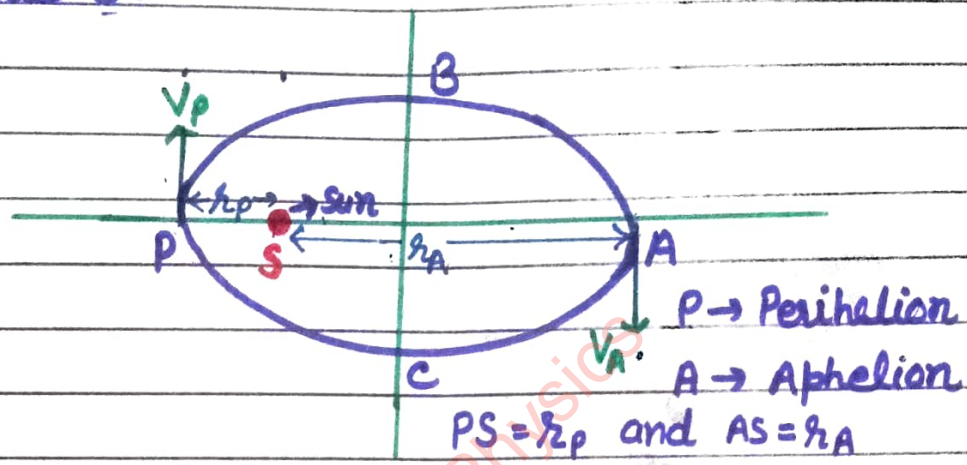
Example 8.1

Let the speed of - - - - - - - - BAC and CPB

Solution :



When there is no torque, angular momentum is constant.

i.e $L = r p \sin\theta$ = Constant , $p \rightarrow$ linear momentum

we can write

$$r_A p_A = r_P p_P \qquad [\because \theta = 90^\circ]$$

or

$$r_A m V_A = r_P m V_P \qquad [\because p = mV]$$

or $$r_A V_A = r_P V_P$$

or

$$\frac{r_A}{r_P} = \frac{V_P}{V_A}$$

Since $r_A > r_P$,

$$V_A < V_P$$

Hence planet will take longer time to traverse BAC than CPB. Which is also verified by Kepler's second law.

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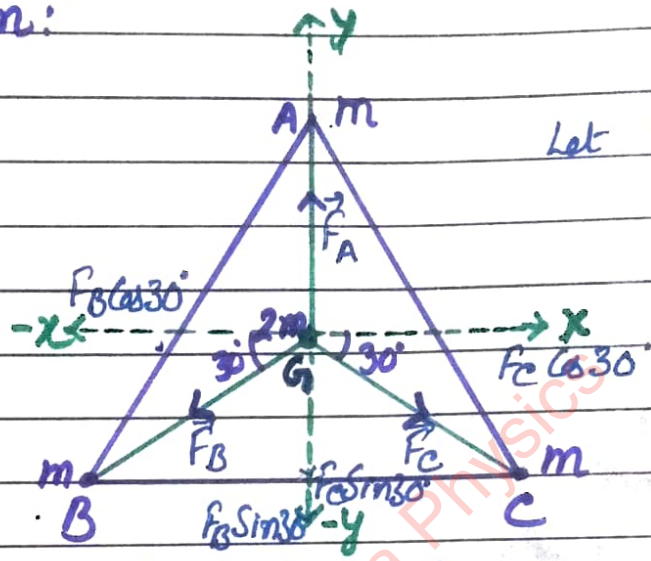
Example 8.2

Three equal masses - - - - - - take $AG = BG = CG = 1m$.

Solution:



Let $\vec{F}_{GA} = \vec{F}_A$
$\vec{F}_{GB} = \vec{F}_B$
$\vec{F}_{GC} = \vec{F}_C$

For equilateral triangle ABC, given that
$AG = BG = CG = 1m$

(a) Force at the centroid G.
Components of $\vec{F}_B$ and $\vec{F}_C$ are shown in fig.

Now.

$$\vec{F}_A = \frac{Gm(2m)}{r^2}\hat{j} = 2Gm^2\hat{j} \qquad [\because r = 1m]$$

$$\vec{F}_B = -F_B\cos 30°\hat{i} - F_B\sin 30°\hat{j}$$

$$= 2Gm^2[-\cos 30\hat{i} - \sin 30\hat{j}]$$

and $$\vec{F}_C = F_C\cos 30°\hat{i} - F_C\sin 30°\hat{j}$$

$$= 2Gm^2[\cos 30\hat{i} - \sin 30\hat{j}]$$

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$$

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$$\vec{F}_{net} = 2Gm^2\hat{j} + 2Gm^2[-\cos 30\,\hat{i} - \sin 30\,\hat{j}] + 2Gm^2[\cos 30\,\hat{i} - \sin 30\,\hat{j}]$$

$$= 2Gm^2[\hat{j} - \cancel{\cos 30\,\hat{i}} - \sin 30\,\hat{j} + \cancel{\cos 30\,\hat{i}} - \sin 30\,\hat{j}]$$

$$= 2Gm^2[\hat{j} - 2\sin 30\,\hat{j}]$$

$$= 2Gm^2[\hat{j} - 2 \times \tfrac{1}{2}\hat{j}] \qquad [\because \sin 30 = \tfrac{1}{2}]$$

$$= 2Gm^2[\hat{j} - \hat{j}]$$

$$\vec{F}_{net} = 0$$

i.e net force acting on mass 2m (at G) is zero.

