Laws of Motion

Force: Force is a push or pull which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body.

Aristotle's Fallacy: According to Aristotelian law of motion an external force is necessary to keep a body moving with uniform velocity.
However Aristotle's view were proved wrong by Galilio Galleio.

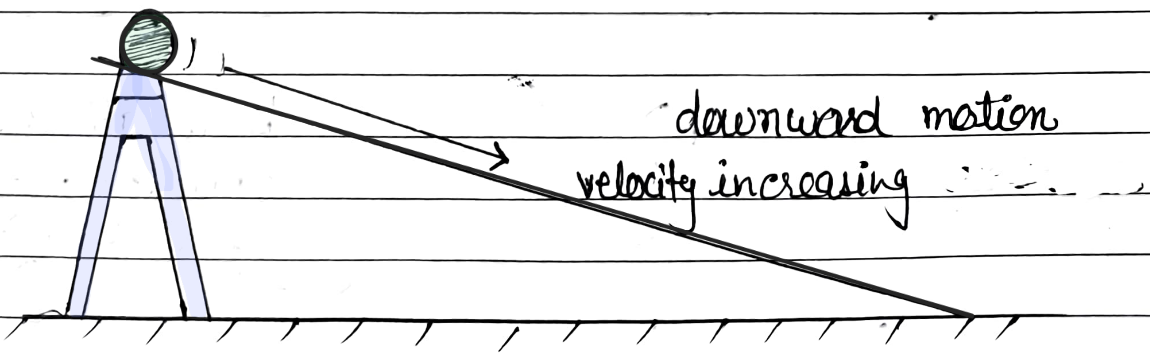
Inertia:
Inertia is the property of a body due to which body resist any changes in its state of rest or motion.

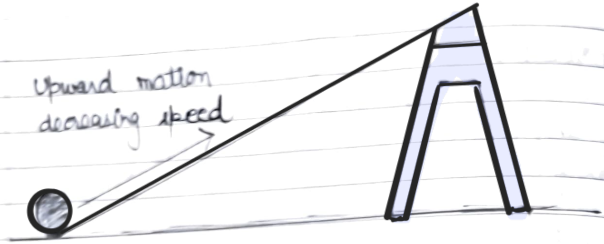
$$\text{Inertia} \propto \text{mass}$$

i.e more mass, more inertia.

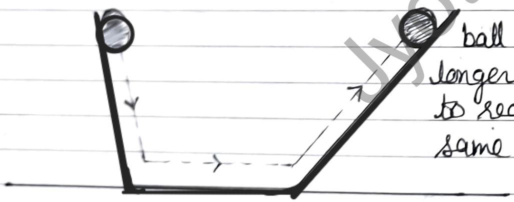
Inertia
- Inertia of rest
- Inertia of direction
- Inertia of motion

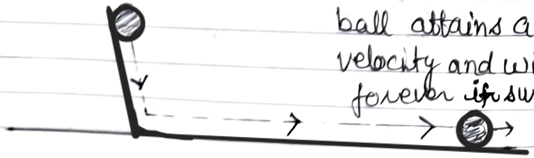
Galileo's Experiment with single inclined Plane





Galilio's observations of motion on a double inclined plane -





Linear Momentum:

Momentum of a body is the quantity of motion possessed by the body.
Linear momentum is equal to the product of mass and velocity of the body.

momentum = mass x velocity

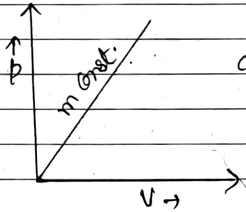
$$\vec{p} = m\vec{v}$$

It is a vector quantity and its dir^n is same as direction of velocity.

SI unit - Kg m/s

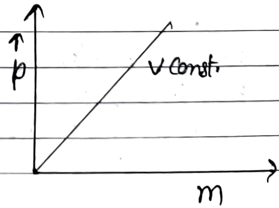
Variation in momentum -

(I) Consider two objects of same mass and different velocities. (say $v_1 > v_2$)



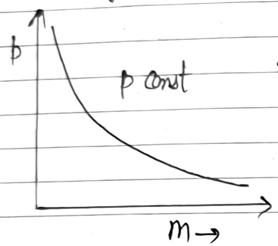
as $v_1 > v_2$, so $p_1 > p_2$

(II) Two bodies of different masses (say $m_1 > m_2$) are moving with same velocity -



~~Cases~~.

