Q1. Find an expression for the electric field strength at a distant point situated (i) on the axis and (ii) along the equatorial line of an electric dipole.

OR

Derive an expression for the electric field intensity at a point on the equatorial line of an electric dipole of dipole moment $\vec{p}$ and length 2a. What is the direction of this field?

Ans:

Consider an electric dipole AB. The charges -q and +q of dipole are situated at A and B respectively as shown in the figure. The separation between the charges is 2a.

Electric dipole moment, p = q.2a

The direction of dipole moment is from -q to +q.

Image

a. **At axial or end-on position:** Consider a point P on the axis of dipole at a distance r from mid-point O of electric dipole.

The distance of point P from charge +q at B is BP = r - a.

And distance of point P from charge -q at A is, AP = r + a.

Let E_1 and E_2 be the electric field strengths at point P due to charges +q and -q respectively.

We know that the direction of electric field due to a point charge is away from positive charge and towards the negative charge. Therefore,

$E_1 = \frac{1}{4\pi\epsilon 0}\frac{q}{(r-a)^2}$ (from B to P)

and $E_2 = \frac{1}{4\pi\epsilon 0}\frac{q}{(r+a)^2}$ (from P to A)

Clearly the directions of electric field strengths $\vec{E}_1$ and $\vec{E}_2$ along the same line but opposite to each other and E_1 > E_2 because positive charge is nearer.

$\therefore$ The resultant electric field due to electric dipole has magnitude equal to the difference of E_1 and E_2 direction from B to P i.e.,

$E = E_1 - E_2$

$= \frac{1}{4\pi\epsilon 0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon 0}\frac{q}{(r+a)^2}$

$= \frac{q}{4\pi\epsilon 0} = \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = \frac{q}{4\pi\epsilon 0}\left[\frac{(r+a)^2-(r-a)^2}{(r^2-a^2)^2}\right]$

$= \frac{q}{4\pi\epsilon 0}\frac{4ra}{(r^2-a^2)^2} = \frac{1}{4\pi\epsilon 0}\frac{2(q2a)r}{(r^2-a^2)^2}$

But q.21 = p (electric dipole moment)

$\therefore\ E = \frac{1}{4\pi\epsilon 0}\frac{2pr}{(r^2-a^2)^2}\ldots(i)$

If the dipole is infinitely small and point P is far away from the dipole, then r >> l, therefore equation (i) may be expressed as,

$E = \frac{1}{4\pi\epsilon 0}\frac{2pr}{r^4}$ or $E = \frac{1}{4\pi\epsilon 0}\frac{2p}{r^3}\ldots(ii)$

This is the expression for the electric field strength at axial position due to a short electric dipole.

ii. **At a point of equatorial line:** Consider a point P on broad side on the position of dipole formed of charges +q and -q at separation 2a. The distance of point P from mid-point (O) of electric dipole is r. Let $\vec{E}_1$ and $\vec{E}_2$ be the electric field strengths due to charges +q and -q of electric dipole.

From fig. AP = BP = $\sqrt{r^2+a^2}$

$\therefore\ \vec{E} = \frac{1}{4\pi\epsilon 0}\frac{q}{r^2+a^2}$ along B to P

$\vec{E}_2 = \frac{1}{4\pi\epsilon 0}\frac{q}{r^2+a^2}$ along P to A

Clearly $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude i.e., $|\vec{E}_1| = |\vec{E}_2|$ or $E_1 = E_2$

To find the resultant of $\vec{E}_1$ and $\vec{E}_2$, we reslove them into rectangular components.

Component of $\vec{E}_1$ Parallel to AB = $E_1 \cos\theta$, in the direction to $\vec{BA}$

Component of $\vec{E}_1$ perpendicular to AB = $E_1 \sin\theta$, along OP

Component of $\vec{E}_2$ Parallel to AB = $E_2 \cos\theta$, in the direction to $\vec{BA}$

Component of $\vec{E}_2$ perpendicular to AB = $E_2 \sin\theta$, along PO

Clearly, comoponents of $\vec{E}_1$ and $\vec{E}_2$ perpendicular to AB = $E_1 \sin\theta$ and $E_2 \sin\theta$ being equal and opposite cancel each other, while the components of $\vec{E}_1$ and $\vec{E}_2$ parallel to AB = $E_1 \cos\theta$ and $E_2 \cos\theta$, being in the same direction add up and give the resultant electric field whose direction is parallel to $\vec{BA}$.

$\therefore$ Resultant electric field at P is $E = E_1 \cos\theta + E_2 \cos\theta$

But $E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)}$

From the figure, $\cos\theta = \frac{OB}{PB} = \frac{l}{\sqrt{r^2 + a^2}} = \frac{l}{(r^2 + a^2)^{1/2}}$

$E = 2E_1 \cos\theta = 2 \times \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \cdot \frac{l}{(r^2 + a^2)^{1/2}}$

$= \frac{1}{4\pi\epsilon_0} \frac{2ql}{(r^2 + a^2)^{3/2}}$

But q.2l = p = electric dipole moment ...(iii)

$\therefore\ E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$

If dipole is infinitesimal and point P is far away, we have a << r, so l^2 may be neglected as compared to r^2 and so equation (iii) gives,

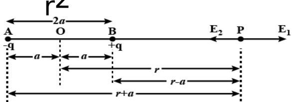
$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$

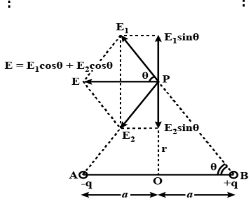
i.e., electric field strength due to a short dipole at broadside on position.

$E = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$ in the direction parallel to $\vec{BA}$ ... (iv)

Its direction is parallel to the axis of dipole from positive to negative charge. It may be noted clearly from equations (ii) and (iv) that electric field strength due to a short dipole at any point is inversely proportional to the cube of its distance from the dipole and the electric field strength at axial position is twice that at broad-side on position for the same distance.

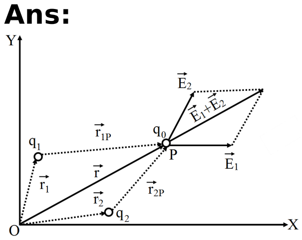
Important: Note the important point that the electric field due to a dipole at large distances falls off as $\frac{1}{r^3}$ and not as $\frac{1}{r^2}$ as in the case of a point charge.





Q2. Consider a system of n charges q_1, q_2, ... q_n with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field $\vec{E}$ at a point P with position vector $\vec{r}_p$, due to this system of charges.

Ans:



Electric field due to a system of point charges.

Consider a system of N point charges $q_1, q_2, \ldots, q_n$, having position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_n$, with respect to origin O.

We wish to determine the electric field at point P whose position vector is $\vec{r}$.

According to Coulomb's law, the force on charge q_0 due to charge q_1 is,

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_0}{r_{1p}^2} \hat{r}_{1p}$$

Where $\hat{r}_{1p}$ is a unit vector in the direction from q_1 to P and r_{1p} is the distance between q_1 and P.

Hence the electric field at point P due to charge q_1 is,

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r_{1p}^2} \hat{r}_{1p}$$

Similarly, electric field at P due to charge q_2 is,

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_2}{r_{2p}^2} \hat{r}_{2p}$$

According to the principle of superposition of electric fields, the electric field at any point due to a group of point charges is equal to the vector sum of the electric fields produced by each charge individually at that point, when all other charges are assumed to be absent.

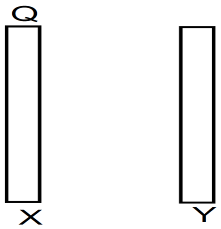
Hence, the electric field at point P due to the system of N charges is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_{1p}^2} \hat{r}_{1p} + \frac{q_2}{r_{2p}^2} \hat{r}_{2p} + \cdots + \frac{q_n}{r_{np}^2} \hat{r}_{np} \right]$$

$$= \frac{1}{4\pi\epsilon_0} \Sigma_{i=1}^{n} \frac{q_i}{r_{ip}^2} \hat{r}_{ip}$$

Q3. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure. The plate X is given a charge Q whereas the other is neutral. Find:



a. The surface charge density at the inner surface of the plate X.
b. The electric field at a point to the left of the plates.
c. The electric field at a point in between the plates.
d. The electric field at a point to the right of the plates.