(b) If ~~force~~ mass at A is doubled, then

$$F_{net} = 2Gm^2[2\hat{j} - \hat{j}]$$

$$= 2Gm^2\hat{j}$$

Ans

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NCERT PHYSICS SOLUTIONS

CHAPTER : ..... CLASS : .....

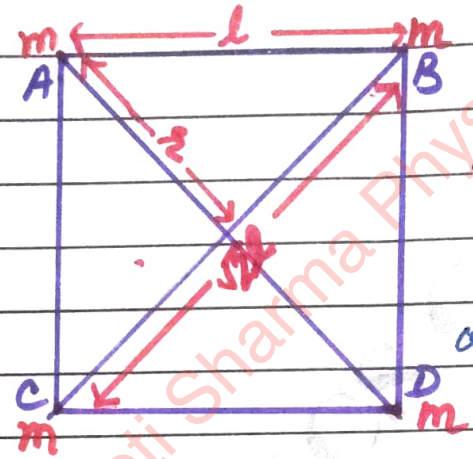
PAGE NO. : ..... DATE : .....

Example 8.3

Find the potential - - - - - - - - - of the square.

Solution:

Consider four masses each of mass m at the corners of a square of side l, as shown in fig.



$AB = BD = DC = CA = l$

$AD = BC = \sqrt{2}\,l$

$r = \frac{\sqrt{2}\,l}{2}$

or $r = \frac{l}{\sqrt{2}}$

We have four mass pair at distance l and two diagonal pair at distance $\sqrt{2}\,l$.

Hence,

Potential energy of the system -

$$U = -4\frac{Gm^2}{l} - 2\frac{Gm^2}{\sqrt{2}\,l} \qquad \left[\because U = -\frac{Gm_1m_2}{r}\right]$$

$$= -\frac{4Gm^2}{l} - \frac{\sqrt{2}\cdot\sqrt{2}\,Gm^2}{\sqrt{2}\,l}$$

$$= -\frac{Gm^2}{l}\left[4+\sqrt{2}\right] = -\frac{Gm^2}{l}\left[4+1.414\right]$$

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$$V = -\frac{Gm^2}{l} \times 5.41$$

or $$V = -5.41\frac{Gm^2}{l}$$ Ans

Now -

Potential at the centre

$$V = -4\frac{Gm}{r}$$

$$= -4\frac{Gm}{l/\sqrt{2}}$$

or $$V = -4\sqrt{2}\frac{Gm}{l}$$ Ans

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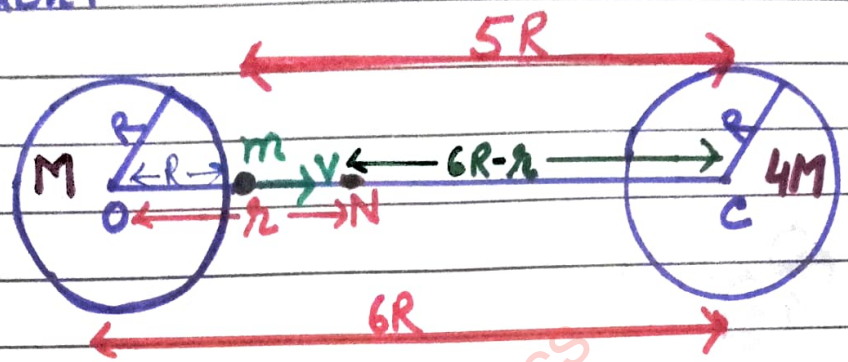
Example 8.4

Two uniform solid sphere - - - - - - second sphere.

Solution:



There are two solid spheres which apply gravitational force on the projectile.