(iii) Two objects having equal linear momentum



$p_1 = p_2$

$m_1 v_1 = m_2 v_2$

$\frac{m_1}{m_2} = \frac{v_2}{v_1}$

here mass and velocities are inversaly proportional to each other. (i.e less mass → more velocity)

Newton's Laws of Motion (NLM)

There are three laws of motion which are called Newton's laws of motion.

First law of Motion (Law of Inertia)

Every body continues in its state of rest or of uniform motion unless any external force is applied on it.
This law is also known as law of inertia.

Examples

(i) Based on law of inertia of rest –
→ Dust is removed from a hanging carpet by beating it.
As the carpet is beaten it suddenly moves but dust particle remains at rest due to inertia of rest. Therefore dust fall off.

→ When we shake the branch of a tree, its fruits and dry leaves fall down.
Due to inertia of rest when branches move on shaking, fruits and leaves tends to remain in rest and get separated from the branches of tree.
→ Coin falls into the tumbler when card is given sudden jerk.

(ii) Based on Inertia of motion

→ A person getting out of a moving bus or train falls in the forward direction. Due to ~~initia~~ inertia of motion his upper part of body moves forward.
→ A ball thrown upward in a moving train comes back into the thrower's hands. When ball thrown it covers horizontal direction also due to inertia of motion and comes into the hands.

(iii) Based on inertia of direction.
→ Mudguards are used over the vehicles wheel as mud flies off tangentially due to inertia of direction.
→ When a dog chases a hare, the hare runs along a zig-zag path.

Vector form of I law of motion

If $\vec{F}_{net} = 0$ [i.e in equilibrium]

$\vec{F}_x = \vec{F}_y = \vec{F}_z = 0$

If vector sum of all the forces acting on a body is zero the the body remains unaccelerated.

Second law of Motion :

According to this law the time rate of change of linear momentum of a body is directly proportional to the net applied force on the body and this change is in direction of the applied force.

Hence,
$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

or
$$\vec{F} = k\frac{d\vec{p}}{dt}$$

where k is the constant of proportionality.

since $\vec{p} = m\vec{v}$

$$\vec{F} = k\frac{d}{dt}(m\vec{v})$$
$$= k\,m\frac{d\vec{v}}{dt}$$

or $\vec{F} = km\vec{a}$ $\quad \left(\frac{d\vec{v}}{dt} = \vec{a}\right)$

If $k=1$
$$\vec{F} = m\vec{a}$$

magnitude of force
$$F = ma$$

Components of force -

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

then $F_x = ma_x$, $F_y = ma_y$ and $F_z = ma_z$

* $\vec{F} = m\vec{a}$ holds good only if mass remains constant.
* If mass of body changes, then

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$
$$= m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

Units of Force -

→ SI unit - Newton (N)

$1N = 1\,kg\text{-}m/s^2$

Thus, force is said to be 1 newton if it produces $1\,m/s^2$ acceleration in a body of 1 kg mass.

→ CGS unit - dyne

$1\,dyne = 1\,g\text{-}cm/s^2$

$1N = 10^5\,dyne$

→ Gravitational unit

$1\,kgf\ (kg\text{-}wt) = 9.8\,N$, $1\,gf = 980\,dyne$

Applications of Newton's Second Law -

1. A cricket player lowers his hands while catching a ball.
2. A person falling on a cemented floor gets injured but falling on a heap of sand is not injured.
3. The vehicles are fitted with shockers.
4. Compartments of a train are provided with buffers.

Impulse - Impulse is the product of average force and the time interval for which the force acts on the body.

$$\text{Impulse} = \text{force} \times \text{time}$$

$$I = F\Delta t$$

It is a vector quantity.
SI unit (N-S)

Expression -
Consider a constant force F which acts on a body for time dt.
The impulse

$$dI = F\,dt$$

or $$\int dI = \int_{t_1}^{t_2} F\,dt$$

or $$I = F[t]_{t_1}^{t_2}$$

or $$I = F(t_2 - t_1)$$

or $$\boxed{I = F \times \Delta t}$$

Vector form $$\boxed{\vec{I} = \vec{F} \times \Delta t}$$

Impulse-momentum Theorem
According to Newton's second law

$$F = \frac{\Delta p}{\Delta t}$$

or $$\Delta p = F\Delta t$$

but we know
$$I = F\Delta t$$

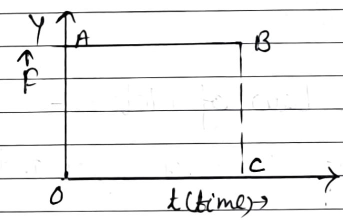
so $$\Delta p = I \text{ (impulse)}$$

or Impulse I = change in momentum.