Ans:
- Given that the charge present on the plate is Q. The other plate will get the same charge Q due to convection.

Let the surface charge densities on both sides of the plate be σ_1 and σ_2.

Now, electric field due to a plate,

$$E = \frac{\sigma}{2\epsilon_0}$$

So, the magnitudes of the electric fields due to this plate on each side = $\frac{\sigma_1}{2\epsilon_0}$ and $\frac{\sigma_2}{2\epsilon_0}$

The plate has two sides, each of area A. So, the net charge given to the plate will be equally distributed on both the sides.This implies that the charge developed on each side will be:

$$q_1 = q_2 = \frac{Q}{2}$$

This implies that the net surface charge density on each side = $\frac{Q}{2A}$

b. Electric field to the left of the plates

On the left side of the plate surface, charge density,

$$\sigma = \frac{Q}{2A}$$

Hence, electric field = $\frac{Q}{2A\epsilon_0}$

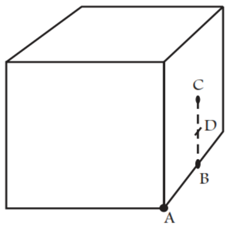
This must be directed towards the left, as 'X' is the positively-charged plate.

c. Here, the charged plate 'X' acts as the only source of electric field, with positive in the inner side. Plate Y is neutral. So, a negative charge will be induced on its inner side. 'Y' attracts the charged particle towards itself. So, the middle portion E is towards the right and is equal to $\frac{Q}{2A\epsilon_0}$.

d. Similarly for the extreme right, the outer side of plate 'Y' acts as positive and hence it repels to the right with E = $\frac{Q}{2A\epsilon_0}$.

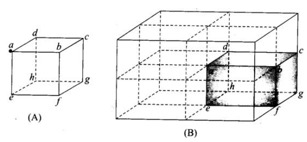
Q4. What will be the total flux through the faces of the cube (Fig.) with side of length a if a charge q is placed at:

i. A: a corner of the cube.
ii. B: mid-point of an edge of the cube.
iii. C: centre of a face of the cube.
iv. D: mid-point of B and C.

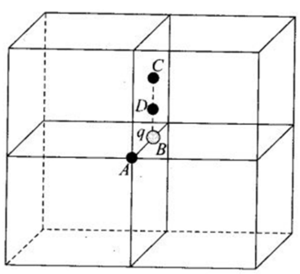


Ans:

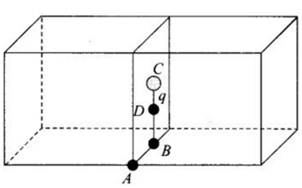
- Use of symmetry consideration may be useful in problems of flux calculation. We can imagine the charged particle is placed at the centre of a cube of side 2a. We can observe that the charge is being shared equally by 8 cubes. Therefore, total flux through the faces of the given cube = $\frac{q}{8\epsilon_0}$.



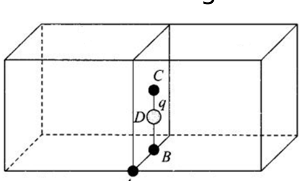
ii. If the charge q is placed at B, middle point of an edge of the cube, it is being shared equally by 4 cubes. Therefore, total flux through the faces of the given cube = $\frac{q}{4\epsilon_0}$.



iii. If the charge q is placed at C, the centre of a face of the cube, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $\frac{q}{2\epsilon_0}$.



iv. Finally, if charge q is placed at D, the mid-point of B and C, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $\frac{q}{2\epsilon_0}$.



Q5. A uniform electric field $\vec{E} = E_x\hat{i}$ N/ C for x > 0 and $\vec{E} = -E_x\hat{i}$ N/ C for x < 0 are given. A right circular cylinder of length l cm and radius r cm has its centre at the origin and its axis along the X-axis. Find out the net outward flux. Using Gauss’s law, write the expression for the net charge within the cylinder.

Ans:

Electric flux through flat surface S_1,

$\Phi_1 = \oint_{S_1} \vec{E}_1.d\vec{S}_1 = \oint_{S_1} (E_x\hat{i}).(dS_1\hat{i}) = E_xS_1$

Electric flux through flat surface S_2,

$\Phi_2 = \oint_{S_2} \vec{E}_2.d\vec{S}_2 = \oint_{S_2} (-E_x\hat{i}).(-dS_1\hat{i}) = \oint_{S_2} E_x dS_2$

$= E_xS_2$

Electric flux through curved surface S_3,

$\Phi_3 = \oint_{S_3} \left(\vec{E}_3.d\vec{S}_3\right) = \oint_{S_3} E_3dS_3 \cos 90^\circ = 0$

$\therefore$ Net electric flux, $\Phi = \Phi_1 + \Phi_2 = E_x(S_1 + S_2)$

But $S_1 = S_2 = \pi(r \times 10^{-2})^2m^2 = \pi r^2 \times 10^{-4}m^2$

$\therefore\ \Phi = E_x2(\pi r^2 \times 10^{-4})$ units

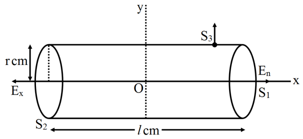
By Gauss's law, $\Phi = \frac{1}{\epsilon_0}q$

$q = \epsilon_0\Phi = \epsilon_0E_x(2\pi r^2 \times 10^{-4})$

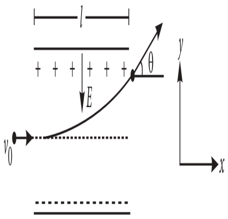
$= 2\pi\epsilon_0E_xr^2 \times 10^{-4} = 4\pi\epsilon_0\left(\frac{E_xr^2 \times 10^{-4}}{2}\right)$

$= \frac{1}{9 \times 10^9}\left[\frac{E_xr^2 \times 10^{-4}}{2}\right]$

$= 5.56E_xr^2 \times 10^{-11}$ coulomb.



Q6. When a charged particle is placed in an electric field, it experiences an electrical force. If this is the only force on the particle, it must be the net force. The net force will cause the particle to accelerate according to Newton's second law. So $\vec{F}_e = q\vec{E} = m\vec{a}$



If $\vec{E}$ is uniform, then $\vec{a}$ is constant and $\vec{a} = q\vec{E}/m$. If the particle has a positive charge, its acceleration is in the direction of the field. If the particle has a negative charge, its acceleration is in the direction opposite to the electric field. Since the acceleration is constant, the kinematic equations can be used.

i. An electron of mass m, charge e falls through a distance h metre in a uniform electric field E. Then time of fall,
   a. $t = \sqrt{\frac{2hm}{eE}}$
   b. $t = \frac{2hm}{eE}$
   c. $t = \sqrt{\frac{2eE}{hm}}$
   d. $t = \frac{2eE}{hm}$
ii. An electron moving with a constant velocity v along X-axis enters a uniform electric field applied along Y-axis. Then the electron moves:
   a. With uniform acceleration along Y-axis
   b. Without any acceleration along Y-axis
   c. In a trajectory represented as $y = ax^2$

d. In a trajectory represented as y = ax

iii. Two equal and opposite charges of masses m_1 and m_2 are accelerated in an uniform electric field through the same distance. What is the ratio of their accelerations if their ratio of masses is $\frac{m_1}{m_2}$ = 0.5?

a. $\frac{a_1}{a_2} = 2$

b. $\frac{a_1}{a_2} = 0.5$

c. $\frac{a_1}{a_2} = 3$

d. $\frac{a_1}{a_2} = 1$

iv. A particle of mass m carrying charge q is kept at rest in a uniform electric field E and then released. The kinetic energy gained by the particle, when it moves through a distance y is:

a. $\frac{1}{2}qEy^2$

b. qEy

c. qEy^2

d. qE^2y

v. A charged particle is free to move in an electric field. It will travel:

a. Always along a line of force.

b. Along a line of force, if its initial velocity is zero.

c. Along a line of force, if it has some initial velocity in the direction of an acute angle with the line of force.

d. None of these.

