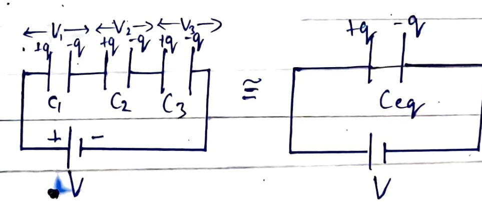
$$C = \frac{q}{V}$$

$$C = \frac{q}{\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}}$$

or $\boxed{C = 4\pi\epsilon_0 r}$ $\quad C \propto r$

* **Combination of capacitors:-**

(1) Series combination -



In series q is constant and

$$V = V_1 + V_2 + V_3$$

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or $\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

* for two capacitors

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

* for three capacitors

$$C_{eq} = \frac{C_1 C_2 C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

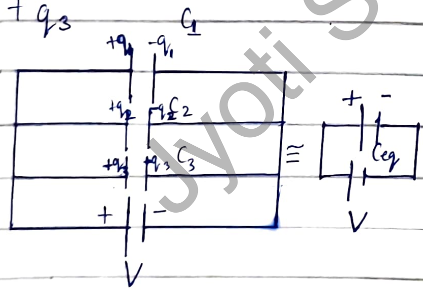
* for n identical capacitors of capacitance c

$$C_{eq} = \frac{c}{n}$$

Parallel combination—

In ‖ combination V is constant and

$q = q_1 + q_2 + q_3$



$q = q_1 + q_2 + q_3$

$C_{eq}V = C_1V + C_2V + C_3V$

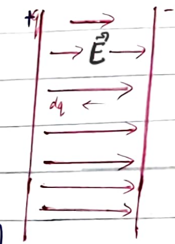
or $\boxed{C_{eq} = C_1 + C_2 + C_3}$

* for n identical capacitor

$$C_{eq} = n \times C$$

Energy stored in a capacitor (Electrostatic Energy)–

Work is done by the battery in transferring the charge dq from one plate to another i.e



$dU = dW = Vdq$

or $\int dU = U = \int V dq$

or $U = \int_0^q \frac{q}{C} dq \quad [\because V = \frac{q}{C}]$

$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$

$\boxed{U = \frac{q^2}{2C}}$

put $q = CV$

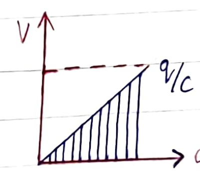
$U = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$

i.e $\boxed{U = \frac{1}{2}CV^2}$

put $CV = q$

$\boxed{U = \frac{1}{2}qV}$

By V-q graph energy stored is the area under V-q graph i.e..



$$U = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} q \times \frac{q}{C}$$
$$U = \frac{q^2}{2C}$$

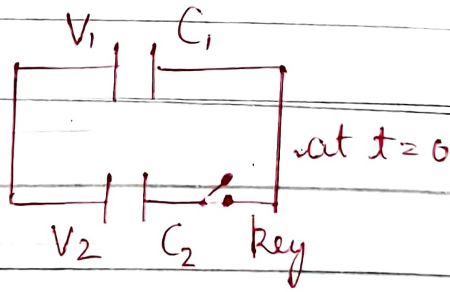
Energy density – Energy stored in a capacitor per unit volume.

Common Potential:

When two capacitors of different capacitance and charged to different potentials are connected in parallel by a conducting wire, charge flows from a capacitor to higher potential to lower potential until potential of both becomes equal. This equal potential is known as common potential.

Total charge on both

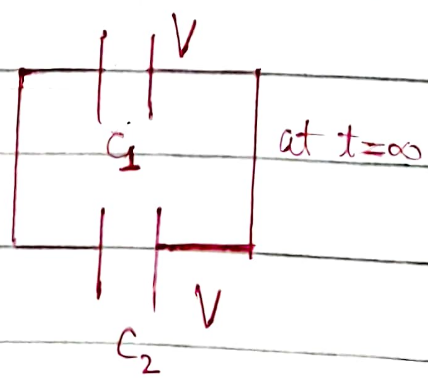
$$q = q_1 + q_2$$
$$q = C_1V_1 + C_2V_2$$



Let V is the common potential

After sharing total charge

$$q' = C_1V + C_2V$$
$$= (C_1 + C_2)V$$



since charge is conserved

$$q = q'$$

$$C_1V_1 + C_2V_2 = (C_1+C_2)V$$

or $$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}}$$

Loss of Energy on sharing charge:-

When two charged capacitors are connected in parallel, no charge is lost. But some energy is lost on sharing the charges. This loss of energy appears as heat loss.