Thus impulse of a force is equal to the change in momentum of the body.

Graphical Method for measurement of Impulse

1. When force is constant

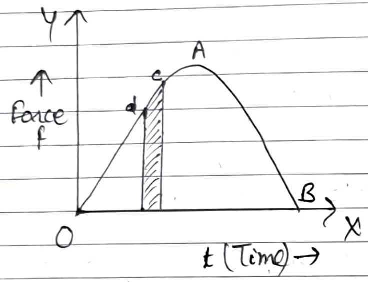


The area of the rectangle OABC
$= OA \times OC$
$= F \times t$
$= I$ (impulse)

Thus for constant force

Impulse = Area under the F-t graph.

2. For variable force



$$I = \int_{t_1}^{t_2} F\,dt$$

$$= F[I]_{t_1}^{t_2}$$

$$= F(t_2 - t_1)$$

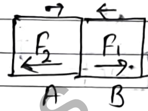
or $\boxed{I = F\Delta t}$

Newton's Third Law of Motion -

To every action there is equal and opposite reaction.

Let two bodies A and B are applying force on each other. They exert equal and opposite forces on each other.

Out of these forces one is called as action and other is called reaction.



$\boxed{\vec{F_1} = -\vec{F_2}}$

or $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$

* A single isolated force cannot exist.
* Forces in nature alway occur in pairs.
* Action and reaction forces always act on different bodies. Therefore they never cancel out each other.

Examples of Newton's Third Law

→ When a ball is dropped it rebounds.

→ A boat moves backward, when a person jumps out of the boat.

→ It is difficult to drive a nail without holding the block.

→ Walking, running swimming etc. are possible due to the reaction force.

→ Launching of a rocket and motion of a vehicle are also an examples of newton's third law.

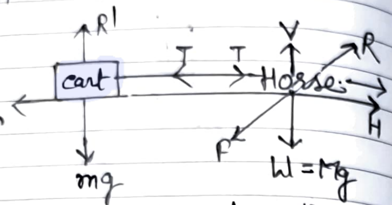
→ **Applications of Newton's laws of motion**

→ (1) Horse-Cart Problem –

Motion of Horse –

$$H - T = Ma \quad -(1)$$

When $H > T$ the horse moves forward



$R \rightarrow$ Normal reaction
$f \rightarrow$ friction force
$H \rightarrow$ Horizontal component of R
$V \rightarrow$ vertical component of R

Motion of Cart –

$$T - f = ma \quad -(2)$$

When Tension T is greater than friction force f the cart moves.

Motion of Horse and cart
On adding (1) and (2)

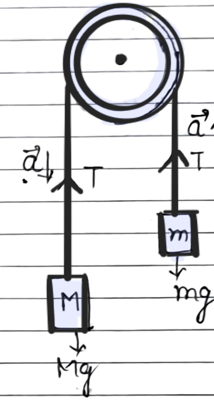
$$H - f = (M+m)a$$

or

$$\boxed{a = \frac{H-f}{(M+m)}}$$

→ (2) **Problem of Masses and Pulley: Connected Motion**

Consider two masses M and m $(M > m)$ connect to the two free ends of an inextensible string which passes over a smooth pulley as shown in fig.



For mass M (Moving downward)

$$F = Mg - T$$

but $F = Ma$, then

$$Ma = Mg - T \quad -(1)$$

For mass m (Moving upward)

$$F = T - mg$$

but $F = ma$, then

$$ma = T - mg \quad -(2)$$

Adding eq^n (1) and (2)

$$(M+m)a = (M-m)g$$

or

$$\boxed{a = \left(\frac{M-m}{M+m}\right)g}$$

put this value of 'a' in eq^n(1)

$$M \times \frac{(M-m)g}{M+m} = Mg - T$$

or $$T = Mg - \frac{(M-m)Mg}{M+m} = Mg\left[1 - \frac{(M-m)}{M+m}\right]$$

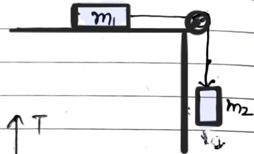
or $$T = Mg\left[\frac{2m}{M+m}\right]$$

or $$\boxed{T = \frac{2Mmg}{M+m}}$$

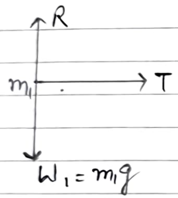
(3) Free Body Diagram (FBD)

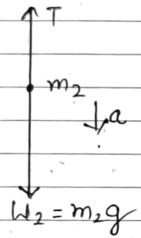
Free body diagram is a diagram showing a given body or object as a point and the various forces acting on it.