Ans:

a. (a) $t = \sqrt{\frac{2hm}{eE}}$

Explanation:

From Newton's law,

$F = m\vec{a}$

Or $qE = m\vec{a}$

$\Rightarrow a = \frac{qE}{m} = \frac{eE}{m}$

Using, $s = ut + \frac{1}{2}at^2$

$\therefore\ h = 0 + \frac{1}{2} \times \frac{eE}{m}t^2$

$\Rightarrow t = \sqrt{\frac{2hm}{eE}}$

ii. (c) In a trajectory represented as $y = ax^2$

iii. (b) $\frac{a_1}{a_2} = 0.5$

Explanation:

Force is same in magnitude for both.

$\therefore m_1a_1 = m_2a_2$;

$\frac{a_1}{a_2} = \frac{m_2}{m_1} = \frac{1}{0.5} = 2$

iv. (b) qEy

Explanation:

Here, u = 0;

$a = \frac{qE}{m};\ s = y$

Using, $v^2 - u^2 = 2as$

$\Rightarrow v^2 = 2\frac{qE}{m}y$

$\therefore\ K.E. = \frac{1}{2}my^2 = qEy$

v. (b) Along a line of force, if its initial velocity is zero.

Explanation:

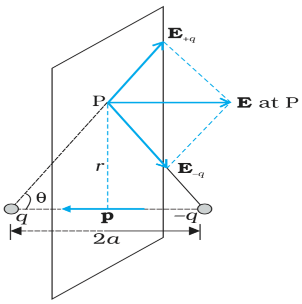
If charge particle is put at rest in electric field, then it will move along line of force.

Q7. a. Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

b. Two identical point charges, q each, are kept 2m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find thea position and nature of Q.

Ans:

a. Electric Field for points on the Equatorial Plane



The magnitudes of the electric field due to the two charges +q and -q are given by,

$E_{+q} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2 + a^2}$ ...(i)

$E_{-q} = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2 + a^2}$ ...(ii)

The directions of E_{+q} and E_{-q} are as shown in the figure. The components normal to the dipole axis cancel away. The components along the dipole axis add up.

$\therefore$ Total electric field, $E = -(E_{+q} + E_{-q})\cos\theta\ \hat{p}$ [Negative sign shows that the field is opposite to $\hat{p}$]

$E = -\frac{2qa}{4\pi\epsilon(r^2 + a^2)^{\frac{3}{2}}}\hat{p}$ ...(iii)

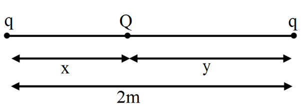
At large distances (r>>a), this reduces to,

$E = -\frac{2qa}{4\pi\epsilon_0 r^3}\hat{p}$ ...(iv)

$\because \vec{p} = q \times \overrightarrow{2a}\hat{p}$

$\therefore E = \frac{-\vec{p}}{4\pi\epsilon_0 r^3}(r >> a)$

b.



$\frac{K(q)(Q)}{x} = \frac{-K(q)(q)}{2}$

$\Rightarrow Q = \frac{-qx}{2}$

$\because \frac{KqQ}{x} = \frac{KQq}{y}$

$x = y$

$x + y = 2$

$\therefore x = y = 1$

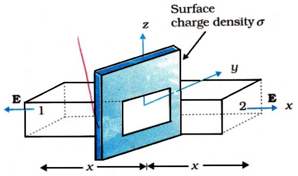
$Q = \frac{-q}{2}$

Q8. i. Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

ii. An infinitely large thin plane sheet has a uniform surface charge density +σ . Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet.

Ans:

a.



$\oint E.ds = \frac{q}{\varepsilon 0}$

The electric field E points outwards normal to the sheet. The field lines are parallel to the Gaussian surface except for surfaces 1 and 2. Hence the net flux = $\oint E.ds$ = EA + EA where A is the area of each of the surface 1 and 2.

$\therefore \quad \oint E.ds = \frac{q}{\varepsilon 0} = \frac{\sigma A}{\varepsilon 0} = 2EA;$

$E = \frac{\sigma}{2\varepsilon 0}$

ii

$W = q\int\limits_{\infty}^{r} \vec{E}.d\vec{r}$

$= q\int_{\infty}^{r}(-Edr)$

$= -q\int_{\infty}^{r}\left(\frac{\sigma}{2 \in 0}\right)dr$

$= \frac{q\sigma}{2 \in}|\infty - r|$

$\implies \quad (\infty)$

Q9. Answer the following Questions.

a. Find expressions for the force and torque on an electric dipole kept in a uniform electric field.

OR

An electric dipole is held in a uniform electric field. (i) Using suitable diagram show that it does not undergo any translatory motion, and (ii) derive an expression for torque acting on it and specify its direction.

b. Derive an expression for the work done in rotating a dipole from the angle $\theta 0$ to $\theta 1$ in a uniform electric field E.

OR

1. Define torque acting on a dipole of dipole moment $\vec{p}$ placed in a uniform electric field $\vec{E}$. Express it in the vector form and point out the direction along which it acts.
2. What happens if the field is non-uniform?
3. What would happen if the external field $\vec{E}$ is increasing (i) parallel to $\vec{p}$ and (ii) anti-parallel to $\vec{p}$?

Ans:

Let an electric dipole be rotated in electric field from angle $\theta 0$ to $\theta 1$ in the direction of electric field. In this process the angle of orientation is changing continuously; hence the torque also changes continuously. Let at any time, the angle between dipole moment p and electric field E be then,

Torque on dipole τ = pE $\sin\theta$

The work done in rotating the dipole a further by small angle dθ is,

dW = Torque × angular displacement = pE $\sin\theta$ dθ

Total work done in rotating the dipole from angle $\theta 0$ to $\theta 1$ is given by

W = $\int\limits_{\theta 0}^{\theta 1}$ pE $\sin\theta$ dθ = pE$[-\cos\theta]_{\theta 0}^{\theta 1}$

$= -$pE$[\cos\theta 1 - \theta 0]$ = pE$(\cos\theta 0 - \cos\theta 1)\ldots$(i)

Special case: If electric dipole is initially in a stable equilibrium position ($\theta 0 = 0^\circ$) and rotated through an angle $\theta(\theta 1 = \theta)$ then work done

W = pE$[\cos 0^\circ - \cos\theta]$ = PE$(1 - \cos\theta)\ldots$(ii)

Q10. Two point charges of $+5 \times 10^{-19}$C and $+20 \times 10^{-19}$C are separated by a distance of 2m. Find the point on the line joining them at which electric field intensity is zero.

Ans:

Let charges $q_1 = +5 \times 10^{-19}$C and $q_2 = +20 \times 10^{-19}$C be placed at A and B respectively. Distance AB = 2m.

As charges are similar, the electric field strength will be zero between the charges on the line joining them. Let P be the point (at a distance x from q_1) at which electric field intensity is zero. Then, AP = x metre, BP = (2 - x) metre.

The electric field strength at P due to charge q_1 is,

$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2}$, along the direction A to P.

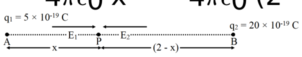
The electric field strength at P due to charge q_2 is,

$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(2-x)^2}$, along the direction B to P.

Clearly, $\vec{E}_1$ and $\vec{E}_2$ and are opposite in direction and for net electric field at P to be zero, $\vec{E}_1$ and $\vec{E}_2$ and must be equal in magnitude.

So, $E_1 = E_2$

$\Rightarrow \frac{1}{4\pi\epsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(2-x)^2}$



Given, $q_1 = 5 \times 10^{-19}$C, $q_2 = 20 \times 10^{-19}$C

Therefore, $\frac{5 \times 10^{-19}}{x^2} = \frac{20 \times 10^{-19}}{(2-x)^2}$

Or $\frac{1}{2} = \frac{x}{2-x}$

Or $x = \frac{2}{3}$m

Q11. A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a:

a. Find the surface charge density on the inner surface and on the outer surface.
b. If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces?
c. Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

Ans: Given:

Amount of charge present at the centre of the hollow sphere = Q

We know that charge given to a hollow sphere will move to its surface.

Due to induction, the charge induced at the inner surface = -Q

Thus, the charge induced on the outer surface = +Q

a. Surface charge density is the charge per unit area, i.e.