Electrostatic potential energy of two capacitors before sharing.

$$U_i = U_1 + U_2$$
$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \quad [U = \tfrac{1}{2}CV^2]$$

After sharing potential of both capacitor becomes V so energy after sharing

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$
$$= \frac{1}{2}(C_1+C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2$$
$$= \frac{1}{2}\cancel{(C_1+C_2)}\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)^{\cancel{2}}}$$
$$U_f = \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{(C_1+C_2)}$$

loss of energy $= U_i = U_f = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) - \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$

$$U_i - U_f = \frac{1}{2}\left[\frac{(C_1V_1^2 + C_2V_2^2)(C_1+C_2) - (C_1V_1+C_2V_2)^2}{C_1+C_2}\right]$$

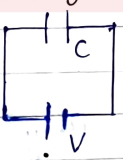
$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_2^2V_2^2 + C_1C_2V_1^2 + C_1C_2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 + 2C_1C_2V_1V_2}{C_1+C_2}\right]$$

$$= \frac{1}{2}\left[\frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{C_1+C_2}\right]$$

$$U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

the RHS is alway positive. Hence $U_i > U_f$ always i.e always there is a loss of energy when charged capacitors are connected together.

Loss of energy when a capacitor is charged-



Before charging the capacitor energy of battery by $W = Vq$

$$U_1 = qV$$

After charging the energy of capacitor

$$U' = \frac{1}{2}CV^2$$
$$= \frac{1}{2}(CV)V$$

or $$= \frac{1}{2}qV \quad [\because q = CV]$$
$$= \frac{1}{2}qV$$

$$U - U' = \frac{1}{2}qV \quad [\text{loss of energy is } 50\%.]$$

Effect of dielectric on various parameter -

Battery disconnect from the capacitor	Battery remains connected
Charge $q = q_0$ (const.)	$q = kq_0$
Potential $V = \frac{V_0}{K}$	V constant
Electric field $E = \frac{E_0}{K}$	E constant
capacitance $C = kC_0$	$C = KC_0$
Energy stored $U = \frac{U_0}{K}$	$U = KU_0$

When battery disconnected —

(I) charge — Charge remains const (q_0)

(II) Electric field - due to dielectric charge induced on dielectric surface which reduces the electric field and becomes $E = \frac{E_0}{K}$

(III) Potential difference -

As E decreases, V is also decreased by

$V = Ed = V = \frac{E_0 . d}{K} = \frac{V_0}{K}$

or $V = \frac{V_0}{K}$

(IV) Capacitance

By $U = \frac{1}{2} CV^2$

$= \frac{1}{2} K C_0 \times \left(\frac{V_0}{K}\right)^2$

$= \frac{1}{2} \frac{K C_0 V_0^2}{K^2} = \frac{1}{2} \frac{C_0 V_0^2}{K}$

$U = \frac{U_0}{K}$

energy ↓ by K

↑ → increases
↓ → decreases

When Battery is connected

(I) Charge —

Charge ↑, K times

by $q = CV$

$= K C_0 V_0$

$q = kq_0$

(II) Electric field — As battery remains connected V remains constant. Hence E is also remains constant.

By $E = \frac{V}{d} = \frac{V_0}{d}$

$E = E_0$ constant

(iii) Potential difference :- As battery is connected V remains constant

$$V = V_0$$

(iv) Capacitance :-

By $C = \frac{q}{V}$

$$= \frac{Kq_0}{V_0} \quad (V = V_0)$$

$$= KC_0$$

Hence, $C \uparrow$ by K times

(v) Energy stored

By $= U = \frac{1}{2}CV^2$

$$= \frac{1}{2}KC_0 \cdot V_0^2$$

$$= K\left(\frac{1}{2}C_0V_0^2\right)$$

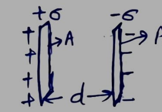
$$U = KU_0$$

Energy $\uparrow$ by K times.

Expression for Energy Density :-

We know

$$U = \frac{1}{2}CV^2$$



$A \to$ Area of plates
$d \to$ distance b/w the plates

put $C = \frac{A\epsilon_0}{d}$ and $V = E \cdot d$

we get,

$$U = \frac{1}{2}\left(\frac{A\epsilon_0}{d}\right) \cdot (Ed)^2$$

$$= \frac{1}{2}\frac{A\epsilon_0}{d} \times E^2d^2 \times \frac{A \cdot d}{Ad}$$

[Vol. = $A\,d$]
Volume b/w plates

or $\frac{U}{Ad} = \frac{1}{2}\epsilon_0E^2$

or $\frac{U}{Vol} = \frac{1}{2}\epsilon_0E^2 =$ Energy density (u)

or $\boxed{u = \frac{1}{2}\epsilon_0E^2}$

* Energy stored in a capacitor per unit volume is called energy density of the capacitor.