Example - calculate the acceleration of each block and tension in string as shown in fig. Assume there is no friction between the surface of block and the surface of table.



Solution -





$$f = m_1 a = T \quad -(1)$$

and $m_2 a = m_2 g - T \quad -(2)$

Adding (1) and (2)

$$m_1 a + m_2 a = m_2 g$$

or $$a = \frac{m_2 g}{m_1 + m_2}$$

from (1)

$$T = \frac{m_1 \times m_2 g}{m_1 + m_2}$$

or $$\boxed{T = \frac{m_1 m_2 g}{m_1 + m_2}}$$

→ **Apparent weight of a person in a lift / Elevator**

(1) When the lift is at rest or moving with constant velocity

$$R = mg$$

$R \rightarrow$ Normal Reaction
$mg \rightarrow$ Weight

i.e. Apparent weight = True weight

(II) When lift accelerated upward

$$f = R - mg \Rightarrow ma = R - mg$$

or $$R = m(a+g)$$

Thus Apparent weight > True weight

(III) When lift accelerates downward.

$$f = mg - R$$
$$ma = mg - R$$
or $$R = m(g-a)$$

Thus apparent weight < True weight

(IV) When body falls freely

$$R = m(g-g)$$
or $$R = 0$$

Thus the apparent weight is zero. This state is known as weightlessness.

→ **Newton's Second Law is real law of Motion**

(1) First law is contained in the second law -

According to Newton's II Law of motion

$$F = ma$$

If $F = 0$ (i.e. no external force acts on the body)

$$ma = 0$$
or $a = 0$
or $v = u$ ⇒ Which verifies I law of Newton.

(ii) Third law is contained in the second law

For an isolated system we consider two particles 1 and 2. Let F_{12} be the force exerted on the body 1 by the body 2, and F_{21} is the force exerted on the body 2 by 1.

Then according to Newton's II law

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \quad -(1)$$

and $$F_{21} = \frac{d\vec{p}_2}{dt} \quad -(2)$$

(1) + (2)

$$\vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt}$$

$$= \frac{d}{dt}(\vec{p}_1 + \vec{p}_2)$$

As no external force is acting on the system

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

or $$\vec{F}_{12} + \vec{F}_{21} = 0$$

or $$\vec{F}_{12} = -\vec{F}_{21}$$

Which is Newton's third law of motion.

As both the I and II laws of motion are contained in the second law, therefore second is the real law.

→ Law of conservation of linear momentum

" The total linear momentum of a system remains constant if the net external force acting on the system is zero.

According to II Law

$$\vec{F} = \frac{d\vec{p}}{dt}$$

If $F = 0$, then

$$\frac{d\vec{p}}{dt} = 0$$

or $\vec{p}$ = Constant

For a system having n particles

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots \vec{p}_n)$$

If $\vec{F}_{ext} = 0$

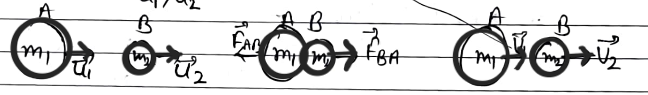
$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots \vec{p}_n) = 0$$

or $$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots \vec{p}_n = \text{Constant}$$

Which is the principle of conservation of linear momentum

→ Conservation of linear momentum by Newton's third law:

$u_1 > u_2$



Total momentum before collision $= m_1\vec{u}_1 + m_2\vec{u}_2$

Total momentum after collision $= m_1\vec{v}_1 + m_2\vec{v}_2$

Change in momentum of body A $= F_{AB}\Delta t$ [by II law]

or $$F_{AB}\Delta t = m_1\vec{v}_1 - m_1\vec{u}_1 \quad -(1)$$

similarly $\vec{F}_{BA}\Delta t = m_2\vec{v}_2 - m_2\vec{u}_2$ — (2)

Now by Newton's III law

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

or $\vec{F}_{AB}\Delta t = -\vec{F}_{BA}\Delta t$ — (3)

From (1) and (2)

$$m_1\vec{v}_1 - m_1\vec{u}_1 = -(m_2\vec{v}_2 - m_2\vec{u}_2)$$

or $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$

or $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

i.e momentum before collision = momentum after collision
This is the law of conservation of linear momentum.