   $\sigma = \frac{\text{Charge}}{\text{Total surface area}}$

   Surface charge density of the inner surface, $\sigma_{in} = \frac{-Q}{4\pi a^2}$

   Surface charge density of the outer surface, $\sigma_{out} = \frac{Q}{4\pi a^2}$

b. Now if another charge q is added to the outer surface, all the charge on the metal surface will move to the outer surface. Thus, it will not affect the charge induced on the inner surface. Hence the inner surface charge density,

   $\sigma_{in} = -\frac{q}{4\pi a^2}$

   As the charge has been added to the outer surface, the total charge on the outer surface will become (Q + q).

   So the outer surface charge density, $\sigma_{out} = \frac{q+Q}{4\pi a^2}$

c. To find the electric field inside the sphere at a distance x from the centre in both the situations,let us assume an imaginary sphere inside the hollow sphere at a distance x from the centre.

   Applying Gauss's Law on the surface of this imaginary sphere,we get:

$\oint E.ds = \frac{Q}{\epsilon 0}$

$E \oint ds = \frac{Q}{\epsilon 0}$

$E(4\pi x^2) = \frac{Q}{\epsilon 0}$

$E = \frac{Q}{\epsilon 0} \times \frac{1}{4\pi x^2} = \frac{Q}{4\pi\epsilon 0 x^2}$

Here, Q is the charge enclosed by the sphere.

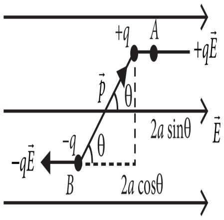
For situation (b):

As the point is inside the sphere, there is no effect of the charge q given to the shell.

Thus, the electric field at the distance x:

$E = \frac{Q}{4\pi\epsilon 0 x^2}$

Q12. When electric dipole is placed in uniform electric field, its two charges experience equal and opposite forces, which cancel each other and hence net force on electric dipole in uniform electric field is zero. However, these forces are not collinear, so they give rise to some torque on the dipole. Since net force on electric dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field. However, some work is done in rotating the dipole against the torque acting on it.



i. The dipole moment of a dipole in a uniform external field $\vec{E}$ is $\vec{P}$. Then the torque 'i acting on the dipole is:
   a. $\vec{\tau} = \vec{P} \times \vec{E}$
   b. $\vec{\tau} = \vec{P} \cdot \vec{E}$
   c. $\vec{\tau} = 2(\vec{P} + \vec{E})$
   d. $\vec{\tau} = (\vec{P} + \vec{E})$

ii. An electric dipole consists of two opposite charges, each of magnitude $1.0\mu C$ separated by a distance of 2.0cm. The dipole is placed in an external field of $10^5 N\ C^{-1}$. The maximum torque on the dipole is:
   a. 0.2×10^{-3}Nm
   b. 1×10^{-3}Nm
   c. 2×10^{-3}Nm
   d. 4×10^{-3}Nm

iii. Torque on a dipole in uniform electric field is minimum when θ is equal to:
   a. 0º
   b. 90º
   c. 180º
   d. Both (a) and (c)

iv. When an electric dipole is held at an angle in a uniform electric field, the net force F and torque t on the dipole are:
   a. $F = 0, \tau = 0$
   b. $F \neq 0, \tau \neq 0$
   c. $F = 0, \tau \neq 0$
   d. $F \neq 0, \tau = 0$

v. An electric dipole of moment pis placed in an electric field of intensity E. The dipole acquires a position such that the axis of the dipole makes an angle θ with the direction of the field. Assuming that the potential energy of the dipole to be zero when $\theta = 90^\circ$ the torque and the potential energy of the dipole will respectively be:

a. $pE\sin\theta, -pE\cos\theta$
b. $pE\sin\theta, -2pE\cos\theta$
c. $pE\sin\theta, 2pE\cos\theta$
d. $pE\cos\theta, -2pE\sin\theta$

Ans:

a. (a) $\vec{\tau} = \vec{P} \times \vec{E}$

Explanation:

As τ = either force × perpendicular distance between the two forces.
$= qaE\sin\theta$ or $\tau = PE\sin\theta$
$(\because qa = P)$
Or $\vec{\tau} = \vec{P} \times \vec{E}$

ii. (c) 2×10^{-3}Nm

Explanation:

The maximum torque on the dipole in an external electric field is given by
$\tau = pE = q(2a) \times E$
Here, $q = 1\mu C = 10^{-6}C$,
$2a = 2cm = 2 \times 10^{-2}m$,
$E = 10^5 N\, C^{-1}$,
τ =?
$\therefore \tau = 10^{-6} \times 2 \times 10^{-2} \times 10^5$
$= 2 \times 10^{-3}$Nm

iii. (d) Both (a) and (c)

Explanation:

When θ is 0 or 180º, the τ minimum, which means the dipole moment should be parallel to the direction of the uniform electric field.

iv. (c) $F = 0, \tau \neq 0$

Explanation:

Net force is zero and torque acts on the dipole, trying to align p with E.

v. (a) $pE\sin\theta, -pE\cos\theta$

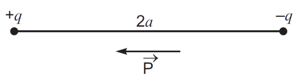
Explanation:

Torque, $\tau = pE\sin\theta$ and potential energy, $U = -pE\cos\theta$.

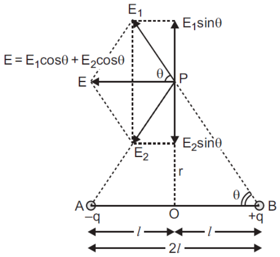
Q13. a. Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.
b. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

Ans:

a.



It is defined as the product of either charge and the distance between the two equal and opposite charges
$|P| = q.|2a|$
It is a vector quantity, so $\vec{p} = q.\overrightarrow{2a}$
Derivation At a point of equatorial plane : Consider a point P on broad side on the position of dipole formed of charges + q and - q at separation 2l. The distance of point P from mid point (O) of electric dipole is r. Let E_1 and E_2 be the electric field strengths due to charges + q and - q of electric dipole.



From fig. AP = BP = $\sqrt{r^2 + l^2}$

$\therefore \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + l^2}$ along B to P

$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + l^2}$ along P to A

Clearly $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude i.e. $|\vec{E}_1| = |\vec{E}_2|$or $E_1 = E_2$

To find the resultant of $\vec{E}_1$ and $\vec{E}_2$, we resolve them along and perpendicular to AB.

Component of $\vec{E}_1$along AB = $E_1 \cos\theta$, parallel to $B\vec{A}$

Component of $\vec{E}_1$perpendicular to AB = $E_1 \sin\theta$ along O to P

Component of $\vec{E}_2$ along AB =$E_2 \cos$, θ parallel to $B\vec{A}$

Component of $\vec{E}_2$ perpendicular to AB = $E_2 \sin\theta$along P to O

Clearly components of $\vec{E}_1$ and $\vec{E}_2$perpendicular to AB : $E_1 \sin\theta$and $E_2 \sin\theta$being equal and opposite cancel each other, while the components of $\vec{E}_1$and $\vec{E}_2$ along AB : $E_1 \cos\theta$and $E_2 \cos\theta$, being in the same direction add up and give the resultant electric field whose direction is parallel to $\overrightarrow{BA}$

$\therefore$ Resultant electric field at P is $E = E_1 \cos\theta + E_2 \cos\theta$

But $E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}$

and $\cos\theta = \frac{OB}{PB} = \frac{l}{\sqrt{r^2 + l^2}} = \frac{l}{(r^2 + l^2)^{1/2}}$

$\therefore E = 2E_1\cos\theta = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)}.\frac{l}{(r^2 + l^2)^{1/2}}$

$= \frac{1}{4\pi\varepsilon_0}\frac{2ql}{(r^2 + l^2)^{3/2}}$

But q.2l = p =electric dipolemoment

$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{P}{(r^2 + l^2)^{3/2}} ------(iii)$

$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

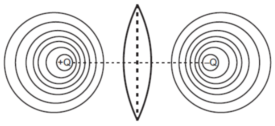
If dipole is infinitesimal and point P is far away, we have l < < r, so l^2 may be neglected as compared to r^2 and so equation (3) gives

$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$

i.e. electric field strength due to a short dipole at broadside on position

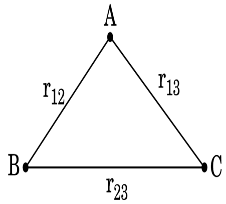
$E = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}$ parallel to $\overrightarrow{BA}$

b.