Let N is the neutral point where the two forces cancel each other exactly. Let $ON = r$ i.e at point N

$$F_M = F_{4M}$$

or $$\frac{GMm}{r^2} = \frac{G(4M)m}{(6R-r)^2}$$

or $$(6R-r)^2 = 4r^2$$

or $$6R-r = 2r$$

or $$r = 2R$$

Now the mechanical energy at the surface of sphere of mass M

$$E_i = \frac{1}{2}mV^2 - \frac{GMm}{R} - \frac{4GMm}{5R}$$

$\left[\because \text{Gravitational P.E} = -\frac{Gm_1m_2}{r}\right]$

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and the mechanical energy at point 'N'

$$E_f = -\frac{GMm}{2R} - \frac{4GMm}{4R} \quad \left[\because \text{at } N, K.E = 0 \text{ as } v = 0\right]$$

By the conservation of mechanical energy

$$E_i = E_f$$

i.e

$$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R} = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R}\left[-\frac{1}{2} - 1 + 1 + \frac{4}{5}\right]$$

or $$\frac{1}{2}v^2 = \frac{GM}{R}\left[-\frac{1}{2} + \frac{4}{5}\right]$$

or $$v^2 = \frac{GM}{R}\left[-1 + \frac{8}{5}\right]$$

or $$v^2 = \frac{3}{5}\frac{GM}{R}$$

or $$v = \sqrt{\frac{3}{5}\frac{GM}{R}}$$

Ans

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Example 8.6

You are given - - - - - in two different ways.

Solution:

(I) We have

$$M_E = \frac{g R_E^2}{G}, \quad M_E \rightarrow \text{Mass of Earth}, \quad R_E \rightarrow \text{Radius of earth}$$

$$= \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$= \frac{9.81 \times 40.57 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$= \frac{398.06}{6.67} \times 10^{23}$$

$$= 59.679 \times 10^{23}$$

$$\simeq 5.97 \times 10^{24} \text{ kg}$$

II The moon is a satellite of the Earth. So we can get mass of Earth by the relation

$$T^2 = \frac{4\pi^2}{G} \frac{R^3}{M_E}$$

$$\text{or} \quad M_E = \frac{4\pi^2}{G} \times \frac{R^3}{T^2}$$

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or $M_E = \dfrac{4\times(3.14)^2\times(3.84)^3\times10^{24}}{6.67\times10^{-11}\times(27.3\times24\times60\times60)^2}$

$= 6.02\times10^{24}$ kg

We get almost the same result by both the methods.

Example 8.7

Express the constant K — — — — time period of revolution in days.

Solution:

Given,

$K = 10^{-13} s^2 m^{-3}$ $\left[K = \dfrac{4\pi^2}{GM_E}\right]$

Distance of moon from earth $= 3.84\times10^5$ Km

We know

1 day $= 24\times60\times60$ sec

so, 1 sec $= \dfrac{1}{24\times60\times60}$ day

and 1 Km $= 1000$ m $= 10^3$ m

so 1 m $= 10^{-3}$ Km

Now

$K = 10^{-13} s^2 m^{-3}$

$= 10^{-13}\times\left(\dfrac{1}{24\times60\times60}\right)^2\times(10^{-3})^{-3}$ day^2 Km^{-3}

$= \dfrac{1}{(24\times3600)^2}\times10^{-4}$

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or $K = \frac{1}{576 \times 1296} \times 10^{-4-4}$

$= \frac{1}{746496} \times 10^{-8}$

$= \frac{1}{0.75} \times 10^{-14}$

$K = 1.33 \times 10^{-14}$ days^2 km^{-3}

Now by using this value of k the time period of moon is

$T^2 = (1.33 \times 10^{-14})(3.84 \times 10^5)^3$ $[\because T^2 = kr^3]$

$= 1.33 \times (3.84)^3 \times 10^{-14+15}$

$= 1.33 \times 56.62 \times 10$

or $T^2 = 753.08$

or $T = 27.4$ days

Ans

Example 8.8

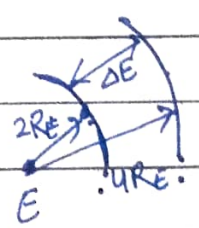
A 400 kg satellite ---------- potential energy

Solution:

Given,

Mass of satellite $m = 400$ kg

Satellite is to be transferred from $2R_E$ to $4R_E$.



Initially total energy

$E_i = \frac{-GM_E m}{2(2R_E)} = \frac{-GM_E m}{4R_E}$ $\left[\because E = \frac{-GMm}{2R}\right]$

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while final energy

$$E_f = -\frac{GM_E m}{2(4R_E)} = -\frac{GM_E m}{8R_E}$$

The change in total energy

$$\Delta E = E_f - E_i$$

$$= -\frac{GM_E m}{8R_E} - \left(-\frac{GM_E m}{4R_E}\right)$$

$$= \frac{GM_E m}{4R_E} - \frac{GM_E m}{8R_E}$$

$$= \frac{GM_E m}{8R_E}$$

$$= \left(\frac{GM_E}{R_E^2}\right) \frac{R_E \cdot m}{8}$$

$$= \frac{g R_E m}{8} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{9.81 \times 6.37 \times 10^6 \times 400}{8}$$

$$\Delta E = 3.13 \times 10^9 J$$

Change in K.E $= K_f - K_i = -\Delta E$

$$= -3.13 \times 10^9 J$$

and change in potential energy

$$\Delta V = V_f - V_i$$

$$= 2\Delta E$$

$$= 2 \times 3.13 \times 10^9 J$$

$$= 6.26 \times 10^9 J$$

Ans

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