Examples of conservation of linear momentum

(i) Recoil of a Gun $\boxed{V = -\frac{mv}{M}}$

$m \rightarrow$ mass of bullet
$M \rightarrow$ " " Gun
$v \rightarrow$ velocity of bullet
$V \rightarrow$ velocity of Gun

(ii) Rocket Propulsion -

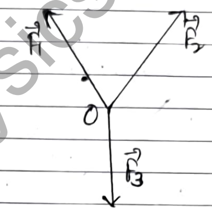
$$dv = -u\frac{dm}{m}$$

$m \rightarrow$ mass of rocket
$dm \rightarrow$ " " fule
$dv \rightarrow$ increase in velocity

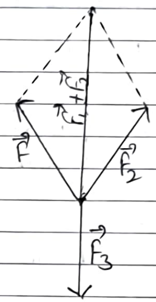
Equilibrium of a particle (concurrent force)

The forces acting simultaneously at a point ~~or a point~~ or a particle are known as concurrent forces.

A point or a particle is said to be in equilibrium if the vector sum of concurrent forces acting on a point or particle is zero.
For example F_1, F_2 and F_3 in given fig are concurrent forces -



(a)



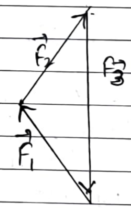
(b)

In fig (b),

$$\vec{F}_3 = -(\vec{F}_1+\vec{F}_2)$$

or $\vec{F}_1+\vec{F}_2+\vec{F}_3 = 0$

i.e the sum of all the forces acting at a point on the particle is zero



$\vec{F}_1+\vec{F}_2+\vec{F}_3 = 0$

(c)

Similarly if a particle is in equilibrium under the n no. of forces, then

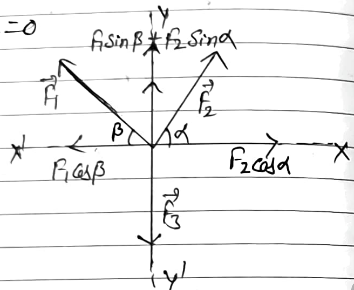
$$F_1+F_2+F_3+ \cdots F_n = 0$$

Condition for the equilibrium of a body in terms of the components of concurrent forces-
The body will be in equilibrium, if

$F_{1x} + F_{2x} + F_{3x} = 0$

$F_{1y} + F_{2y} + F_{3y} = 0$

and $F_{1z} + F_{2z} + F_{3z} = 0$



also,
We resolve $\vec{F_1}$ and $\vec{F_2}$
as body is in equilibrium
i.e

$\Sigma F_x = 0$

or $F_1\cos\beta + F_2\cos\alpha = 0$

or $F_1\cos\beta = F_2\cos\alpha$

and $\Sigma F_y = 0$

i.e $F_1\sin\beta + F_2\sin\alpha - F_3 = 0$

or $F_1\sin\beta + F_2\sin\alpha = F_3$

Intertial and Non Intertial frame of references

Intertial frame → A frame of reference which is either rest or moving with constant velocity is called intertial frame.
e.g. A car is moving with constant velocity.

Non-Inetertial frame → A frame of reference which is accelerated is called non-intertial frame of referance.

e.g. A vehicle is accelerating, the passengers are in non intertial frame.

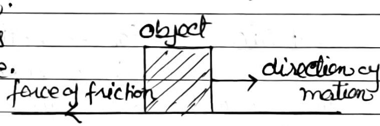
Pseudo force - A fictitious force or pseudo force is a force that appears to act on a mass which is in non intertial frame.
e.g. → centrifugal force.
→ In a lift, moving up an additional force is experience to the weight, this force is a pseudo force.

Common forces in Mechanics-

Contact force → When two bodies are in contact and exert force on each other. The forces are called contact force.
e.g friction force

Friction - Friction is the force which opposes the relative motion or has tendency to opposes the motion.

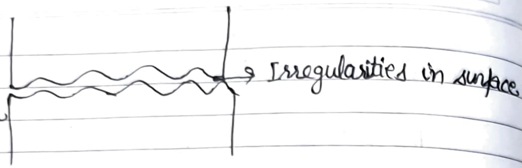
** force of friction is non-conservative force.