Electric potential is zero at all points in the plane passing through the dipole equator.

Q14. a. Define electrostatic potential at a point. Write its S.I. unit. Three point charges q_1, q_2 and q_3 are kept respectively at points A, B and C as shown in the figure. Derive the expression for the electrostatic potential energy of the system.



b. Depict the equipotential surfaces due to.

i. An electric dipole,

ii. Two identical positive charges separated by a distance.

Ans: • Electrostatic potential: Work done by an external force in bringing a unit positive charge from infinity to the given point SI unit- volt or J/C) Net work done in moving charges q_1. q_2 & q_3 from infinity to A, B and C respectively. $W = 0 + q_2 V_{13} + q_3 (V_{13} V_{23})$

$$\frac{1}{4\pi\varepsilon\circ}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon\circ}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

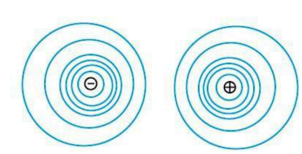
But potential energy of the system is equal to the work done.

$$\therefore u = w = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{23}} + \frac{q_2 q_3}{r_{23}}\right)$$

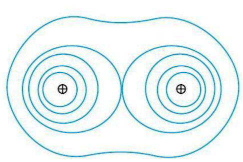
b. Equipotential surface due to

i. An electric dipole.

(i) An electric dipole



(ii) Two identical positive changes



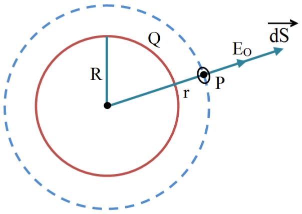
ii. Two identical positive changes.

Q15. a. Use Gauss's law to show that due to a uniformly charged spherical shell of radius R, the electric field at any point situated outside the shell at a distance r from its centre is equal to the electric field at the same point, when the entire charge on the shell were concentrated at its centre. Also plot the graph showing the variation of electric field with r, for r ≤ R and r ≥ R.

b. Two point charges of $+1\mu C$ and $+4\mu C$ are kept 30cm apart. How far from the $+1\mu C$ charge on the line joining the two charges, will the net electric field be zero?

Ans:

a.



Consider a spherical Gaussian surface of radius r ≤ R, concentric with given shell. If $\vec{E}$ is electric field outside the shell, then by symmetry, electric field strength has same magnitude $\vec{E_0}$ on the Gaussian surface and is directed radially outward. Also the directions of normal at each point is radially outward, so angle between $\vec{E_0}$ and $\vec{ds}$ is zero at each point. Hence, electric flux through Gaussian.

$= \phi_S \vec{E_0} \cdot \vec{ds}$

$\phi_S E_0 ds \cos 0°$

Surface = $E_0 4\pi r^2$

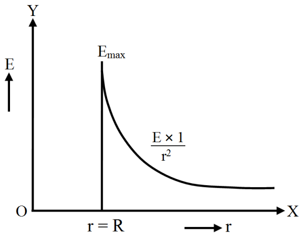
Now, Gaussian surface is outside the given charged shell, so charge enclosed by the Gaussian surface is Q.

Hence, by Gauss's theorem,

$\phi_S \vec{E_0} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \times$ charge enclosed

$\Rightarrow E_0 \cdot = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r^2}$

Thus, electric field outside a charged thin spherical shell is same as if the whole charge Q is concentrated at the centre. Graphically,



For r < R, there is no strength of electric field inside a charged spherical shell.

For r > R, electric field outside a charged thin spherical shell is same as if the whole charge Q is concentrated at the centre.

b. Given that,

First point charge = $+1\mu C$

Second point charge = $+4\mu C$

Distance = 30cm

Let us consider the net electric field zero at x distance from first charge.

We need to calculate the electric field.

Using formula of electric field,

$E_x = E_{30-x}$

$\frac{kq}{r^2} = \frac{kq}{r^2}$

Put the value into the formula,

$\frac{1 \times 10^{-6}}{x^2} = \frac{4 \times 10^{-6}}{(30-x)^2}$

$\frac{(30-x)^2}{x^2} = \frac{4 \times 10^{-6}}{(30-x)^2}$

$\frac{(30-x)^2}{x^2} = \frac{4 \times 10^{-6}}{1 \times 10^{-6}}$

$\frac{30-x}{x} = \sqrt{\frac{4}{1}}$

$30 - x = 2x$

x = 10cm

Hence, the net electric field will be zero at distance 10cm from $+1\mu C$.

Q16. Check that the ratio $ke^2/G\ m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Ans:

The given ratio is $\frac{ke^2}{Gm_e m_p}$.

Where,

G = Gravitational constant

Its unit is Nm^2kg^{-2}

m_e and m_p = Masses of electron and proton.

Their unit is kg.

e = Electric charge.

Its unit is C.

$\in_0$ = Permittivity of free space

Its unit is $Nm^2\ C^{-2}$

Therefore unit of the given ratio $\frac{ke^2}{Gm_e m_p}$

$$= \frac{\left[Nm^2C^{+2}\right]\left[C^{+2}\right]}{\left[Nm^2kg^{-2}\right]\left[kg\right]\left[kg\right]}$$

$= M^0L^0T^0$

Hence, the given ratio is dimensionless.

$e = 1.6 \times 10^{-19}$ C

$G = 6.67 \times 10^{-11}\ Nkg^{-2}$

$me = 9.1 \times 10^{-31}$ kg

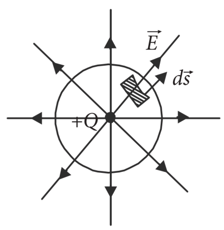
$mp = 1.66 \times 10^{-27}$ kg

Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_e m_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-3} \times 1.67 \times 10^{-22}} \approx 2.3 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

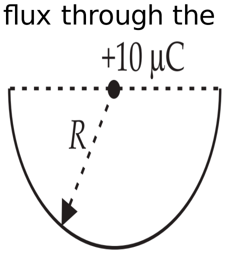
Q17. Gauss's law and Coulomb's law, although expressed in different forms, are equivalent ways of describing the relation between charge and electric field in static conditions. Gauss's law is $\epsilon 0\phi = q_{end}$, when q_{encl} is the net charge inside an imaginary closed surface called Gaussian surface. $\phi = \oint \vec{E} \cdot d\vec{A}$ gives the electric flux through the Gaussian surface. The two equations hold only when the net charge is in vacuum or air.



i. If there is only one type of charge in the universe, then ($\vec{E} \rightarrow$ Electric field, $d\vec{s} \rightarrow$ Area vector).
   a. $\oint \vec{E} \cdot d\vec{s} = 0$ on any surface.
   b. $\oint \vec{E} \cdot d\vec{s}$ could not be defined.
   c. $\oint \vec{E} \cdot d\vec{s} = \infty$ if charge is inside.
   d. $\oint \vec{E} \cdot d\vec{s} = 0$ if charge is outside, $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon 0}$ if charge is inside.

ii. What is the nature of Gaussian surface involved in Gauss law of electrostatic?
   a. Magnetic.
   b. Scalar.
   c. Vector.
   d. Electrical.

iii. A charge 10μC is placed at the centre of a hemisphere of radius R = 10cm as shown. The electric flux through the hemisphere (in MKS units) is:



   a. 20×10^5
   b. 10×10^5
   c. 6×10^5

d. 2×10^5

iv. The electric flux through a closed surface area S enclosing charge Q is ϕ. If the surface area is doubled, then the flux is:

a. 2ϕ

b. $\frac{\phi}{2}$

c. $\frac{\phi}{4}$

d. ϕ

v. A Gaussian surface encloses a dipole. The electric flux through this surface is:

a. $\frac{q}{\epsilon_0}$

b. $\frac{2q}{\epsilon_0}$

c. $\frac{q}{2\epsilon_0}$

d. Zero

Ans:

a. (d) $\oint \vec{E} \cdot d\vec{s} = 0$ if charge is outside, $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$ if charge is inside.