Cause of friction -

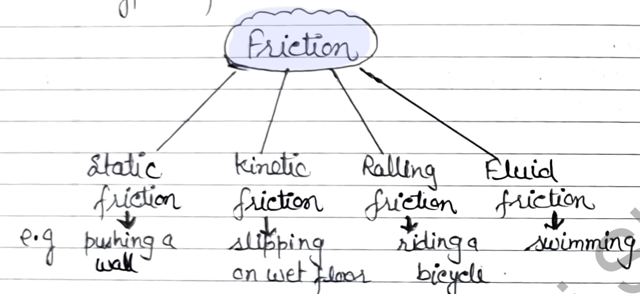
When one surface is placed over another surface the molecules at the surface of contact get interlocked. When one surface slides over the other the bonds between the molecules are continuously broken and re-build at other points. The force required to break bonds is called force of friction.

Roughness of the surface causes friction

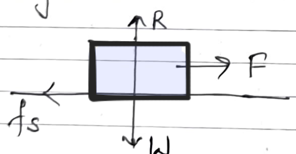


** Normal reaction and force of friction are due to the intermolecular forces which are electrical in nature.

Types of Friction :-



Static friction – static friction is a self adjusting force because it comes into play when the body is lying over the another body, without any motion.
static friction is the friction between two surfaces when the body is rest.



$F \rightarrow$ external force
$f_s \rightarrow$ static friction

the frictional force f_s counter balance the applied force F.
In this case

$$R - W = 0 \quad \text{or} \quad R = W$$

and $F - f_s = 0$ or $F = f_s$

Thus applied force = static friction

Limiting Friction :-
The maximum value of static friction when the body is just going to start sliding is called limiting friction.

Laws of friction

1. The direction of friction is always opposite to the direction of motion.
2. The force of friction acts tangentially along the surface of contact.
3. The magnitude of limiting friction f is directly proportional to the normal reaction R
$$f \propto R$$
4. Friction is independent of area of contact as long as normal reaction is constant.
5. Friction depends on the nature of the surface. i.e on roughness and smoothness of the surface.

Dynamic or Kinetic Friction:

Kinetic friction is defined as the force that acts between the moving surfaces in the opposite direction of motion.

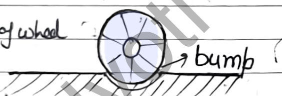
Types of kinetic friction

→ Sliding Friction – The force of friction acting between two surfaces when one body is sliding over the other body.
e.g. When a wooden block is pulled or pushed.

→ Rolling Friction – The force of friction acting when one body rolls over the other body is called rolling friction.
e.g. When wheel rolls over the road.

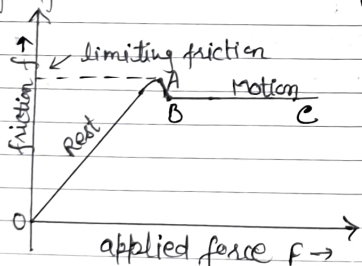
Cause of Rolling friction – Rolling friction takes place due to the deformation of the surfaces.

* $f_r \propto \frac{1}{r}$, $r \rightarrow$ radius of wheel



* $\boxed{f_r = \frac{\mu_r}{r}}$

Variation of force with the applied force –



In the region OA, when applied forces increases, static friction also increases.
At point A static friction is maximum i.e it represents the limiting friction.
When applied force increases beyond limiting friction, the body starts moving. At this state static friction is converted into kinetic friction which is less than the static friction.
Region BC represents the kinetic friction.

Co-efficient of Static friction

The limiting friction (max. static friction)

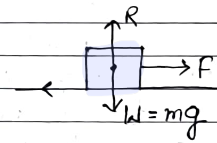
$$\text{i.e } (f_s)_{max} \propto R$$

$$\text{or } (f_s)_{max} = \mu_s R$$

where μ_s is called co-efficient of static friction and $R = mg$

$$\mu_s = \frac{(f_s)_{max}}{R}$$

$$\mu_s = \frac{(f_s)_{max}}{mg}$$



Unit – Unitless

Co-efficient of kinetic Friction –

The kinetic force of friction (f_k).

$$f_k \propto R$$

$$f_k = \mu_k R$$

$$\mu_k = \frac{f_k}{R}$$

where μ_k is called co-efficient of kinetic friction.

and $R = mg$

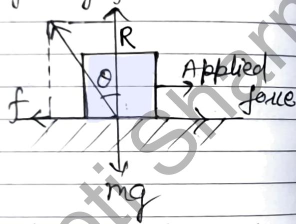
or

$$\mu_k = \frac{f_k}{mg}$$

* If $F < (f_s)_{max}$ body remains at rest
* If $F > (f)_{max}$, body moves
* If $F = (f_k)_{max}$, the body is either at rest or moves with constant velocity.