Explanation:

If there is only one type of charge in the universe, then it will produce electric field somehow. Hence, Gauss's law is valid.

ii. (c) Vector.

iii. (c) 6×10^5

Explanation:

According to Gauss's theorem,

Electric flux through the sphere = $\frac{q}{\epsilon_0}$

$\therefore$ Electric flux through the hemisphere = $\frac{1}{2}\frac{q}{\epsilon_0}$

$= \frac{10 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}} = 0.56 \times 10^6$ Nm^2 C^{-1}

= 0.6×10^6 Nm^2 C^{-1} = 6×10^5 Nm^2 C^{-1}

iv. (d) ϕ

Explanation:

As flux is the total number of tines passing through the surface, for a given charge, it is always the charge enclosed $\frac{Q}{\epsilon_0}$. If area is doubled, the flux remains the same.

v. (d) Zero

Explanation:

As net charge on a dipole is (- q + q) = 0

Thus, when agaussian surface encloses a dipole, as per Gauss's theorem, electric flux through the surface,

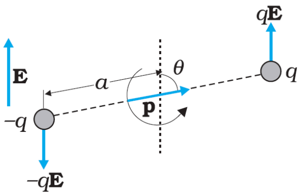
$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} = 0$

Q18. i. An electric dipole of dipole moment $\vec{p}$ is placed in a uniform electric field $\vec{E}$ at an angle θ with it. Derive the expression for torque ($\vec{\tau}$) acting on it. Find the orientation of the dipole relative to the electric field for which torque on it is (1) maximum, and (2) half of maximum.

ii. Two point charges $q_1 = +1\mu C$ and $q_2 = +4\mu C$ are placed 2m apart in air. At what distance from q_1 along the line joining the two charges, will the net electric field be zero?

Ans:

a.



Dipole in a uniform electric field.

From digaram

Magnitude of Torque = (qE)(2a sin θ)

= (2qa)(E sin θ)

= pE sin θ

For direction $\vec{\tau} = \vec{p} \times \vec{E}$

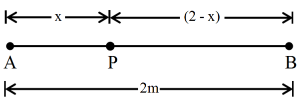
1. For maximum Torque, dipole should be placed perpendicular to the direction of electric field.

   $\theta = 90^\circ = \frac{\pi}{2}$

2. For the torque to be half the maximum,

   $\theta = 30^\circ = \frac{\pi}{6}$

ii.



$E_{PA} = E_{PB}$; $E = \frac{kq}{r^2}$

$\frac{kq_A}{x^2} = \frac{kq_B}{(2-x)^2}$

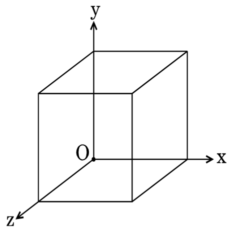
$\frac{1}{x^2} = \frac{4}{(2-x)^2}$

$\frac{1}{x} = \frac{2}{2-x}$

$x = \frac{2}{3}$m

Q19. a. Two point charges q_1 and q_2 are kept r distance apart in a uniform external electric field $\vec{E}$. Find the amount of work done in assembling this system of charges.

b. A cube of side 20cm is kept in a region as shown in the figure. An electric field $\vec{E}$. exists in the region such that the potential at a Point is given by V = 10x + 5, where V is in volt and x is in m.



Find the:

i. Electric field $\vec{E}$.

ii. Total electric flux through the cube.

Ans:

a. Let the charge q_1 travels r_1 distance and q_1 travels.

The work done in bringing the charge q_1 in the field to

$W_1 = F_1 \times r_1$

$= q_1E \times r_1$

The work done in bringing the second charge,

$W_2 = F_2 \times r_2$

$= q_2E \times r_1$

And the work is also done to overcome the force of the charge on one-another.

$W_3 = \frac{1}{4\pi \in_0} \cdot \frac{q_1 q_2}{r^2}$

So, total work = $q_1 Er_1 + q_2 Er_2 + \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$

b.

i. Given that,

$V = 10x + 5$

We know

$E = -\frac{dv}{dx}$

$V = 10x + 5$

$\frac{dV}{dx} = \frac{d}{dx}(10x + 5)$

$= 10\frac{d}{dx}x + 0$

$= 10$

$E = -\frac{10N}{C}$

$\vec{E} = -\frac{10\hat{i}N}{C}$

ii. Since electric field is constant in negative x-direction.

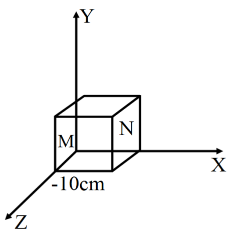
So, flux enter in the cube will be same as flux come out through the cube,

$\phi in = \phi out$

So Net flux from the cube = 0.

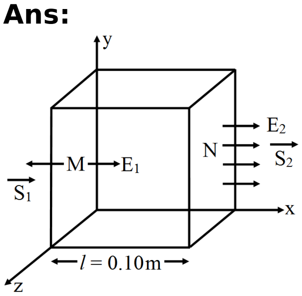
Q20. Electric field in the given figure is directed along +X direction and given by Ex = $5A_x + 2B$, where E is in NC^{-1} and x is in metre, A and B are constants with dimensions. Taking A = $10NC^{-1}$ m^{-1} and B = $5NC^{-1}$, calculate:

i. The electric flux through the cube.



ii. Net charge enclosed within the cube.

Ans:



Given $E_x = 5A_x + 2B$

The electric field at face M where x = 0 is,

$E_1 = 2B$

The electric field at face N where x = 10cm = 0.10m is,

$E_2 = 5A \times 0.10 + 2B = 0.5A + 2B$

The electric flux through face M is,

$\Phi_1 = \vec{E}_1.\vec{S}_1 = E_1 S_1 \cos\pi = -E_1 S_1$

= $-2B \times l^2$ where l = 10cm = 0.01m

The electric flux through face N,

$\Phi_2 = \vec{E}_2.\vec{S}_2 = E_2 S_2 \cos 0 = (0.5A + 2B)l^2$

Net electric flux, $\Phi = \Phi_1 + \Phi_2$

$= -2Bl^2 + (0.5A + 2B)l^2 = 0.5Al^2$

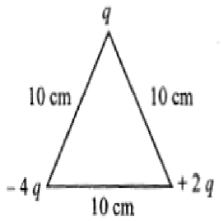
$= 0.5 \times 10 \times (0.10)^2 = 5 \times 10^{-2}$ V m

Q21. a. Using Gauss' law, derive an expression for the electric field intensity at any point outside a uniformly charged thin spherical shell of radius *R* and charge density σ C/m^2. Draw the field lines when the charge density of the sphere is (i) positive, (ii) negative.

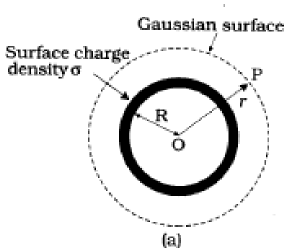
b. A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 μC/m^2. Calculate the

i. Charge on the sphere.

ii. Total electric flux passing through the sphere.



Ans: •

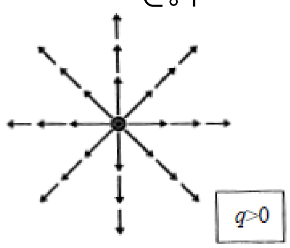


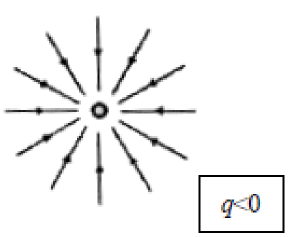
$\oint \vec{E} . \vec{ds} = \frac{q}{\epsilon_\circ}$

$\oint$ Eds $\cos\theta = \frac{\sigma 4\pi R^2}{\epsilon_0}$

$E 4\pi r^2 = \frac{\sigma 4\pi R^2}{\epsilon_\circ}$

$\therefore E = \frac{\sigma R^2}{\epsilon_\circ r^2}$





b. .

i. $q = \sigma 4\pi R^2$

$= 100 \times 10^{-6} \times 4 \times 3.14 \times \left(\frac{2.5}{2}\right)^2$

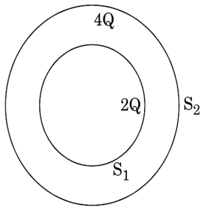
$= 19.6 \times 10^{-4}$ C

ii. flux, $\phi = \frac{q}{\epsilon_\circ}$

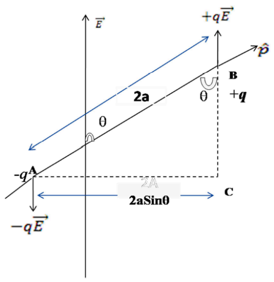
$$= \frac{19.6 \times 10^{-4}}{8.85 \times 10^{-12}}$$
$$= 2.2 \times 10^{-4} Nm^2C^{-1}.$$

Q22. a. Deduce the expression for the torque acting on a dipole of dipole moment $\vec{P}$ in the presence of a uniform electric field $\vec{E}$.

b. Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in the figure. (i) Find out the ratio of the electric flux through them. (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



Ans: • The forces, acting on the two charges of the dipole, are $+q\vec{E}$ and $-q\vec{E}$



The net force on the dipole is zero. The two forces are, however, equivalent to a torque having a magnitude.