Angle of Friction -

The angle between the normal reaction and the resultant of limiting force of friction and normal reaction is called angle of friction.



$$\tan\theta = \frac{f}{R}$$

but $\frac{f}{R} = \mu_s$

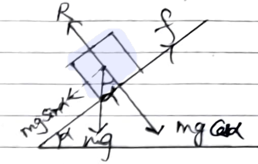
so $\tan\theta = \mu_s$

Thus coefficient of static friction is numerically equal to the tangent of angle of friction.

Angle of Repose

The minimum angle made by the inclined plane with the horizontal surface for which when an object placed on it, just start sliding is called angle of repose.

When the body is in equilibrium



$f = mg\sin\alpha$

and $R = mg\cos\alpha$

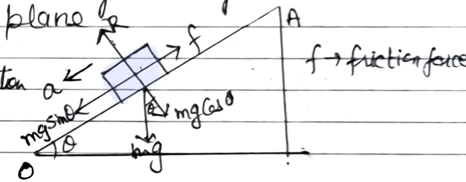
but $\frac{f}{R} = \mu_s$

or $\frac{f}{R} = \frac{mg\sin\alpha}{mg\cos\alpha} = \mu_s$

i.e. $\tan\alpha = \mu_s = \tan\theta$ $[\because \mu_s = \tan\theta]$

so, angle of friction = Angle of repose.

Acceleration of a body sliding down a rough Inclined plane



Let a be the acceleration produced in the body.

Now from fig we can balance different forces as -

$R = mg\cos\theta$

and net force

$F = mg\sin\theta - f$

but $F = ma$, then

$ma = mg\sin\theta - f$

but $f = \mu_k R$, we get

$ma = mg\sin\theta - \mu_k R$

or $ma = mg\sin\theta - \mu_k mg\cos\theta$ $[\because R = mg\cos\theta]$

or

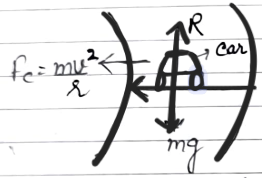
$$a = g(\sin\theta - \mu_k\cos\theta)$$

This is the acceleration of the body when sliding down.

Examples of Circular motion -

(1) Car on a level circular road.
Consider a car of mass m moving with a constant speed v on a circular road of radius r

From fig.



$R - mg = 0$

or $R = mg$

since the car is moving in a circular path, the centripetal force is provided by force of friction. i.e

$$\frac{mv^2}{r} = f = \mu R$$

or $$\frac{mv^2}{r} = \mu mg$$

or $$v = \sqrt{\mu r g}$$

Thus the car will negotiated the flate rough circular path with a maximum speed $\sqrt{\mu r g}$.

Banking of a road -

The outer edge of the circular road is raised a little above the inner edge. This is called banking of a road.

Need of banking - A vehicle can not remain on a flat circular road if its speed exceeds the speed $v = \sqrt{\mu r g}$. i.e force of friction can not provide necessary centripetal force.

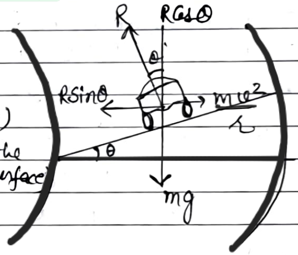
To overcome this problem banking of roads is required. The outer edge of road is raised by an angle θ.

→ Car on a banked road - (without friction consideration)

In fig the various forces acting on the car are

(i) Weight of car $W = mg$ (↓)

(ii) Normal reaction R (⊥ to the surface)



resolve R into $R\cos\theta$ and $R\sin\theta$.

$R\sin\theta$ will provide the centripetal force. i.e

$$R\sin\theta = \frac{mv^2}{r} \quad -(1)$$

and $$R\cos\theta = mg \quad -(2)$$

(1) ÷ (2)

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$$

or $\tan\theta = \frac{v^2}{rg}$

or $\boxed{v = (rg\tan\theta)^{1/2}}$

which is the safe speed of the car to negotiate a banked road of radius r.

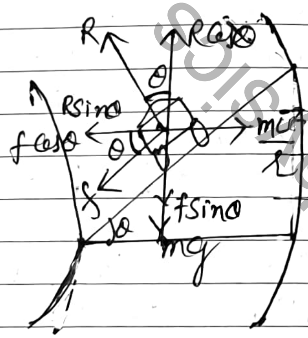
→ **Motion of a car on a banked circular road (taking friction)**

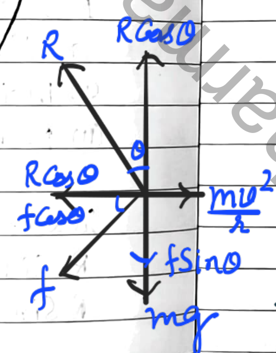
The various forces are-

(i) Weight of car $W = mg$ in downward direction

(ii) Normal reaction R

(iii) Force of friction F between road and tyres





Now, from fig -

$$\therefore R\cos\theta = mg + f\sin\theta$$

or $R\cos\theta - f\sin\theta = mg$ —(1)

and $R\sin\theta + f\cos\theta = \frac{mv^2}{r}$ —(2)

(2) ÷ (1)

$$\frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{mv^2/r}{mg}$$

or $$\frac{R\sin\theta + \mu R\cos\theta}{R\cos\theta - \mu R\sin\theta} = \frac{v^2}{rg} \quad [\because f = \mu R]$$

or $$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{rg}$$

multiply and divide by $\cos\theta$ on L.H.S

$$\frac{\frac{\sin\theta}{\cos\theta} + \frac{\mu\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} - \frac{\mu\sin\theta}{\cos\theta}} = \frac{v^2}{rg}$$

or $$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{v^2}{rg}$$

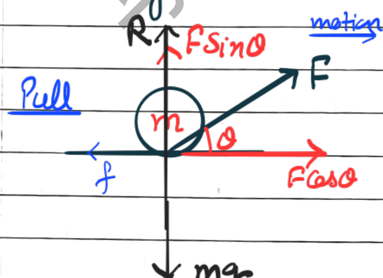
or $$\boxed{v = \left[\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}\right]^{1/2}}$$

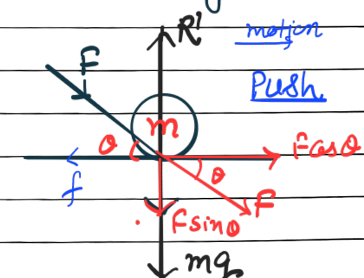
If $\mu = 0$

$$v = [rg\tan\theta]^{1/2}$$

There will be minimum wear and tear of the tyres of the car moving with this speed.

* **Pulling is easier than pushing**





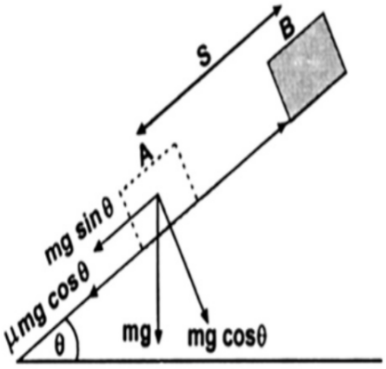
$R = R_{pull} = mg - F\sin\theta$ $\quad$ $R' = R_{push} = mg + F\sin\theta$

i.e $R' > R$ or $\mu R' > \mu R$

i.e $f' > f \Rightarrow f_{push} > f_{pull}$ So pulling is easier.

Work done when a body moving up on a rough inclined plane

Let a body of mass m be placed on a rough inclined plane and displaced from A to B by a distance S.



(i) mg sinθ is the component down the inclined plane.

(ii) mgcosθ is the component perpendicular to plane.

Since body is in equilibrium perpendicular to the plane,

$$R = mg\cos\theta$$

Since the body is displaced up the inclined plane, therefore force of friction will be down the inclined plane and is $f = \mu R = \mu\, mg\cos\theta$

∴ Work done against friction is,

$$W = \vec{f}.\vec{S} = \mu mg\cos\theta$$

→ Friction is called necessary evil

- Advantages of friction:
  1. Friction enables us to walk freely.
  2. It helps to support ladder against wall.
  3. It becomes possible to transfer one form of energy to another.
  4. Objects can be piled up without slipping.
  5. Breaks of vehicles work due to friction.

- Disadvantages of friction:
  1. It always resists the motion, so extra energy is required to overcome it.
  2. It causes wear and tear of machines.
  3. It decreases the life expectancy of moving parts of vehicles.
  4. Since friction is very useful in some cases while harmful in some cases, friction is called a necessary evil.