$\tau =$ (qE)AC

$= qE. 2a \sin\theta$

$= pE \sin\theta$

The direction of this torque is that of the cross product $(\vec{p} \times \vec{E})$. Hence, the torque acting on the dipole, is given by

$\vec{\tau} = \vec{p} \times \vec{E})$.

b. As per Guass's Theorem

Electric Flux $= \oint_s \vec{E}.\vec{dS} = \frac{q\ enclosed}{\in_o}$

$\therefore$For sphere S_1, flux enclosed $= \phi 1 = \frac{2Q}{\in_o}$

For sphere S_2, flux enclosed $= \phi 2 = \frac{2Q + 4Q}{\in_o} = \frac{6Q}{\in_o}$

$\therefore \frac{\phi 1}{\phi 2} = \frac{1}{3}$

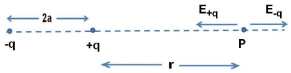
When a medium of dielectric consistent $\in_r$ is introduced in sphere S1 the flux through S_1would be $\phi_1' = \frac{2Q}{\in_r}$.

Q23. i. Derive an expression for the electric field E due to a dipole of length '2a' at a point distant r from the centre of the dipole on the axial line.

ii. Draw a graph of E versus r for r >> a.

iii. If this dipole were kept in a uniform external electric field E0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases.

Ans: •



Electric field at P due to charge $(+q)$ = $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$

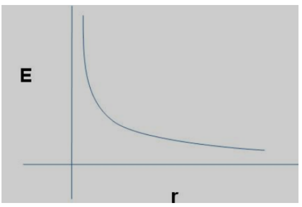
Electric field at P due to charge $(-q)$ = $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$

Net electric Field at P = $E_1 - E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$

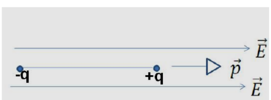
$= \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2-a^2)^2}$ $\quad (p = q.2a)$

Its direction is parallel to $\vec{p}$

ii.



iii.

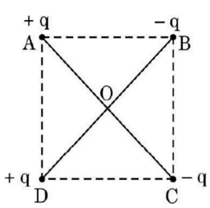


Stable equilibrium

Unstable equilibrium

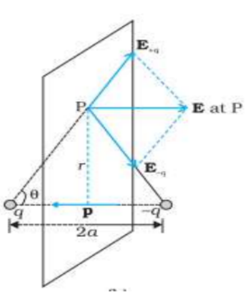
Torque = 0 for (i) as well as case (ii).

Q24. Derive an expression for the electric field at a point on the equatorial plane of an electric dipole consisting of charges q and - q separated by a distance 2a. The distance of a far off point on the equatorial plane of an electric dipole is halved. How will the electric field be affected for the dipole? Two identical electric dipoles are placed along the diagonals of a square ABCD of side $\sqrt{2}$ m as shown in the figure. Obtain the magnitude and direction of the net electric field at the centre (O) of the square.



Ans:

1. Derivation of electric field
2. Effect on electric field
3. Finding magnitude and direction of electric field



E+q = $\frac{q}{4\pi \in_o} \times \frac{1}{r^2 + a^2}$

E-q = $\frac{q}{4\pi \in_o} \times \frac{1}{r^2 + a^2}$

The components normal to dipole axis cancel away. The components along the dipole axis add up. Total electric field is opposite to dipole moment.

$\vec{E} = -(E_{+q} + E_{-q}) \cos\theta \hat{p}$

$= \frac{-2qa}{4\pi \in_o (r^2 + a^2)^{\frac{3}{2}}} \hat{p}$

$= \frac{-\vec{p}}{4\pi \in_0 (r^2 + a^2)^{\frac{3}{2}}}$

Deduct ½ mark if the expression of electric field is not in vector form.

At far off point r >> a

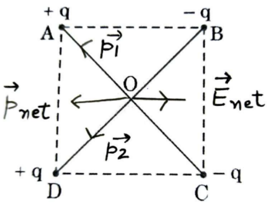
$\vec{E} = \frac{-\vec{p}}{4\pi \in_0 r^3}$

When distance is halved.

$\vec{E} = \frac{-\vec{p}}{4\pi \in_0 (\frac{r}{2})^3}$

$= \frac{-8\vec{p}}{4\pi \in_0 r^3}$

$\vec{E}$ becomes 8 times



p_1 q × 2Cm (along OA)

p_2 q × 2Cm (along OD)

$p_{net} = \sqrt{p_1^2 + p_2^2}$

$= 2\sqrt{2}$q cm

Electric field at centre O

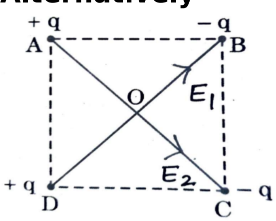
$E = \frac{kp_{net}}{(r^2 + a^2)^{\frac{3}{2}}}$

at point O, r = 0, a = 1m

$E = \frac{k \times 2\sqrt{2}q}{1^3} = 2\sqrt{2}kq = \frac{2\sqrt{2}q}{4\pi \in_o}$

Along DC

Alternatively



$E = \frac{kq}{r^2}$

AC = BD = 2m

r = OA = OB = OC = OD = 1m

Electric field at O due to charges at B and D

$E_1 = E_B + E_D$

$E_1 = \frac{kq}{1^2} + \frac{kq}{1^2}$ along OB

= 2kq

Electric field at O due to charges at A and C

$E_2 = E_A + E_C$

$E_2 = \frac{kq}{1^2} + \frac{kq}{1^2}$ along OB

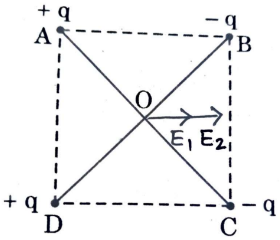
$= 2kq$

$E_{net} = \sqrt{E_1^2 + E_2^2}$

$= 2\sqrt{2}kq = \frac{2\sqrt{2}q}{4\pi \in_o}$

Along DC

Alternatively



Considering AB as dipole, electric field at O

$$E_1 = \frac{2kq \times a}{\left(\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right)^{\frac{3}{2}}} = \frac{2kqa}{\left(\frac{1}{2} + \frac{1}{2}\right)^{\frac{3}{2}}} = 2kqa$$

Similarly considering DC as another dipole, electric field at O

$$E_2 = \frac{2kq \times a}{\left(\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2\right)^{\frac{3}{2}}} = \frac{2kqa}{\left(\frac{1}{2} + \frac{1}{2}\right)^{\frac{3}{2}}} = 2kqa$$

$E_{net} = E_1 + E_2 = 4kqa = \frac{1}{4\pi \in_o} \times 4 \times \frac{1}{\sqrt{2}} \times q$

$= 2\sqrt{2}kq = \frac{2\sqrt{2}q}{4\pi \in_o}$

Along DC

Q25. A long cylindrical wire carries a positive charge of linear density $2.0 \times 10^{-8} Cm^{-1}$. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

Ans: Let the linear charge density of the wire be λ.

The electric field due to a charge distributed on a wire at a perpendicular distance r from the wire,

$E = \frac{\lambda}{2\pi\epsilon_0 r}$

The electrostatic force on the electron will provide the electron the necessary centripetal force required by it to move in a circular orbit. Thus,

$qE = \frac{mv^2}{r}$

$\Rightarrow mv^2 = qEr \ \ldots (1)$

Kinetic energy of the electron, $K = \frac{1}{2}mv^2$

From (1),

$K = \frac{qEr}{2}$

$K = \frac{qr}{2}\frac{\lambda}{2\pi\epsilon_0 r} \left[\because E = \frac{\lambda}{2\pi\epsilon_0 r}\right]$

$K = \left(1.6 \times 10^{-19}\right) \times \left(2 \times 10^{-8}\right) \times \left(9 \times 10^{9}\right) J$

$K = 2.88 \times 10^{-17} J$

Q26. Four equal charges $2.0 \times 10^{-6} C$ each are fixed at the four corners of a square of side 5cm. Find the Coulomb force experienced by one of the charges due to the rest three.

Ans: $q^1 = q^2 = q^3 = q^4 = 2 \times 10^{-6}C$

$v = 5cm = 5 \times 10^{-2}m$

So force on $\bar{C} = \bar{F}_{CA} + \bar{F}_{CB} + \bar{F}_{CD}$

So Force along × Component $= \bar{F}_{CD} + \bar{F}_{CA} \cos 45^\circ + 0$

$= \frac{k(2 \times 10^{-6})^2}{(5 \times 10^{-2})^2} + \frac{k(2 \times 10^{-6})^2}{(5 \times 10^{-2})^2} \frac{1}{2\sqrt{2}}$

$= kq^2\left(\frac{1}{25 \times 10^{-4}} + \frac{1}{50\sqrt{2} \times 10^{-4}}\right)$

$\frac{9 \times 10^9 \times 4 \times 10^{12}}{24 \times 10^{-4}}\left(1 + \frac{1}{2\sqrt{2}}\right)$

$= 1.44(1.35) = 19.49$ Force along % component = 19.49

So, Resultant $R = \sqrt{Fx^2 + Fy^2}$

$= 19.49\sqrt{2}$

$= 27.56$

Q27. Answer the following questions:

i. Define electric flux. Write its SI unit.

ii. Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

iii. How is the field directed if:

- The sheet is positively charged.
- Negatively charged?

Ans:

a. **Electric flux:** It is defined as the total number of electric field lines passing through an area normal to its surface.

$\Phi = \oint \vec{E}.d\vec{S}$

The SI unit is Nm^2/ C or volt-metre.

ii. Image

Let electric charge be uniformly distributed over the surface of a thin, non-conducting infinite sheet. Let the surface charge density (i.e., charge per unit surface area) be σ. We need to calculate the electric field strength at any point distant r from the sheet of charge.

To calculate the electric field strength near the sheet, we now consider a cylindrical Gaussian surface bounded by two plane faces A and B lying on the opposite sides and parallel to the charged sheet and the cylindrical surface perpendicular to the sheet (fig). By symmetry the electric field strength at every point on the flat surface is the same and its direction is normal outwards at the points on the two plane surfaces and parallel to the curved surface.

Total electric flux,

or $\oint_S \vec{E}.d\vec{S} = \oint_{S_1} \vec{E}.d\vec{S}_1 + \oint_{S_2} \vec{E}.d\vec{S}_2 + \oint_{S_3} \vec{E}.d\vec{S}_3$

$\oint_S \vec{E}.d\vec{S} = \oint_{S_1} E\, dS_1 \cos 0^\circ + \oint_{S_2} E\, dS_2 \cos 0^\circ + \oint_{S_3} E\, dS_3 \cos 90^\circ$

$= E\oint dS_1 + E\oint dS_2 = Ea + Ea = 2Ea$

$\therefore$ Total electric flux = 2Ea

As σ is charge per unit area of sheet and a is the intersecting area, the charge enclosed by Gaussian surface = σa

Total electric flux $= \frac{1}{\epsilon_0} \times$ (total charge enclosed by the surface)

i.e., $2Ea = \frac{1}{\epsilon_0}(\sigma a)$

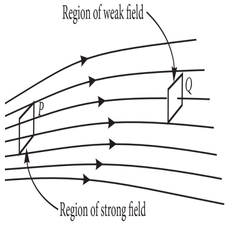
$\therefore\ E = \frac{p}{2\epsilon_0}$.

Thus electric field strength due to an infinite flat sheet of charge is independent of the distance of the point.

iii.

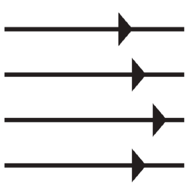
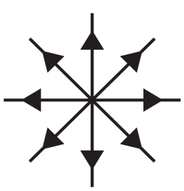
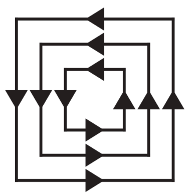
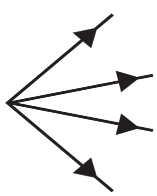
- If σ is positive, $\vec{E}$ points normally outwards/ away from the sheet.
- If σ is negative, $\vec{E}$ points normally inwards/ towards the sheet.

Q28. Electric field strength is proportional to the density of lines of force i.e., electric field strength at a point is proportional to the number of lines of force cutting a unit area element placed normal to the field at that point. As illustrated in the given figure, the electric field at P is stronger that at Q.



i. Electric lines of force about a positive point charge are:
   a. Radially outwards.
   b. Circular clockwise.
   c. Radially inwards.
   d. Parallel straight lines.

ii. Which of the following is false for electric lines of force?
   a. They always start from positive charges and terminate on negative charges.
   b. They are always perpendicular to the surface of a charged conductor.
   c. They always form closed loops.
   d. They are parallel and equally spaced in a region of uniform electric field.

iii. Which one of the following pattern of electric line of force in not possible in filed due to stationary charges?
   a.
   
   b.
   
   c.
   
   d.
   

iv. Electric lines of force are curved:

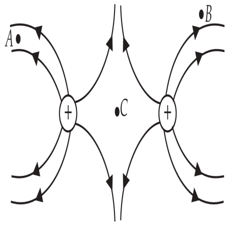
a. In the field of a single positive or negative charge.

b. In the field of two equal and opposite charges.

c. In the field of two like charges.

d. Both (b) and (c).

v. The figure below shows the electric field lines due to two positive charges. The magnitudes E_A, E_B and E_C of the electric fields at points A, Band C respectively are related as:



a. $E_A > E_B > E_C$

b. $E_B > E_A > E_C$

c. $E_A = E_B > E_C$

d. $E_A > E_B = E_C$

Ans:

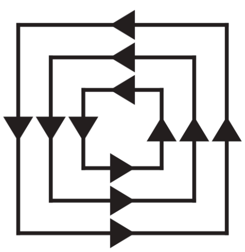
a. (a) Radially outwards.

b. (c) They always form closed loops.

Explanation:

Electric lines of force do not form any closed loops.

iii. (c)



Explanation:

Electric field tines can't be closed.

iv. (d) Both (b) and (c).

v. (a) $E_A > E_B > E_C$

Q29. A pendulum bob of mass 80mg and carrying a charge of 2×10^{-8}C is at rest in a uniform, horizontal electric field of $20kVm^{-1}$. Find the tension in the thread.

Ans: $E = 20Kv/m = 20 \times 10^3 v/m,$

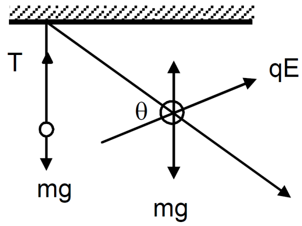
$m = 80 \times 10^{-5} kg,\ c = 20 \times 10^{-5} C$

$\tan\theta = \left(\frac{qE}{mg}\right)^{-1} \left[T\sin\theta = mg,\ T\cos\theta = qe\right]$

$\tan\theta = \left(\frac{2 \times 10^{-8} \times 20 \times 10^3}{80 \times 10^{-6} \times 10}\right)$

$= \left(\frac{1}{2}\right)^{-1}$

$1 + \tan^2\theta = \frac{1}{4} + 1 = \frac{5}{4}\ \left[\cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}\right]$



$T \sin\theta = mg$

$\Rightarrow T \times \frac{2}{\sqrt{5}} = 80 \times 10^{-6} \times 10$

$\Rightarrow T = \frac{8 \times 10^{-4} \times \sqrt{5}}{2} = 4 \times \sqrt{5} \times 10^{-4}$

$= 8.9 \times 10^{-4}$